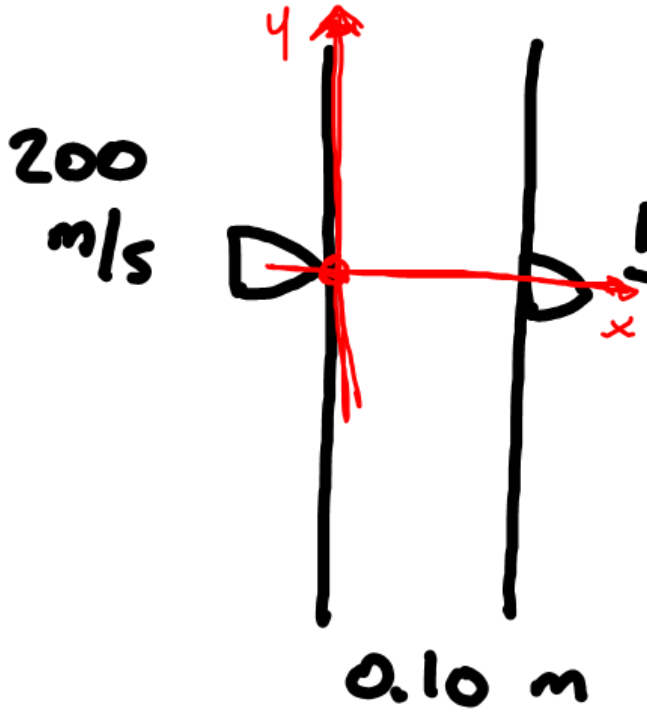
Aplicaciones a la Ecuación No Lineal



$$\frac{dV}{dt} = -kV^2$$

$t = ?$

$$\frac{dV}{dt} + kV^2 = 0$$

$$V(0) = 200 \ \frac{m}{s}$$

$$V(t_f) = 50 \ \frac{m}{s}$$

$$x(t_f) = 0.10 \ m$$

Método de Variables separables

$$\frac{dV}{V^2} + k\,dt = 0$$

$$\int \frac{dV}{V^2} + k\int dt = C_1$$

$$\frac{V^{-1}}{-1} + kt = C_1$$

$$V^{-1} - kt = -C_1$$

$$V^{-1} = -C_1 + kt$$

$$V = \frac{1}{-C_1 + kt}$$

$V(0) = 200$

$$\frac{1}{-C_1 + k(0)} = 200$$

$$-\frac{1}{200} = C_1$$

$$V_p = \frac{1}{\left(\frac{1}{200}\right) + kt}$$

$$\frac{dx(t)}{dt} = \frac{1}{\left(\frac{1}{200}\right) + kt}$$

$$dx(t) = \frac{dt}{\left(\frac{1}{200}\right) + kt}$$

Método Variables Separables.

$$\int dx = \int \frac{dt}{\left(\frac{1}{200}\right) + kt} + C_2.$$

$u = \frac{1}{200} + kt \qquad du = k\,dt$

$$\int dx = \frac{1}{k}\int \frac{k\,dt}{\left(\frac{1}{200}\right) + kt} + C_2$$

$$x = \frac{1}{k} L\left(\frac{1}{200} + kt\right) + C_2 \qquad \boxed{\begin{matrix} t=0 \\ x=0 \end{matrix}}$$

$$kx = L\left(\frac{1}{200} + kt\right) + kC_2 \qquad 0 = \frac{1}{k} L\left(\frac{1}{200}\right) + C_2$$

$$L\left(\frac{1}{200} + kt\right) = k(x - C_2) \qquad C_2 = \frac{-1}{k} L\left(\frac{1}{200}\right)$$

$$\frac{1}{200} + kt = e^{-kC_2} e^{kx} \qquad e^{-kC_2} = e^{L\left(\frac{1}{200}\right)}$$

$x(t_f) = 0.10$

$V(t_f) = 50$

$$kt = C_{20} e^{kx} - \frac{1}{200}$$

$$t = \frac{C_{20} e^{kx} - \frac{1}{200}}{k}$$

$$t = \frac{e^{L\left(\frac{1}{200}\right)} e^{kx} - \frac{1}{200}}{k}$$

$$t = \frac{\frac{1}{200} e^{kx} - \frac{1}{200}}{k}$$

$$V = \frac{1}{\frac{1}{200} + kt} \qquad V = \frac{1}{\frac{1}{200} + \left(\frac{1}{200} e^{kx} - \frac{1}{200}\right)}$$

$$V = \frac{200}{e^{kx}}$$

$$50 = \frac{200}{e^{k/10}}$$

$$\frac{50}{200} = \frac{1}{e^{k/10}}$$

$$\frac{200}{50} = e^{k/10}$$

$$k/10 = L\left(\frac{200}{50}\right)$$

$$k = 10\, L\left(\frac{200}{50}\right) \Rightarrow 13.9$$

$$t_p = \frac{\frac{1}{200} e^{\frac{13.9}{10}} - \frac{1}{200}}{13.9} \Rightarrow 1{,}08 \times 10^{-3}\,(s.)$$

$0{,}00108$ s

$V(0) = 200$

$V(t_{f_2}) = 20$