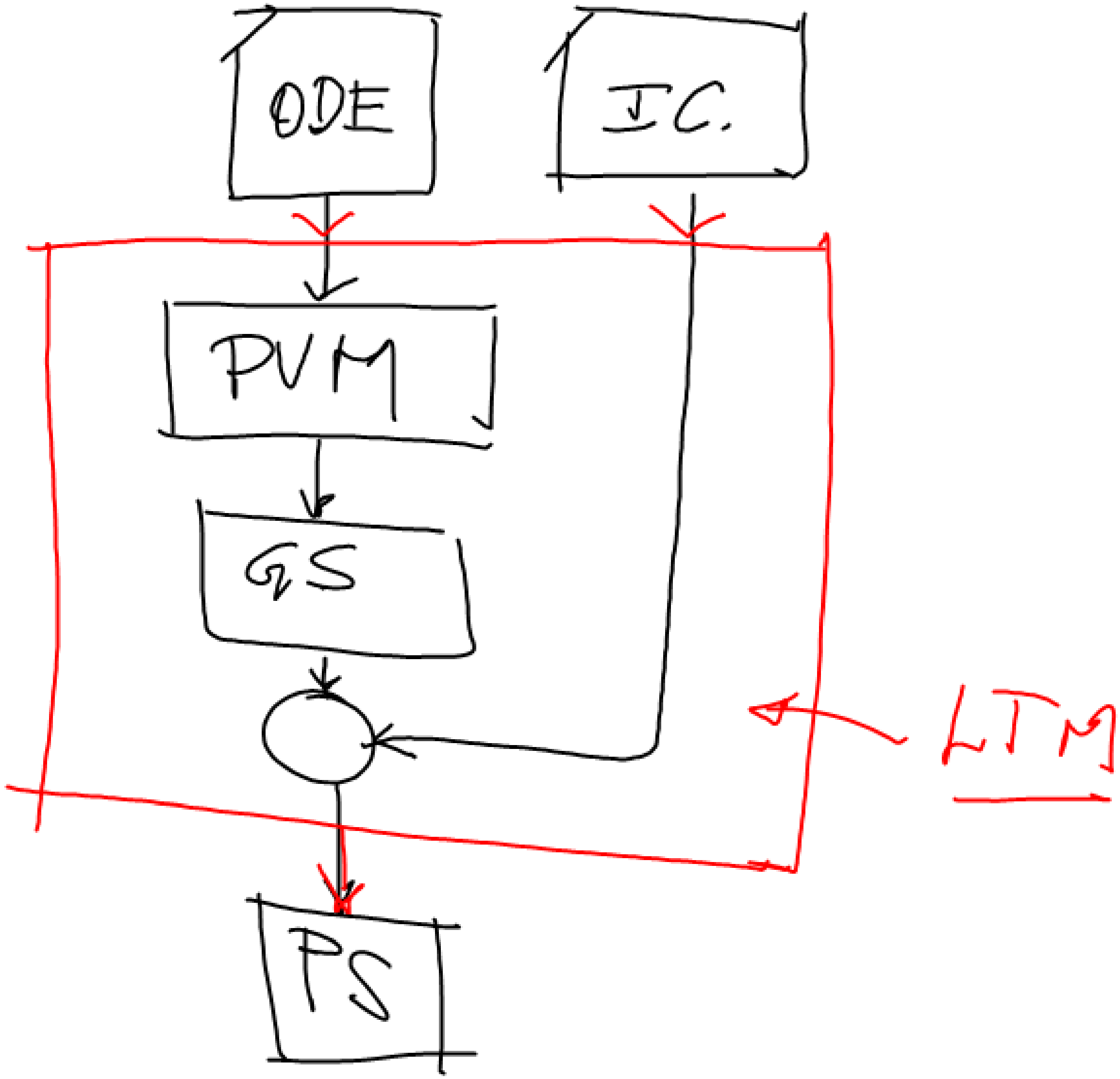
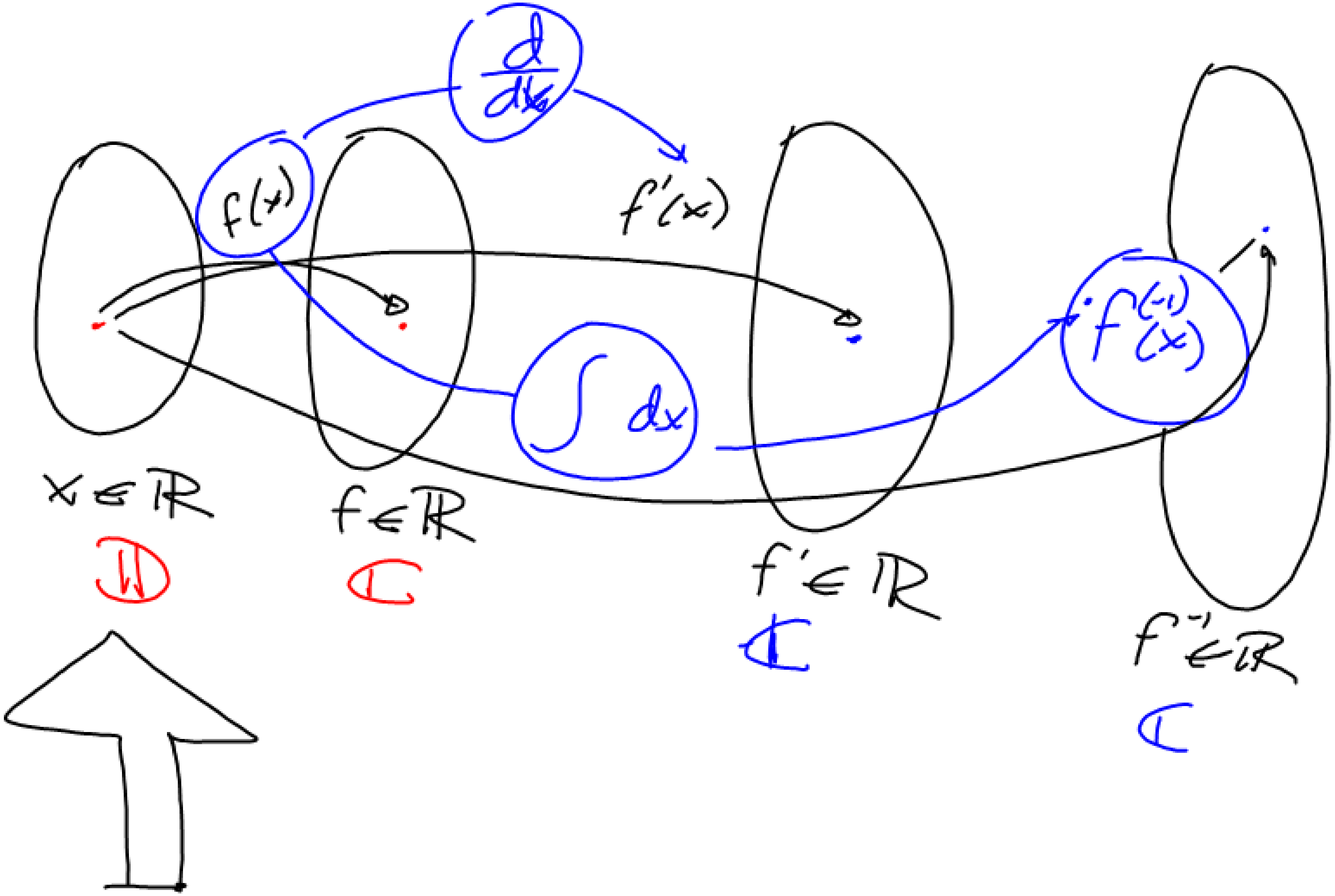
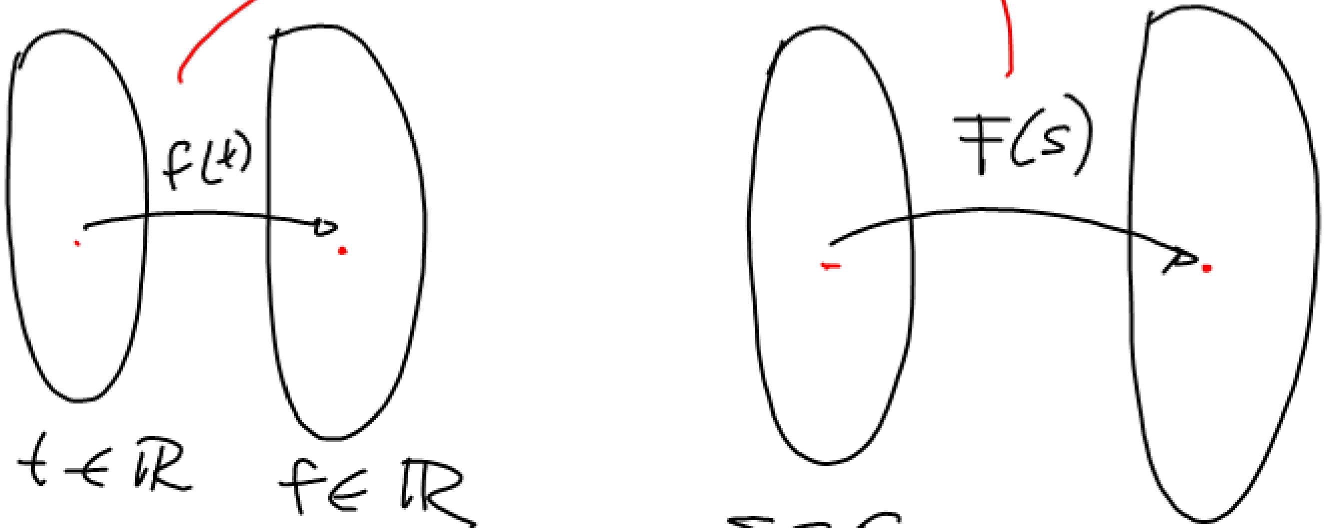
Chapter 4.- Laplace Transform as a method of solution for initial condition problems.

The result is particular solution.


ODE
I.C.
PVM
GS
PS
LTM


$\frac{d}{dx}$
$f(x)$
$f'(x)$
$\int dx$
$f^{(-1)}(x)$
$x \in \mathbb{R}$
$f \in \mathbb{R}$
$f' \in \mathbb{R}$
$f^{-1} \in \mathbb{R}$

$$T\{f(t)\} \Rightarrow F(s)$$



$t \in \mathbb{R}$ $\quad f \in \mathbb{R}$ $\quad s \in \mathbb{C}$ $\quad F \in \mathbb{R}$

$$a, b \in \mathbb{R} \quad af(t) + bg(t) \longrightarrow aF(s) + bG(s)$$

$$f'(t) \longrightarrow sF(s) - f(0)$$

$$\int f(t)\,dt \longleftarrow \frac{F(s)}{s}$$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$

$50 \longrightarrow 100\ p.$

Transform Definition

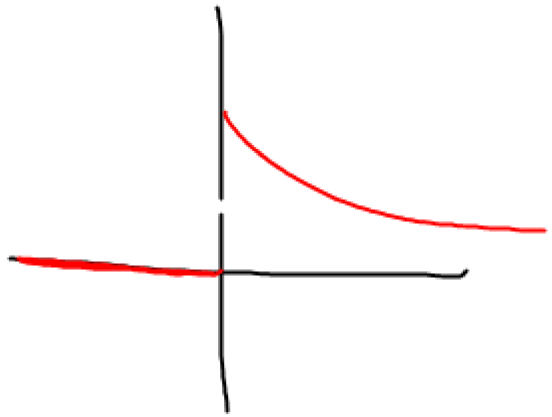
$$T\{f(t)\} = \int_{-\infty}^{\infty} N(t,s)\, f(t)\, dt \Rightarrow F(s)$$

$t \in \mathbb{R}$ $s \in \mathbb{C}$

$f \in \mathbb{R}$ $F \in \mathbb{R}$

Laplace Transform

$$N(t,s) = \begin{cases} 0 \ ; & t < 0 \\ e^{-st} \ ; & t \geq 0 \end{cases}$$



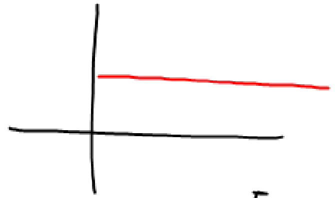
$$\mathcal{L}\left\{ f(t) \right\} = F(s)$$

gothical "L"

$$\boxed{\mathcal{L}\left\{ f(t) \right\} = \int_0^{\infty} e^{-st} f(t)\, dt}$$

LAPLACE TRANSFORM.

$f(t) = 1$



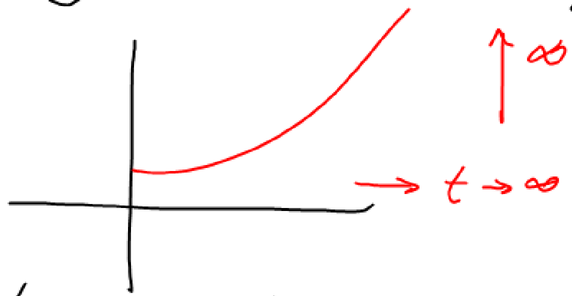
$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}(1)\,dt \Rightarrow \left[\int e^{-st}\,dt\right]_0^\infty$$

$$= \left[-\frac{e^{-st}}{s}\right]_0^\infty \Rightarrow -\frac{1}{s}\left(\cancelto{0}{\lim_{t\to\infty} e^{-st}} - 1\right)$$

$$\lim_{t\to\infty} e^{-st} = \lim_{t\to\infty} \frac{1}{e^{st}} \Rightarrow \lim_{b\to\infty} \frac{1}{b} \Rightarrow 0$$

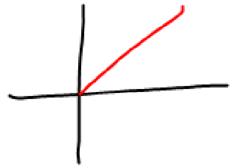
zero

$$\lim_{t\to\infty} e^{st} \longrightarrow \infty$$



$$\mathcal{L}\{1\} = -\frac{1}{s}(-1) \Rightarrow \frac{1}{s}$$

$f(t) = t$



$$\mathcal{L}\{t\} = \int_0^{\infty} e^{-st}\, t\, dt$$

$$\int e^{-st} t\, dt = -\frac{te^{-st}}{s} - \int -\frac{e^{-st}}{s}\, dt$$

$$\boxed{\begin{array}{ll} u = t & du = dt \\ dv = e^{-st} dt & v = -\dfrac{e^{-st}}{s} \end{array}}$$

$$= -\frac{te^{-st}}{s} + \frac{1}{s}\int e^{-st} dt$$

$$= -\frac{te^{-st}}{s} + \frac{1}{s}\left(-\frac{e^{-st}}{s}\right)$$

$$= -\frac{te^{-st}}{s} - \frac{1}{s^2} e^{-st}$$

$$\mathcal{L}\{t\} = \left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= -\frac{1}{s}\left(\cancelto{0}{\lim_{t\to\infty} te^{-st}} - \cancelto{0}{(0)(1)}\right) - \frac{1}{s^2}\left(\cancelto{0}{\lim_{t\to\infty} e^{-st}} - (1)\right)$$

$$\lim_{t\to\infty} te^{-st} = \lim_{t\to\infty} t \cdot \lim_{t\to\infty} \frac{1}{e^{st}} = 0$$

$$\mathcal{L}\{t\} = -\frac{1}{s^2}(-1) \Rightarrow \frac{1}{s^2}$$

$$\mathcal{L}\{e^{bt}\} = \int_0^\infty e^{-st} e^{bt}\,dt$$

$$= \left[\int e^{-(s-b)t}\,dt\right]_0^\infty$$

$$= \left[-\frac{e^{-(s-b)t}}{(s-b)}\right]_0^\infty$$

$$= \frac{-1}{(s-b)}\left[\lim_{t\to\infty} e^{-(s-b)t} - (1)\right]$$

$$\mathcal{L}\{e^{bt}\} = -\frac{1}{s-b}(-1) \Rightarrow \frac{1}{s-b}$$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-a}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\mathrm{Sen}(bt)$	$\frac{b}{s^2+b^2}$

$a \in \mathbb{R}$

$b \in \mathbb{R}$

$0! = 1$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{e^{-4t}\} = \frac{1}{s+4}$$