```
> restart
```

PROBLEMA DE LA CUERDA DE GUITARRA

```
> Ecuacion := diff(y(x,t),t$2) = c··2·diff(y(x,t),x$2)
```

$$Ecuacion := \frac{\partial^2}{\partial t^2} y(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t) \right) \quad (1)$$

```
> c := 1
```

$$c := 1 \quad (2)$$

```
> Ecuacion;
```

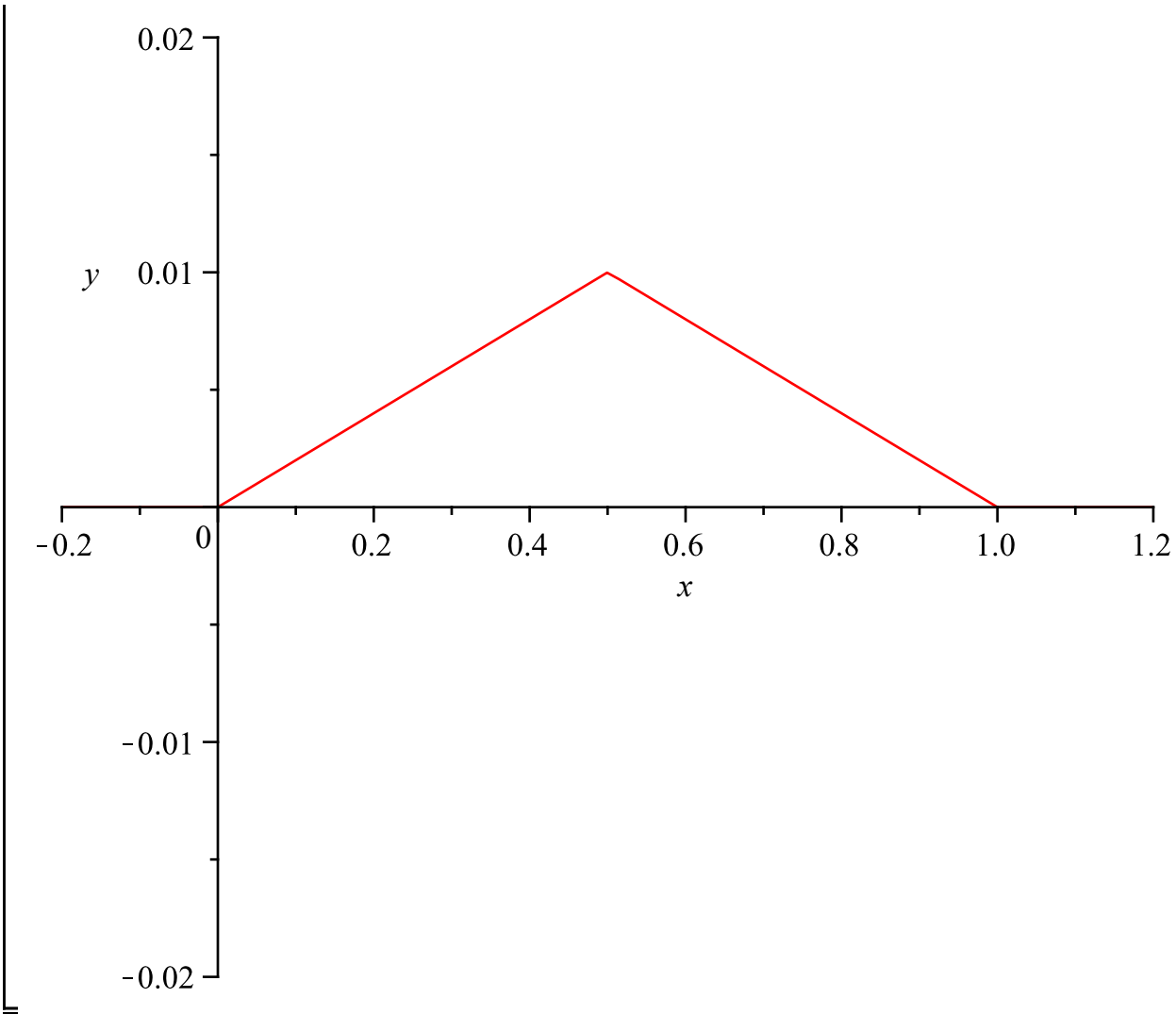
$$\frac{\partial^2}{\partial t^2} y(x,t) = \frac{\partial^2}{\partial x^2} y(x,t) \quad (3)$$

```
> CondicionesFrontera := y(0,t) = 0, y(1,t) = 0
```

$$CondicionesFrontera := y(0,t) = 0, y(1,t) = 0 \quad (4)$$

```
> CondicionInicialTrayectoria := f = (1/100)/(5/10)·x·Heaviside(x) - 2·(1/100)/(5/10)·(x - 5/10)
  ·Heaviside(x - 5/10) + (1/100)/(5/10)·(x - 1)·Heaviside(x - 1);
  plot(rhs(CondicionInicialTrayectoria), x = -0.2..1.2, y = -0.02..0.02)
```

$$CondicionInicialTrayectoria := f = \frac{1}{50} x \, \text{Heaviside}(x) - \frac{1}{25} \left(x - \frac{1}{2} \right) \text{Heaviside}\left(x - \frac{1}{2} \right) + \frac{1}{50} (x - 1) \, \text{Heaviside}(x - 1)$$



> $CondicionInicialVelocidad := DerYcero = 0;$

$$CondicionInicialVelocidad := DerYcero = 0 \quad \textbf{(5)}$$

>

MÉTODO DE SEPARACIÓN DE VARIABLES

> $Ecuacion$

$$\frac{\partial^2}{\partial t^2} y(x,t) = \frac{\partial^2}{\partial x^2} y(x,t) \quad \textbf{(6)}$$

> $EcuacionInicial := eval(subs(y(x,t) = F(x)\cdot G(t), Ecuacion))$

$$EcuacionInicial := F(x)\left(\frac{d^2}{dt^2} G(t)\right) = \left(\frac{d^2}{dx^2} F(x)\right) G(t) \quad \textbf{(7)}$$

> $EcuacionSeparada := \frac{lhs(EcuacionInicial)}{F(x)\cdot G(t)} = \frac{rhs(EcuacionInicial)}{F(x)\cdot G(t)}$

$$EcuacionSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{\frac{d^2}{dx^2} F(x)}{F(x)} \quad \textbf{(8)}$$

> $EcuacionX := rhs(EcuacionSeparada) = \text{alpha}; EcuacionT := lhs(EcuacionSeparada) = \text{alpha}$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$$

$$EcuacionT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (9)$$

>

para alpha=0

> $SolucionXcero := dsolve(subs(\text{alpha} = 0, EcuacionX))$

$$SolucionXcero := F(x) = \_C1\, x + \_C2 \quad (10)$$

> $CondicionesFronteraX := F(0) = 0, F(1) = 0;$

$$CondicionesFronteraX := F(0) = 0, F(1) = 0 \quad (11)$$

> $SolucioParticularX := dsolve(\{subs(\text{alpha} = 0, EcuacionX), CondicionesFronteraX\})$

$$SolucioParticularX := F(x) = 0 \quad (12)$$

>

para alpha positivo

> $SolucionXpos := dsolve(subs(\text{alpha} = \text{beta}\cdot\cdot 2, EcuacionX))$

$$SolucionXpos := F(x) = \_C1\, e^{-\beta x} + \_C2\, e^{\beta x} \quad (13)$$

> $SolucionPositivaX := dsolve(\{subs(\text{alpha} = \text{beta}\cdot\cdot 2, EcuacionX), CondicionesFronteraX\})$

$$SolucionPositivaX := F(x) = 0 \quad (14)$$

>

para alpha negativa

> $SolucionXneg := dsolve(subs(\text{alpha} = -\text{beta}\cdot\cdot 2, EcuacionX))$

$$SolucionXneg := F(x) = \_C1 \sin(\beta x) + \_C2 \cos(\beta x) \quad (15)$$

> $SolucionNegativaX := F(x) = \sin(n\cdot \text{Pi}\cdot x)$

$$SolucionNegativaX := F(x) = \sin(n\,\pi\,x) \quad (16)$$

> $SolucionNegativaT := dsolve(subs(\text{alpha} = -n\cdot\cdot 2\cdot \text{Pi}\cdot\cdot 2, EcuacionT))$

$$SolucionNegativaT := G(t) = \_C1 \sin(n\,\pi\,t) + \_C2 \cos(n\,\pi\,t) \quad (17)$$

> $SolucionNegativaGeneral := y(x,t) = rhs(SolucionNegativaX)\cdot rhs(SolucionNegativaT)$

$$SolucionNegativaGeneral := y(x,t) = \sin(n\,\pi\,x)\,(\_C1 \sin(n\,\pi\,t) + \_C2 \cos(n\,\pi\,t)) \quad (18)$$

> $SolucionGeneral := y(x,t) = Sum(\sin(n * \text{Pi} * x)\cdot(b_n\cdot\cos(n\cdot\text{Pi}\cdot t) + a_n\cdot\sin(n\cdot\text{Pi}\cdot t)), n = 1..infinity)$

$$SolucionGeneral := y(x,t) = \sum_{n=1}^{\infty} \sin(n\,\pi\,x)\,(b_n \cos(n\,\pi\,t) + a_n \sin(n\,\pi\,t)) \quad (19)$$

> $SolucionParticularInicial := eval(subs(t = 0, SolucionGeneral))$

$$SolucionParticularInicial := y(x,0) = \sum_{n=1}^{\infty} \sin(n\,\pi\,x)\, b_n \quad (20)$$

> $b_n := \left(\frac{1}{\left(\frac{5}{10}\right)}\right)\cdot int\left(rhs(CondicionInicialTrayectoria)\cdot\sin\left(\frac{n\cdot\text{Pi}\cdot x}{1}\right), x = 0..1\right)$

$$b_n := \frac{1}{25}\ \frac{-\sin(n\,\pi) + 2\sin\left(\frac{1}{2}\,n\,\pi\right)}{n^2\,\pi^2} \tag{21}$$

> $a_n := 0;$

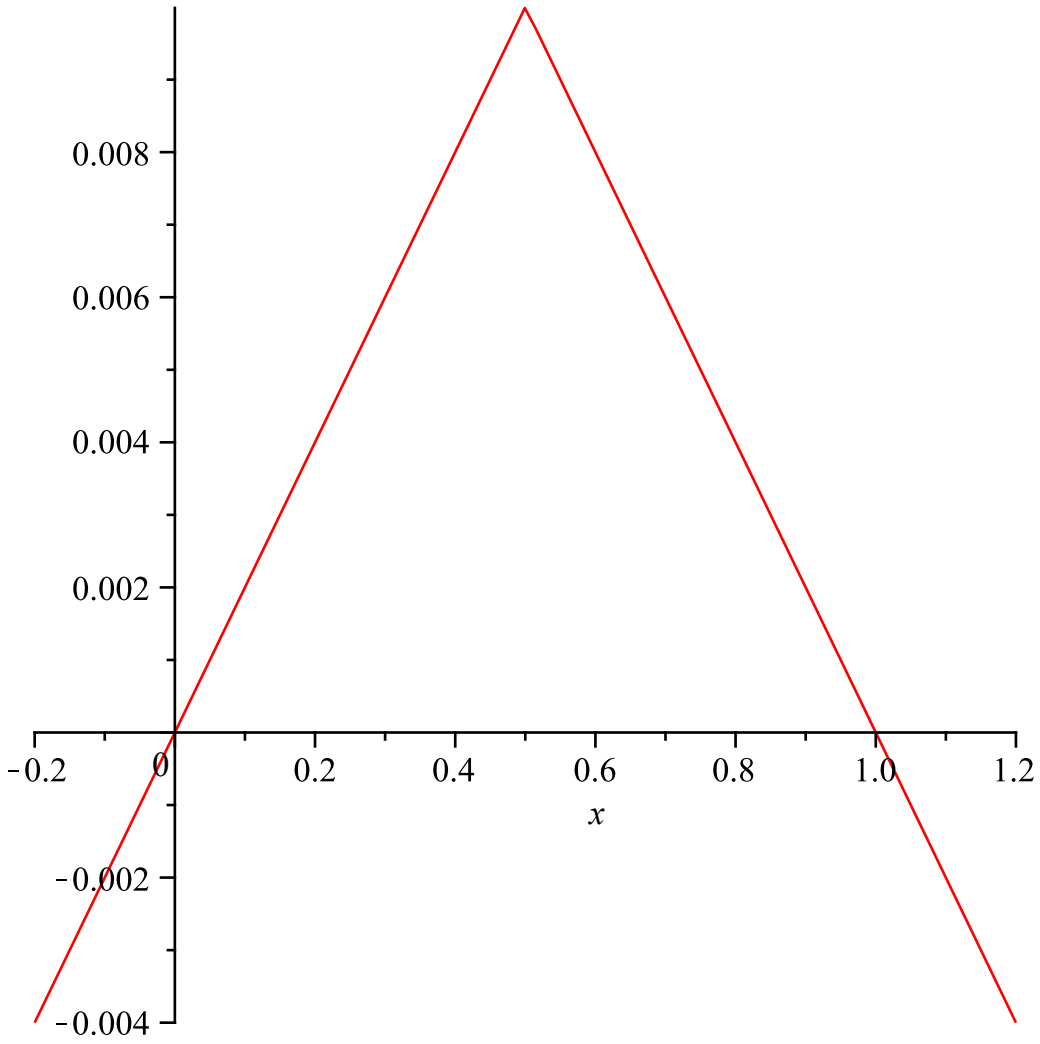
$$a_n := 0 \tag{22}$$

> *SolucionGeneral*;

$$y(x,t) = \sum_{n=1}^{\infty} \frac{1}{25}\ \frac{\sin(n\,\pi\,x)\left(-\sin(n\,\pi) + 2\sin\left(\frac{1}{2}\,n\,\pi\right)\right)\cos(n\,\pi\,t)}{n^2\,\pi^2} \tag{23}$$

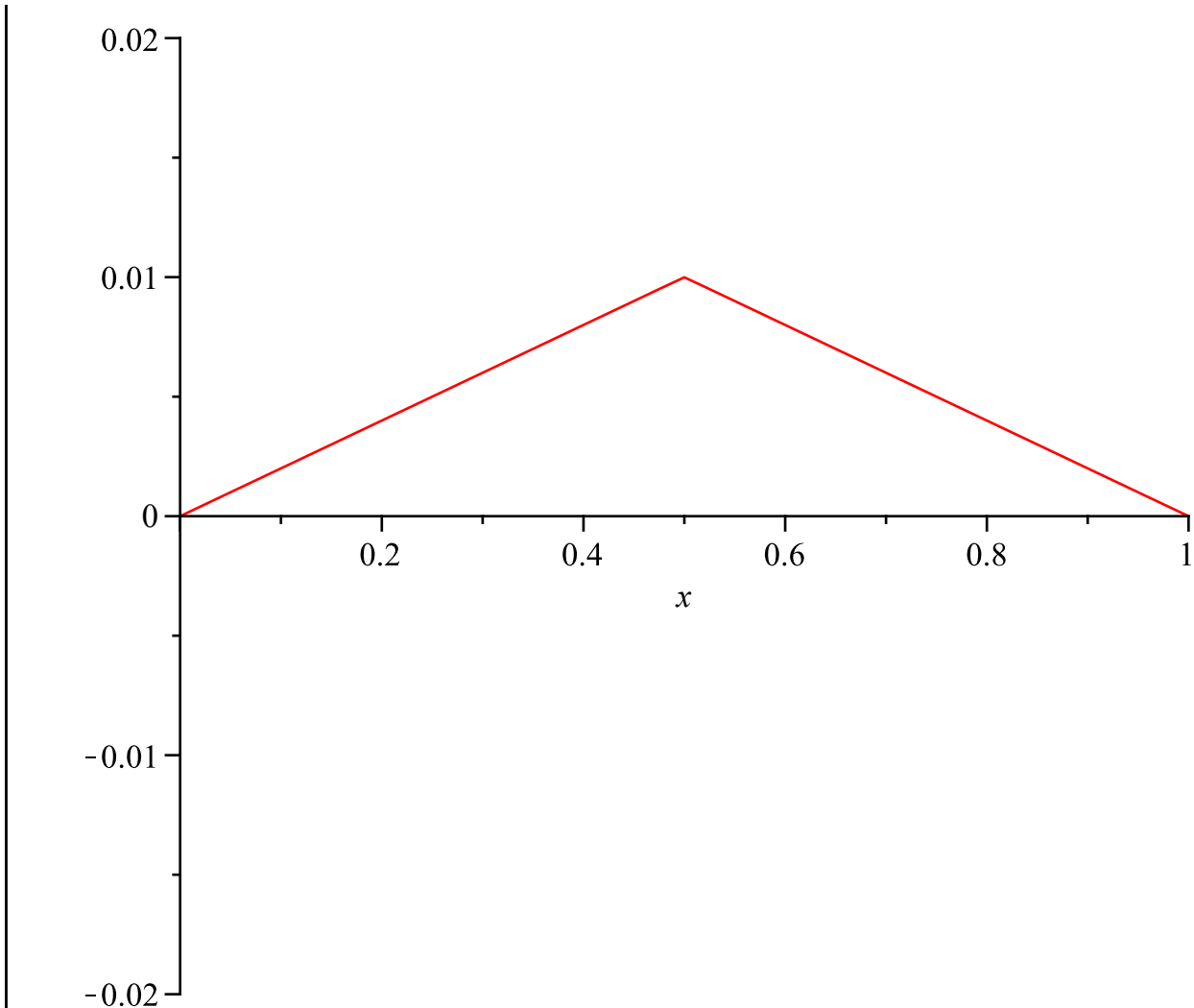
> $SolucionGeneral_{500} := \sum_{n=1}^{500} \frac{1}{25}\ \frac{\sin(n\,\pi\,x)\left(-\sin(n\,\pi) + 2\sin\left(\frac{1}{2}\,n\,\pi\right)\right)\cos(n\,\pi\,t)}{n^2\,\pi^2}$:

> $plot(subs(t=0, SolucionGeneral_{500}), x=-0.2..1.2)$



> $with(plots)$:

> $animate(SolucionGeneral_{500}, x=0..1, t=0..4, frames=150, view=[0..1, -0.02..0.02])$


0.02
0.01
0
-0.01
-0.02
0.2
0.4
0.6
0.8
1
x

>
>
>
>
>