

EDO(n)L

$$a_0(x) \frac{d^{\hat{n}} y}{dx^n} + a_1(x) \frac{d^{\hat{n}-1} y}{dx^{n-1}} + \dots + a_{n-1}(x) \frac{dy}{dx} + a_n(x) y = Q(x)$$

LINEALES: $\left\{ \begin{array}{l} \text{HOMOGÉNEAS} \\ \text{NO-HOMOGÉNEAS} \end{array} \right\}$ $\left\{ \begin{array}{l} \text{coeficientes Variable} \\ \text{coeficientes constantes} \end{array} \right\}$

* $\left\{ \begin{array}{l} \text{Si } Q(x) = 0 \text{ — homogénea} \\ \text{Si } Q(x) \neq 0 \text{ — no-homogénea} \end{array} \right.$

* $\left\{ \begin{array}{l} \forall i \in \mathbb{N} \quad \forall a_i(x) = k_i \text{ coef. Const.} \\ \text{una } a_i(x) \neq k_i \text{ coef. Variables} \end{array} \right.$

$$\frac{d^2 y}{dx^2} - x \frac{dy}{dx} + x^2 y = 8 \cos(2x).$$

EDO(2) L. cv. N-H.

$$\frac{dy}{dx} = y \Rightarrow \frac{dy}{dx} - y = 0$$

EDO(1) L. cc. H.

$$\frac{dy}{dx} - \frac{y}{x} = 0$$

$$a_0(x) = 1 \quad a_1(x) = -\frac{1}{x} \quad Q(x) = 0$$

EDO(1) L. cv. H.

$$\frac{d^3 y}{dt^3} - 5 \frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} - 6y = 8e^{3t} + t^2$$

EDO(3) L. cc. NH.

$$\mathbb{E} \text{Do}(1) \subset \text{cc } H.$$

$$a_0 \frac{dy}{dx} + a_1 y = 0$$

Regla de Oro: El coeficiente de la derivada de mayor orden debe ser siempre la unidad.

$$\frac{dy}{dx} + \frac{a_1}{a_0} y = 0 \rightarrow \frac{dy}{dx} + k_1 y = 0$$

$$\frac{dy}{dx} = -k_1 y \rightarrow dy = -k_1 y dx$$

$$\frac{dy}{y} = -k_1 dx \rightarrow \begin{cases} \int \frac{dy}{y} = -k_1 \int dx \\ Ly + C_1 = -k_1 (x + C_2) \\ Ly = -k_1 x + (-k_1 C_2 - C_1) \\ y = e^{(-k_1 x + (-k_1 C_2 - C_1))} \end{cases}$$

$$\mathbb{E} \text{Do}(1) \subset \text{cc } H.$$

$$\boxed{\frac{dy}{dx} + k_1 y = 0}$$

$$y = e^{(-k_1 x + (-k_1 C_2 - C_1))}$$

$$y = e^{(-k_1 C_2 - C_1)} \cdot e^{-k_1 x}$$

$$\boxed{y = C e^{-k_1 x}} \quad \text{Solución General}$$

$$\frac{dy}{dx} + \sqrt{3}y = 0 \longrightarrow y = Ce^{-\sqrt{3}x}$$

$$\frac{dy}{dx} - \frac{y}{2} = 0 \longrightarrow y = Ce^{\frac{1}{2}x}$$

$$\frac{dy}{dx} + my = 0 \longrightarrow y = Ce^{-mx}$$

$$y = Ce^{5x} \longrightarrow \frac{dy}{dx} - 5y = 0$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad \text{EDO}(2) \text{ L. co H.}$$

$$\frac{dy}{dx} - my = 0 \rightarrow y = C e^{mx} \quad \begin{array}{c} \uparrow \\ y_{PF} \end{array}$$

$$y = C_1 y_1 + C_2 y_2$$

$$\boxed{y_{PF} = e^{mx}} \rightarrow \frac{dy}{dx} = e^{mx} \cdot m \rightarrow \frac{dy}{dx} = m e^{mx}$$

$$\frac{d^2 y}{dx^2} = m \cdot (m e^{mx}) \rightarrow \frac{d^2 y}{dx^2} = m^2 e^{mx}$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$[m^2 e^{mx}] + a_1 [m e^{mx}] + a_2 [e^{mx}] = 0$$

$$(m^2 + a_1 m + a_2) e^{mx} = 0 \quad (\text{I.A.})$$

$$e^{mx} = 0 \quad \forall x \quad y = 0 \quad \text{trivial EDO}(n) \text{ L.}$$



$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad \text{EDO}(2) L \subset H.$$

$$\text{si } y_p = e^{mx}$$

$$m^2 + a_1 m + a_2 = 0 \quad \left\{ \begin{array}{l} \text{ECUACIÓN} \\ \text{CARACTERÍSTICA.} \end{array} \right.$$

$$\left. \begin{array}{l} m_1 \\ m_2 \end{array} \right\} \text{raíces solución.}$$

$$m_1^2 + a_1 m_1 + a_2 = 0 \longrightarrow 0 \equiv 0$$

$$m_2^2 + a_1 m_2 + a_2 = 0 \longrightarrow 0 \equiv 0$$

$$y_{PF} = e^{m_1 x}$$

$$y_{PF} = e^{m_2 x}$$

$$y_g = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0 \quad \text{H.} \quad y = e^{mx}$$

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0$$

$$m_1 = 2 \quad y = e^{2x}$$

$$m_2 = 3 \quad y = e^{3x}$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0 \quad y = e^{2x}$$

$$\frac{dy}{dx} = 2e^{2x} \quad \frac{d^2 y}{dx^2} = 4e^{2x}$$

$$[4e^{2x}] - 5[2e^{2x}] + 6[e^{2x}] = 0$$

$$(4-10+6)e^{2x} = 0 \quad 0 \equiv 0$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0 \quad y = e^{3x}$$

$$\frac{dy}{dx} = 3e^{3x} \quad \frac{d^2 y}{dx^2} = 9e^{3x}$$

$$[9e^{3x}] - 5[3e^{3x}] + 6[e^{3x}] = 0$$

$$(9-15+6)e^{3x} = 0 \quad \underline{0 \equiv 0}$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0 \quad y = e^x$$

$$\frac{dy}{dx} = e^x \quad \frac{d^2 y}{dx^2} = e^x$$

$$[e^x] - 5[e^x] + 6[e^x] = 0$$

$$(1-5+6)e^x = 0$$

$$2e^x \neq 0.$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

$$y_g = C_1 e^{2x} + C_2 e^{3x}$$

$$y_g = C_1 e^{-4x} + C_2 e^{-3x} + C_3 e^{-2x}$$

$$(m+4)(m+3)(m+2) = 0$$

$$(m+4)(m^2 + 5m + 6) = 0$$

$$m^3 + 9m^2 + 26m + 24 = 0$$

$$\frac{d^3 y}{dx^3} + 9 \frac{d^2 y}{dx^2} + 26 \frac{dy}{dx} + 24y = 0$$

EDO(3) LCC H

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad \text{EDO}(2) L \subset \mathbb{H}.$$

$$m^2 + a_1 m + a_2 = 0 \quad \left\{ \begin{array}{l} m_1 \\ m_2 \end{array} \right.$$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$W = \begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0$$

$$m_2 e^{m_2 x} e^{m_1 x} - m_1 e^{m_1 x} e^{m_2 x} \neq 0$$

$$(m_2 - m_1) e^{m_1 x} e^{m_2 x} \neq 0 \quad \begin{array}{l} e^{m_1 x} \neq 0 \\ e^{m_2 x} \neq 0 \end{array}$$

$$(m_2 - m_1) \neq 0 \quad m_2 \neq m_1$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0 \quad m_1 = m_2 = -1$$

$$y = c_1 e^{-x} + c_2 e^{-x} \Rightarrow (c_1 + c_2) e^{-x}$$

$$y = c_3 e^{-x}$$

$$m^2 + a_1 m + a_2 = 0 \quad m_1 = m_2$$

$$(m - m_1)^2 = 0$$

$$\frac{d}{dm} \left(2(m - m_1) = 0 \right)$$

$$2m + a_1 = 0$$

$$m^2 + a_1 m + a_2 = 0 \quad m_1 \neq m_2$$

$$(m - m_1)(m - m_2) = 0$$

$$(m - m_1) + (m - m_2) = 0$$

$$e^{mx} \xrightarrow{m=m_1} e^{m_1 x}$$

$$\frac{d}{dm} \left(x e^{mx} \xrightarrow{m=m_1} x e^{m_1 x} \right)$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \quad \left\{ \begin{array}{l} m_1 = m_2 \end{array} \right.$$

$$y = x e^{m_1 x}$$

$$\frac{dy}{dx} = m_1 x e^{m_1 x} + e^{m_1 x}$$

$$\frac{d^2 y}{dx^2} = m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x}$$

$$[m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x}] + a_1 [m_1 x e^{m_1 x} + e^{m_1 x}] + a_2 [x e^{m_1 x}] = 0$$

$$(m_1^2 + a_1 m_1 + a_2) x e^{m_1 x} + (2m_1 + a_1) e^{m_1 x} = 0$$

$$(0) x e^{m_1 x} + (0) e^{m_1 x} = 0$$

$$\underline{\underline{Q.E.D.}}$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \quad m_1 = m_2$$

$$y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$$

$$\frac{d^2 y}{dx^2} = 0 \quad \text{EDO}(2) \text{ L cc H.}$$

$$m^2 = 0 \quad m_1 = m_2 = 0$$

$$y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}$$

$$y = C_1 + C_2 x$$