Solución General

$$x(t) = c_1 e^{2t} + c_2 t e^{2t} + c_3 t^2 e^{2t}$$

$$(m-2)^3 = 0$$ Ecuación Característica

$$m^3 - 6m^2 + 12m - 8 = 0$$

$$\frac{d^3x}{dt^3} - 6\frac{d^2x}{dt^2} + 12\frac{dx}{dt} - 8x = 0$$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0 \quad \text{EDO}(2)\text{LCCH.}$$

CASO I:- $m^2 + a_1 m + a_2 = 0$ $\Big\}$ $m_1 \neq m_2 \in \mathbb{R}$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

CASO II:- $m^2 + a_1 m + a_2 = 0$ $\Big\}$ $m_1 = m_2 \in \mathbb{R}$

$$y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}.$$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

caso III:

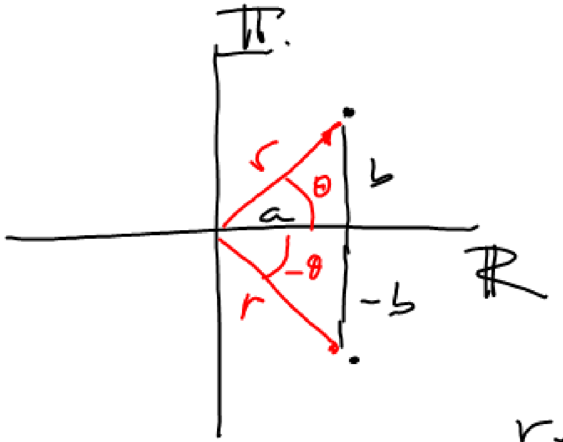
$$m^2 + a_1 m + a_2 = 0 \qquad \begin{cases} m_1 = a + bi \\ m_2 = a - bi \end{cases} \qquad m_1, m_2 \in \mathbb{C}$$

$$y = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x} \qquad m_1 \neq m_2$$

$$y \in \mathbb{R} \ \forall x \in \mathbb{R} \qquad C_1, C_2 \in \mathbb{C}.$$

Teorema Euler

$$e^{\pi i} + 1 = 0$$



$a+bi$

$a-bi$

$$re^{\theta i} = a+bi$$
$$re^{-\theta i} = a-bi$$

$$re^{\theta i} = r\cos\theta + (r\,\mathrm{sen}\,\theta)i$$
$$re^{-\theta i} = r\cos\theta - (r\,\mathrm{sen}(\theta))i$$

$$\left.\begin{aligned} e^{\theta i} &= \cos(\theta) + (\mathrm{sen}(\theta))i \\ e^{-\theta i} &= \cos(\theta) - (\mathrm{sen}(\theta))i \end{aligned}\right\} \text{conversión Euler}$$

$$e^{\pi i} = -1 \rightarrow e^{\pi i} + 1 = 0$$

$$y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x}$$

$$= c_1 e^{ax} e^{bxi} + c_2 e^{ax} e^{-bxi}$$

$$= e^{ax}\left(c_1 e^{bxi} + c_2 e^{-bxi}\right)$$

$$= e^{ax}\left(c_1\left[\cos(bx) + (\operatorname{sen}(bx))i\right] + c_2 \cdot \left[\cos(bx) - (\operatorname{sen}(bx)i)\right]\right)$$

$$= e^{ax}\left([c_1 + c_2]\cos(bx) + (c_1 i - c_2 \cdot i)\operatorname{sen}(bx)\right)$$

$$= e^{ax}\left(c_{10}\cos(bx) + c_{20}\operatorname{sen}(bx)\right) \quad x \in \mathbb{R}, \; y \in \mathbb{R}$$

$$y_g = c_{10} e^{ax}\cos(bx) + c_{20} e^{ax}\operatorname{sen}(bx) \quad c_{10}, c_{20} \in \mathbb{R}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

$$m^2 - 2m + 2 = 0 \qquad m_{1,2} = \frac{2 \pm \sqrt{4-4(2)}}{2}$$

$$m_{1,2} = \frac{2 \pm \sqrt{-4}}{2} \Rightarrow 1 \pm i$$

$$y = C_1 e^x \cos(x) + C_2 e^x \operatorname{sen}(x) \qquad \begin{array}{l} a = 1 \\ b = 1 \end{array}$$

$$\frac{dy}{dx} = C_1\left(-e^x \operatorname{sen}(x) + e^x \cos(x)\right) + C_2\left(e^x \cos(x) + e^x \operatorname{sen}(x)\right)$$

$$\frac{dy}{dx} = (C_1 + C_2) e^x \cos(x) + (C_2 - C_1) e^x \operatorname{sen}(x)$$

$$\frac{d^2y}{dx^2} = (C_1 + C_2)\left(-e^x \operatorname{sen}(x) + e^x \cos(x)\right) + (C_2 - C_1)\left(e^x \cos(x) + e^x \operatorname{sen}(x)\right)$$

$$= (2C_2) e^x \cos(x) + (-2C_1) e^x \operatorname{sen}(x)$$

$$\frac{d^2y}{dx^2} \leftrightarrow 2C_2 e^x \cos(x) - 2C_1 e^x \operatorname{sen}(x)$$

$$+$$

$$-2\frac{dy}{dx} \leftrightarrow (-2C_1 - 2C_2) e^x \cos(x) + (-2C_2 + 2C_1) e^x \operatorname{sen}(x)$$

$$+$$

$$+2y \leftrightarrow 2C_1 e^x \cos(x) + 2C_2 e^x \operatorname{sen}(x)$$

$$=$$

$$0 \leftrightarrow (0) e^x \cos(x) + (0) e^x \operatorname{sen}(x)$$

$$0 = 0$$

CASO III :-

$$m^2 + a_1 m + a_2 = 0 \quad \begin{cases} m_1 = a + bi \\ m_2 = a - bi \end{cases} \quad m_{1,2} \in \mathbb{C}.$$

$$y_g = C_1 e^{ax} \cos(bx) + C_2 e^{ax} \operatorname{sen}(bx) \quad \begin{matrix} a \in \mathbb{R} \\ b \in \mathbb{R}^+ \end{matrix}$$

III bis $\dfrac{d^2y}{dx^2} + 9y = 0$.

$$m^2 + 9 = 0 \quad m^2 = -9 \quad m = \pm\sqrt{-9} \quad m = \pm 3i$$

$$y_g = C_1 \cos(3x) + C_2 \operatorname{sen}(3x)$$

EDO(n) L cc H.

Ec. caract. $\begin{cases} \text{I. } m_1 \neq m_2 \in \mathbb{R} \\ \text{II. } m_1 = m_2 \in \mathbb{R} \\ \text{III. } m_1, m_2 \in \mathbb{C}. \end{cases}$

$$y_{pp} = \begin{cases} e^{ax} \\ x^n \\ \text{pa} \begin{cases} \cos(bx) \\ \operatorname{sen}(bx) \end{cases} \end{cases}$$

$$y_g = \underset{m_1 \in \mathbb{R}}{C_1 e^{2x}} + \underbrace{C_2 e^{-2x} + C_3 x e^{-2x}}_{m_2 = m_2 \in \mathbb{R}} + \underbrace{C_4 e^{5x} \cos(3x) + C_5 e^{5x} \sin(3x)}_{m_4,\, m_5,\, m \in \mathbb{C}}$$

$$(m-2)(m+2)^2(m-(5-3i))(m-(5+3i)) = 0$$