

$$\begin{aligned} &> \text{restart} \\ &> \text{Solucion} := x(t) = C_1 \cdot \exp(5t) + C_2 \cdot \exp(-2t) \cdot \cos(4t) + C_3 \cdot \exp(-2t) \cdot \sin(4t) + 6 \cdot \exp(3t) \\ &\quad \text{Solucion} := x(t) = C_1 e^{5t} + C_2 e^{-2t} \cos(4t) + C_3 e^{-2t} \sin(4t) + 6 e^{3t} \end{aligned} \quad (1)$$

$$\begin{aligned} &> \text{SolucionHom} := x(t) = C_1 e^{5t} + C_2 e^{-2t} \cos(4t) + C_3 e^{-2t} \sin(4t) \\ &\quad \text{SolucionHom} := x(t) = C_1 e^{5t} + C_2 e^{-2t} \cos(4t) + C_3 e^{-2t} \sin(4t) \end{aligned} \quad (2)$$

$$\begin{aligned} &> \text{SolucionPart} := x(t) = 6 \exp(3t) \\ &\quad \text{SolucionPart} := x(t) = 6 e^{3t} \end{aligned} \quad (3)$$

$$\begin{aligned} &> \text{EcuacionCaract} := \text{expand}((m-5) \cdot (m - (-2 + 4 \cdot I)) \cdot (m - (-2 - 4 \cdot I))) = 0 \\ &\quad \text{EcuacionCaract} := m^3 - m^2 - 100 = 0 \end{aligned} \quad (4)$$

$$\begin{aligned} &> \text{EcuacionHom} := \text{diff}(x(t), t\$3) - \text{diff}(x(t), t\$2) - 100 \cdot x(t) = 0 \\ &\quad \text{EcuacionHom} := \frac{d^3}{dt^3} x(t) - \left(\frac{d^2}{dt^2} x(t) \right) - 100 x(t) = 0 \end{aligned} \quad (5)$$

$$\begin{aligned} &> Q := \text{eval}(\text{subs}(x(t) = \text{rhs}(\text{SolucionPart}), \text{lhs}(\text{EcuacionHom}))) \\ &\quad Q := -492 e^{3t} \end{aligned} \quad (6)$$

$$\begin{aligned} &> \text{EcuacionNoHom} := \text{lhs}(\text{EcuacionHom}) = Q \\ &\quad \text{EcuacionNoHom} := \frac{d^3}{dt^3} x(t) - \left(\frac{d^2}{dt^2} x(t) \right) - 100 x(t) = -492 e^{3t} \end{aligned} \quad (7)$$

$$\begin{aligned} &> \text{Solucion} \\ &\quad x(t) = C_1 e^{5t} + C_2 e^{-2t} \cos(4t) + C_3 e^{-2t} \sin(4t) + 6 e^{3t} \end{aligned} \quad (8)$$

$$\begin{aligned} &> \text{SolGral} := \text{simplify}(\text{dsolve}(\text{EcuacionNoHom})) \\ &\quad \text{SolGral} := x(t) = 6 e^{3t} + _C1 e^{5t} + _C2 e^{-2t} \cos(4t) + _C3 e^{-2t} \sin(4t) \end{aligned} \quad (9)$$

MÉTODO DE PARÁMETROS VARIABLES (O VARIACIÓN DE PARÁMETROS)

$$\begin{aligned} &> \text{restart} \\ &> \text{Ecuacion} := \frac{d^3}{dt^3} x(t) - \left(\frac{d^2}{dt^2} x(t) \right) - 100 x(t) = -492 e^{3t} \\ &\quad \text{Ecuacion} := \frac{d^3}{dt^3} x(t) - \left(\frac{d^2}{dt^2} x(t) \right) - 100 x(t) = -492 e^{3t} \end{aligned} \quad (10)$$

$$\begin{aligned} &> \text{EcuacionHomogenea} := \text{lhs}(\text{Ecuacion}) = 0 \\ &\quad \text{EcuacionHomogenea} := \frac{d^3}{dt^3} x(t) - \left(\frac{d^2}{dt^2} x(t) \right) - 100 x(t) = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} &> Q := \text{rhs}(\text{Ecuacion}) \\ &\quad Q := -492 e^{3t} \end{aligned} \quad (12)$$

$$\begin{aligned} &> \text{EcuacionCaracteristica} := m \cdot 3 - m \cdot 2 - 100 = 0 \\ &\quad \text{EcuacionCaracteristica} := m^3 - m^2 - 100 = 0 \end{aligned} \quad (13)$$

$$\begin{aligned} &> \text{Raiz} := \text{solve}(\text{EcuacionCaracteristica}) \\ &\quad \text{Raiz} := 5, -2 + 4I, -2 - 4I \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{Sol}_1 := x(t) = \exp(\text{Raiz}_1 \cdot t) \end{aligned} \quad (15)$$

$$Sol_1 := x(t) = e^{5t} \quad (15)$$

$$\begin{aligned} > Sol_2 := x(t) = \exp(\operatorname{Re}(Raiz_2) \cdot t) \cdot \cos(\operatorname{Im}(Raiz_2) \cdot t) \\ Sol_2 &:= x(t) = e^{-2t} \cos(4t) \end{aligned} \quad (16)$$

$$\begin{aligned} > Sol_3 := x(t) = \exp(\operatorname{Re}(Raiz_2) \cdot t) \cdot \sin(\operatorname{Im}(Raiz_2) \cdot t) \\ Sol_3 &:= x(t) = e^{-2t} \sin(4t) \end{aligned} \quad (17)$$

$$\begin{aligned} > SolucionHomogenea &:= x(t) = C_1 \cdot rhs(Sol_1) + C_2 \cdot rhs(Sol_2) + C_3 \cdot rhs(Sol_3) \\ SolucionHomogenea &:= x(t) = C_1 e^{5t} + C_2 e^{-2t} \cos(4t) + C_3 e^{-2t} \sin(4t) \end{aligned} \quad (18)$$

$$\begin{aligned} > SolucionNoHomogenea &:= x(t) = AA \cdot rhs(Sol_1) + BB \cdot rhs(Sol_2) + DD \cdot rhs(Sol_3) \\ SolucionNoHomogenea &:= x(t) = AA e^{5t} + BB e^{-2t} \cos(4t) + DD e^{-2t} \sin(4t) \end{aligned} \quad (19)$$

> with(linalg) :

$$\begin{aligned} > WW &:= \operatorname{wronskian}([rhs(Sol_1), rhs(Sol_2), rhs(Sol_3)], t) \\ WW &:= \begin{bmatrix} e^{5t} & e^{-2t} \cos(4t) & e^{-2t} \sin(4t) \\ 5e^{5t} & -2e^{-2t} \cos(4t) - 4e^{-2t} \sin(4t) & -2e^{-2t} \sin(4t) + 4e^{-2t} \cos(4t) \\ 25e^{5t} & -12e^{-2t} \cos(4t) + 16e^{-2t} \sin(4t) & -12e^{-2t} \sin(4t) - 16e^{-2t} \cos(4t) \end{bmatrix} \end{aligned} \quad (20)$$

$$\begin{aligned} > BBB &:= \operatorname{array}([0, 0, Q]) \\ BBB &:= \begin{bmatrix} 0 & 0 & -492 e^{3t} \end{bmatrix} \end{aligned} \quad (21)$$

$$\begin{aligned} > Parametro &:= \operatorname{linsolve}(WW, BBB) : Aprima := \operatorname{simplify}(Parametro_1); Bprima \\ &:= \operatorname{simplify}(Parametro_2); Dprima := \operatorname{simplify}(Parametro_3); \end{aligned}$$

$$Aprima := -\frac{492}{65} e^{-2t}$$

$$Bprima := \frac{123}{65} e^{5t} (-7 \sin(4t) + 4 \cos(4t))$$

$$Dprima := \frac{123}{65} e^{5t} (7 \cos(4t) + 4 \sin(4t)) \quad (22)$$

$$\begin{aligned} > AA &:= \operatorname{simplify}(\operatorname{int}(Aprima, t) + C_1); BB := \operatorname{simplify}(\operatorname{int}(Bprima, t) + C_2); DD \\ &:= \operatorname{simplify}(\operatorname{int}(Dprima, t) + C_3) \end{aligned}$$

$$AA := \frac{246}{65} e^{-2t} + C_1$$

$$\begin{aligned} BB &:= \frac{84}{65} e^{5t} \cos(4t) - \frac{21}{13} e^{5t} \sin(4t) + \frac{96}{13} e^{5t} \cos(t)^4 + \frac{384}{65} e^{5t} \cos(t)^3 \sin(t) \\ &\quad - \frac{96}{13} e^{5t} \cos(t)^2 - \frac{192}{65} e^{5t} \cos(t) \sin(t) + \frac{12}{13} e^{5t} + C_2 \end{aligned}$$

$$\begin{aligned} DD &:= \frac{168}{13} e^{5t} \cos(t)^4 + \frac{672}{65} e^{5t} \cos(t)^3 \sin(t) - \frac{168}{13} e^{5t} \cos(t)^2 - \frac{336}{65} e^{5t} \cos(t) \sin(t) \\ &\quad + \frac{21}{13} e^{5t} - \frac{48}{65} e^{5t} \cos(4t) + \frac{12}{13} e^{5t} \sin(4t) + C_3 \end{aligned} \quad (23)$$

$$> SolucionNoHomogenea$$

$$\begin{aligned}
 x(t) = & \left(\frac{246}{65} e^{-2t} + C_1 \right) e^{5t} + \left(\frac{84}{65} e^{5t} \cos(4t) - \frac{21}{13} e^{5t} \sin(4t) + \frac{96}{13} e^{5t} \cos(t)^4 \right. \\
 & + \frac{384}{65} e^{5t} \cos(t)^3 \sin(t) - \frac{96}{13} e^{5t} \cos(t)^2 - \frac{192}{65} e^{5t} \cos(t) \sin(t) + \frac{12}{13} e^{5t} + C_2 \Big) \\
 & e^{-2t} \cos(4t) + \left(\frac{168}{13} e^{5t} \cos(t)^4 + \frac{672}{65} e^{5t} \cos(t)^3 \sin(t) - \frac{168}{13} e^{5t} \cos(t)^2 \right. \\
 & - \frac{336}{65} e^{5t} \cos(t) \sin(t) + \frac{21}{13} e^{5t} - \frac{48}{65} e^{5t} \cos(4t) + \frac{12}{13} e^{5t} \sin(4t) + C_3 \Big) \\
 & e^{-2t} \sin(4t)
 \end{aligned} \tag{24}$$

$$\begin{aligned}
 & \text{> } \text{Solucion} := x(t) = C_1 e^{5t} + C_2 e^{-2t} \cos(4t) + C_3 e^{-2t} \sin(4t) + 6 e^{3t} \\
 & \quad \text{Solucion} := x(t) = C_1 e^{5t} + C_2 e^{-2t} \cos(4t) + C_3 e^{-2t} \sin(4t) + 6 e^{3t}
 \end{aligned} \tag{25}$$

$$\begin{aligned}
 & \text{> } \text{comprobacion} := \text{simplify}(\text{rhs}(\text{SolucionNoHomogenea}) - \text{rhs}(\text{Solucion})) = 0 \\
 & \quad \text{comprobacion} := 0 = 0
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 & \text{> } \text{SolucionNoHomogeneaParticular} := \text{subs}(C_1 = 1, C_2 = 1, C_3 = 1, \text{SolucionNoHomogenea}) \\
 & \text{SolucionNoHomogeneaParticular} := x(t) = \left(\frac{246}{65} e^{-2t} + 1 \right) e^{5t} + \left(\frac{84}{65} e^{5t} \cos(4t) \right. \\
 & \quad - \frac{21}{13} e^{5t} \sin(4t) + \frac{96}{13} e^{5t} \cos(t)^4 + \frac{384}{65} e^{5t} \cos(t)^3 \sin(t) - \frac{96}{13} e^{5t} \cos(t)^2 \\
 & \quad - \frac{192}{65} e^{5t} \cos(t) \sin(t) + \frac{12}{13} e^{5t} + 1 \Big) e^{-2t} \cos(4t) + \left(\frac{168}{13} e^{5t} \cos(t)^4 \right. \\
 & \quad + \frac{672}{65} e^{5t} \cos(t)^3 \sin(t) - \frac{168}{13} e^{5t} \cos(t)^2 - \frac{336}{65} e^{5t} \cos(t) \sin(t) + \frac{21}{13} e^{5t} \\
 & \quad \left. - \frac{48}{65} e^{5t} \cos(4t) + \frac{12}{13} e^{5t} \sin(4t) + 1 \right) e^{-2t} \sin(4t)
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 & \text{> } \text{SolucionParticular} := \text{subs}(C_1 = 1, C_2 = 1, C_3 = 1, \text{Solucion}) \\
 & \quad \text{SolucionParticular} := x(t) = e^{5t} + e^{-2t} \cos(4t) + e^{-2t} \sin(4t) + 6 e^{3t} \\
 & \text{> } \text{plot}([\text{rhs}(\text{SolucionNoHomogeneaParticular}), \text{rhs}(\text{SolucionParticular}) + 1], t = -1 .. 0.5)
 \end{aligned} \tag{28}$$

