

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 2e^{3x}$$

$$\text{I.H. } \frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

$$Q = 2e^{3x}$$

$$m^2 - 5m + 6 = 0 \quad (m-2)(m-3) = 0 \quad \left. \begin{matrix} m_1 = 2 \\ m_2 = 3 \end{matrix} \right\} \text{I.}$$

$$y = C_1 e^{2x} + C_2 e^{3x}$$

$$y = A(x)e^{2x} + B(x)e^{3x}$$

$$\begin{cases} \frac{dy}{dx} = 2A(x)e^{2x} + 3B(x)e^{3x} + C_0 \\ \frac{d^2 y}{dx^2} = 4Ae^{2x} + 9Be^{3x} + 2e^{3x} \end{cases} \begin{cases} A'e^{2x} + B'e^{3x} = 0 \\ 2A'e^{2x} + 3B'e^{3x} = 2e^{3x} \end{cases}$$

$$\begin{aligned} 2A'e^{2x} + 3B'e^{3x} &= 2e^{3x} \\ -2A'e^{2x} - 2B'e^{2x} &= 0 \end{aligned}$$

$$C_0 + B'e^{3x} = 2e^{3x}$$

$$A'e^{2x} = -B'e^{3x} \quad \left| \begin{matrix} B' = 2 \\ B = 2x + C_2 \end{matrix} \right. \rightarrow B = 2x + C_2$$

$$A'e^{2x} = -2e^{3x}$$

$$A' = -2e^x$$

$$A = -2 \int e^x dx + C_1 \rightarrow A = -2e^x + C_1$$

$$y = (-2e^x + C_1)e^{2x} + (2x + C_2)e^{3x}$$

$$y = C_1 e^{2x} + (C_2 - 2)e^{3x} + 2xe^{3x}$$

$$y = C_1 e^{2x} + C_2 e^{3x} + 2xe^{3x}$$