$$e^{at} = \frac{(at)^0}{0!} + \frac{at}{1!} + \frac{(at)^2}{2!} + \dots + \frac{(at)^k}{k!} + \dots$$

$$\left[e^{At}\right] = I + \frac{A}{1!}t + \frac{A^2}{2!}t^2 + \dots + \frac{A^k}{k!}t^k + \dots$$

$$A_{n\times n} \Rightarrow \det\left(A - \lambda I\right) = 0$$

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n = 0$$

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$$A^n + a_1A^{n-1} + a_2A^{n-2} + \dots + a_nI = [0]$$

$$A^n = -a_nI - a_{n-1}A - \dots - a_1A^{n-1}$$

$$e^{At} = I + \frac{A}{1!}t + \frac{A^2}{2!}t^2 + \dots + \frac{A^{n-1}}{n-1!}t^{n-1} + \frac{A^n}{n!}t^n + \frac{A^{n+1}}{n+1!}t^{n+1} + \dots + \frac{A^k}{k!}t^k + \dots$$

$$\left[e^{At}\right] = \underbrace{I\,B_0(t) + A\,B_1(t) + A^2B_2(t) + A^3B_3(t) + \dots + A^{n-1}B_{n-1}(t)}_{n \text{ "términos"}}$$

$A_{n\times n}$

$$e^{\lambda_i t} = B_0(t) + \lambda_i B_1(t) + \lambda_i^2 B_2(t) + \dots + \lambda_i^{n-1}B_{n-1}(t)$$

$\lambda_i \in$ Valores Caract.

$$A=\begin{bmatrix}0 & 1 & -1\\ 2 & 0 & -2\\ -1 & 1 & 0\end{bmatrix} \qquad e^{At}=\beta_0(t)I+\beta_1(t)A+\beta_2(t)A^2$$

3×3 $\quad \det(A-\lambda I)=0 \quad \begin{vmatrix}-\lambda & 1 & -1\\ 2 & -\lambda & -2\\ -1 & 1 & -\lambda\end{vmatrix}=0$

$$-\lambda\begin{vmatrix}-\lambda & -2\\ 1 & -\lambda\end{vmatrix}-1\begin{vmatrix}2 & -2\\ -1 & -\lambda\end{vmatrix}-1\begin{vmatrix}2 & -\lambda\\ -1 & 1\end{vmatrix}=0$$

$$-\lambda(\lambda^2+2)-(-2\lambda-2)-(2-\lambda)=0$$

$$-\lambda^3\cancel{-2\lambda}+\cancel{2\lambda}+\cancel{2}-\cancel{2}+\lambda=0$$

$$-\lambda^3+\lambda=0 \qquad \lambda(-\lambda^2+1)=0 \qquad \lambda(\lambda+1)(\lambda-1)=0$$

$$\lambda_1=0 \quad \lambda_2=-1 \quad \lambda_3=1$$

$$e^{0t}=\beta_0(t)(1)+(0)+(0) \Rightarrow \boxed{\beta_0(t)=1}$$

$$e^{-t}=\beta_0(t)(1)+(-1)\beta_1(t)+(-1)^2\beta_2(t) \Rightarrow \beta_0(t)-\beta_1(t)+\beta_2(t)=e^{-t}$$

$$e^{t}=\beta_0(t)(1)+(1)\beta_1(t)+(1)^2\beta_2(t) \qquad \beta_0(t)+\beta_1(t)+\beta_2(t)=e^{t}$$

$$+\quad -\beta_1(t)+\beta_2(t)=e^{-t}-1$$

$$\beta_1(t)+\beta_2(t)=e^{t}-1$$

$$2\beta_2(t)=e^{t}+e^{-t}-2$$

$$\beta_2(t)=\frac{1}{2}\left(e^{t}+e^{-t}-2\right)$$

$$\beta_1(t)=(e^{t}-1)-\beta_2(t)$$

$$\beta_1(t)=e^{t}-1-\frac{1}{2}e^{t}-\frac{1}{2}e^{-t}+1$$

$$\beta_1(t)=\frac{1}{2}e^{t}-\frac{1}{2}e^{-t}$$

$$\beta_1(t)=\frac{1}{2}\left(e^{t}-e^{-t}\right)$$

$$\left[e^{At}\right] = (1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{1}{2}\left(e^{t} - e^{-t}\right)\begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & -2 \\ -1 & 1 & 0 \end{bmatrix} +$$

$$+ \frac{1}{2}\left(e^{t} + e^{-t} - 2\right)\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$$

$$\left[e^{At}\right] = \begin{bmatrix} -2 & 1 & 2 \\ -2 & 1 & 2 \\ -2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} & 0 & -\frac{3}{2} \\ 2 & 0 & -2 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} e^{t} + \begin{bmatrix} \frac{3}{2} & -1 & -\frac{1}{2} \\ 0 & 0 & 0 \\ \frac{3}{2} & -1 & -\frac{1}{2} \end{bmatrix} e^{-t}$$

$$MMatEExp := \begin{bmatrix} -2 & 1 & 2 \\ -2 & 1 & 2 \\ -2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} & 0 & -\frac{3}{2} \\ 2 & 0 & -2 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \mathrm{e}^{t} + \begin{bmatrix} \frac{3}{2} & -1 & -\frac{1}{2} \\ 0 & 0 & 0 \\ \frac{3}{2} & -1 & -\frac{1}{2} \end{bmatrix} \mathrm{e}^{-t}$$

$$\frac{d^3x(t)}{dt^3} - 6\frac{d^2x(t)}{dt^2} + 12\frac{dx(t)}{dt} - 8x(t) = 0$$

$$x(t) \Rightarrow x_1(t)$$

$$\frac{dx}{dt} \Rightarrow \frac{dx_1(t)}{dt} = x_2(t)$$

$$\frac{d^2x}{dt^2} \Rightarrow \frac{dx_2(t)}{dt} = x_3(t)$$

$$\frac{d^3x}{dt^3} \Rightarrow \frac{dx_3(t)}{dt} = 8x_1(t) - 12x_2(t) + 6x_3(t)$$

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 8 & -12 & 6 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$