Laplace Transform

of derivate

$$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st} f'(t)\,dt$$

$$= \left[\int e^{-st} f'(t)\,dt\right]_0^\infty$$

$$\int e^{-st} f'(t)\,dt = f(t)e^{-st} + \frac{1}{s}\int e^{-st} f(t)\,dt$$

$u = e^{-st}$ $\quad dv = f'(t)\,dt$

$du = -se^{-st}\,dt$ $\quad v = f(t)$

$$= \left[e^{-st} f(t)\right]_0^\infty + s\int_0^\infty e^{-st} f(t)\,dt$$

$$= \left(\lim_{t\to\infty} e^{-st} \cdot \lim_{t\to\infty} f(t) - e^{-s(0)} f(0)\right) +$$

$$+ s\,\mathcal{L}\{f(t)\}$$

$$\boxed{\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0)}$$

$$\mathcal{L}\left\{\frac{d^n}{dt^n} f(t)\right\} = s^n \mathcal{L}\{f(t)\} - s^{n-1}\cdot f(0) - s^{n-2}\cdot f'(0) -$$

$$- s^{n-3}\cdot f''(0) = \ldots\ldots = f^{(n-1)}(0)$$

$$\frac{d^2y(t)}{dt^2} - 7\frac{dy(t)}{dt} + 12y(t) = 4e^{2t} \qquad y(0)=2,\ y'(0)=-3$$

Linear over add (subtract) and multiply by constant

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y\right\} = \mathcal{L}\left\{4e^{2t}\right\}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 7\mathcal{L}\left\{\frac{dy}{dt}\right\} + 12\mathcal{L}\{y\} = 4\mathcal{L}\left\{e^{2t}\right\}$$

$$\left[s^2\bar{Y}(s) - s(2) - (-3)\right] - 7\left[s\bar{Y}(s) - (2)\right] + 12\bar{Y}(s) = 4\left(\frac{1}{s-2}\right)$$

$$\left(s^2 - 7s + 12\right)\bar{Y}(s) - 2\cdot s + 17 = \frac{4}{s-2}$$

$$\left(s^2 - 7s + 12\right)\bar{Y}(s) = \frac{4}{s-2} + 2\cdot s - 17$$

$$= \frac{4 + (2s-17)(s-2)}{(s-2)}$$

$$\left(s^2 - 7s + 12\right)\bar{Y}(s) = \frac{2s^2 - 21s + 38}{(s-2)}$$

$$\bar{Y}(s) = \frac{2s^2 - 21s + 38}{(s-2)(s^2 - 7s + 12)}$$

$$= \frac{2s^2 - 21\cdot s + 38}{(s-2)(s-3)(s-4)}$$

$$\frac{2s^2-21s+38}{(s-2)(s-3)(s-4)} = \frac{A}{s-2} + \frac{B}{(s-3)} + \frac{C}{(s-4)}$$

$$2s^2-21s+38 = A(s-3)(s-4)+B(s-2)(s-4)+C(s-2)(s-3)$$

if $\boxed{s=2}$

$$2(2)^2-21(2)+38 = A(-1)(-2)+(0)+(0)$$

$$2A = 8-42+38 \Rightarrow 2A=4 \Rightarrow \boxed{A=2}$$

if $\boxed{s=3}$

$$2(3)^2-21(3)+38 = (0)+B(1)(-1)+(0)$$

$$-B = 18-63+38 \Rightarrow -B=-7 \quad \boxed{B=7}$$

if $\boxed{s=4}$

$$2(4)^2-21(4)+38 = (0)+(0)+C(2)(1)$$

$$2C = 32-84+38 \Rightarrow 2C=-14 \quad \boxed{C=-7}$$

$$\bar{Y}(s) = \frac{2}{(s-2)} + \frac{7}{(s-3)} - \frac{7}{(s-4)}$$

$$\mathcal{L}^{-1}\{\bar{Y}(s)\} = 2\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + 7\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} - 7\mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\}$$

$$\boxed{y(t) = 2e^{2t} + 7e^{3t} - 7e^{4t}}$$ Particular Solution

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