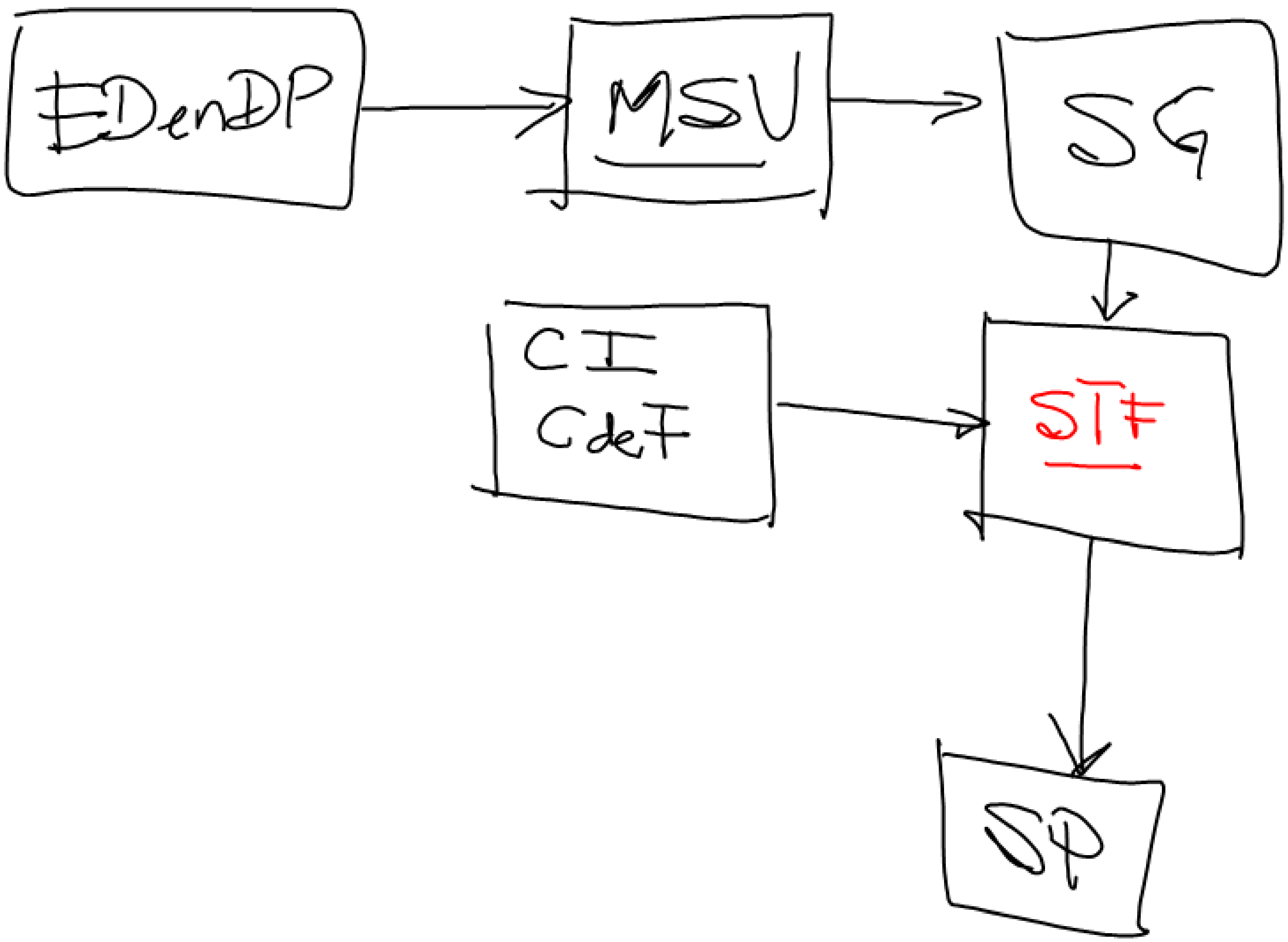
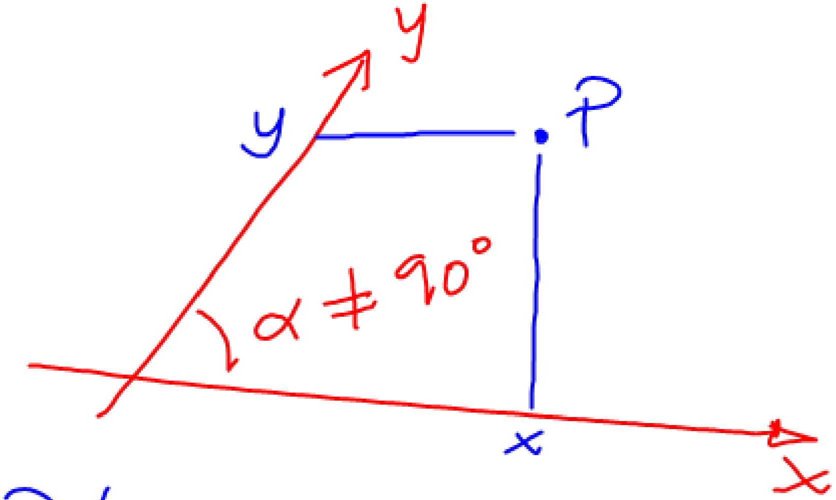
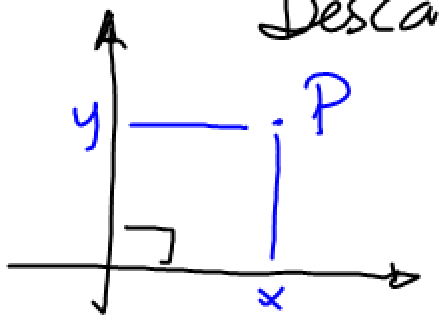

EDenDP
MSU
SG
CI
CdeF
STF
SP

+ SERIE TRIGONOMÉTRICA FOURIER

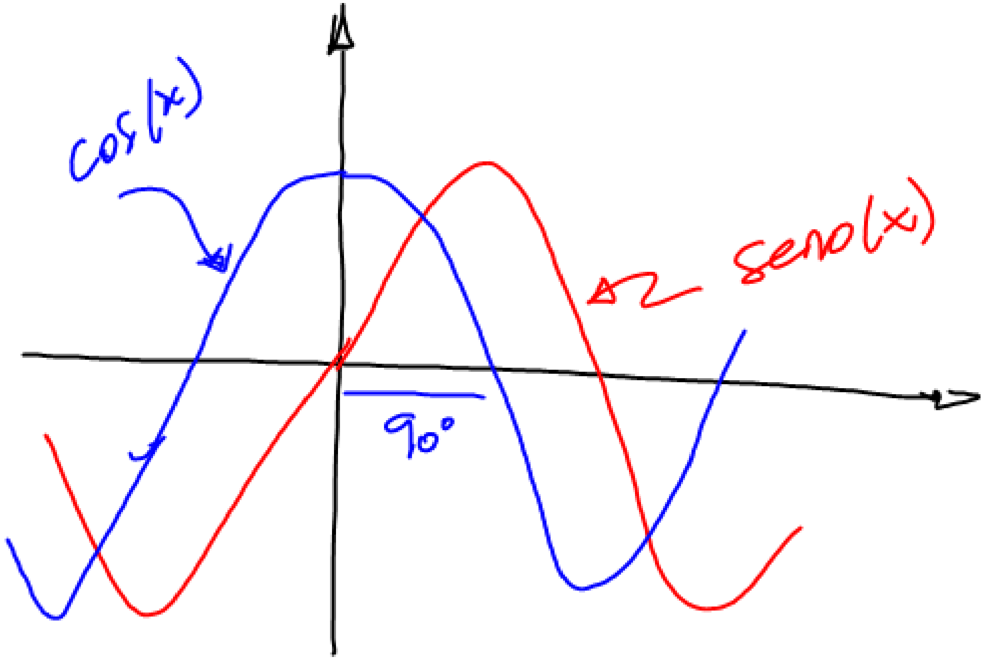


$$P(\underbrace{x, y, \alpha}_{\text{3 datos}})$$

Descartes



$$P(\underbrace{x, y}_{\text{2 datos}})$$



$g := x^2$

$h := e^{3x}$

$$f(x) = c + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \operatorname{sen} \frac{n\pi}{L} x \right)$$

$a < x < b$

$L = \frac{b-a}{2}$

$f(x)$ $\quad L$

$$C = \frac{a_0}{2} \qquad a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{1}{L}\int_{a}^{b} f(x)\,dx$$

$$a_n = h(n) = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

$$b_n = j(n) \rightarrow = \frac{1}{L}\int_{-L}^{L} f(x)\,\mathrm{sen}\left(\frac{n\pi}{L}x\right)dx$$

$f(x) = x^2 - 4 \qquad -2 < x < 2 \qquad L = 2$

$a_0 = \frac{1}{2}\int_{-2}^{2} (x^2-4)\,dx \Rightarrow \frac{1}{2}\left[\int x^2 dx - 4\int dx\right]_{-2}^{2}$

$= \frac{1}{2}\left[\frac{x^3}{3} - 4x\right]_{-2}^{2} \Rightarrow \frac{1}{2}\left[\frac{8}{3} - 8 - \left(-\frac{8}{3}+8\right)\right]$

$a_0 = \frac{1}{2}\left(\frac{16}{3} - 16\right) \Rightarrow \frac{8}{3} - 8 \Rightarrow -\frac{16}{3} \qquad \boxed{C = \frac{a_0}{2} \Rightarrow -\frac{8}{3}}$

$a_n = \frac{1}{2}\int_{-2}^{2} (x^2-4)\cos\left(\frac{n\pi}{2}x\right)dx$

$a_n = \frac{1}{2}\left[\int x^2\cos\left(\frac{n\pi}{2}x\right)dx - 4\int \cos\left(\frac{n\pi}{2}x\right)dx\right]_{-2}^{2}$

$\int \cos\left(\frac{n\pi}{2}x\right)dx = \frac{2}{n\pi}\int \cos\left(\frac{n\pi}{2}x\right)\left(\frac{n\pi}{2}dx\right)$

$= \frac{2}{n\pi}\,\text{sen}\left(\frac{n\pi}{2}x\right)$

$\int x^2\cos\left(\frac{n\pi}{2}x\right)dx = \frac{2}{n\pi}x^2\,\text{sen}\left(\frac{n\pi}{2}x\right) - \frac{4}{n\pi}\int x\,\text{sen}\left(\frac{n\pi}{2}x\right)dx$

$u = x^2 \quad du = 2x\,dx$

$dV = \cos\left(\frac{n\pi}{2}x\right)dx \quad V = \frac{2}{n\pi}\text{sen}\left(\frac{n\pi}{2}x\right)$

$\int x\,\text{sen}\left(\frac{n\pi}{2}x\right)dx = -\frac{2}{n\pi}x\cos\left(\frac{n\pi}{2}x\right) + \frac{2}{n\pi}\int \cos\left(\frac{n\pi}{2}x\right)dx$

$u = x \quad du = dx$

$dV = \text{sen}\left(\frac{n\pi}{2}x\right)dx \quad V = -\frac{2}{n\pi}\cos\left(\frac{n\pi}{2}x\right)$

$\int x\,\text{sen}\left(\frac{n\pi}{2}x\right)dx = -\frac{2}{n\pi}x\cos\left(\frac{n\pi}{2}x\right) + \frac{4}{n^2\pi^2}\left(\text{sen}\left(\frac{n\pi}{2}x\right)\right)$

$\int x^2\cos\left(\frac{n\pi}{2}x\right)dx = \frac{2}{n\pi}x^2\,\text{sen}\left(\frac{n\pi}{2}x\right) - \frac{4}{n\pi}\left(-\frac{2}{n\pi}x\cos\left(\frac{n\pi}{2}x\right) + \frac{4}{n^2\pi^2}\text{sen}\left(\frac{n\pi}{2}x\right)\right)$

$\int x^2\cos\left(\frac{n\pi}{2}x\right)dx = \frac{2}{n\pi}x^2\,\text{sen}\left(\frac{n\pi}{2}x\right) + \frac{8}{n^2\pi^2}x\cos\left(\frac{n\pi}{2}x\right) - \frac{16}{n^3\pi^3}\text{sen}\left(\frac{n\pi}{2}x\right)$

$a_n = \frac{1}{2}\left(\frac{8}{n\pi}\left(\text{sen}(n\pi) + \text{sen}(n\pi)\right) + \frac{16}{n^2\pi^2}\left(\cos(n\pi) + \cos(n\pi)\right) - \right.$

$\left. -\frac{16}{n^3\pi^3}\left(\text{sen}(n\pi) + \text{sen}\,n\pi\right)\right) - 4\left(\frac{2}{n\pi}\left(\text{sen}(n\pi) + \text{sen}(n\pi)\right)\right)$

$a_n = \left(\frac{8}{n\pi} - \frac{16}{n^3\pi^3}\right)\text{sen}(n\pi) + \frac{16}{n^2\pi^2}\left(\cos(n\pi)\right) - \frac{8}{n\pi}\left(\text{sen}(n\pi)\right)$

$n \in \mathbb{Z} \qquad \text{sen}(n\pi) = 0 \qquad \cos(n\pi) = (-1)^n$

$\boxed{a_n = \frac{16}{n^2\pi^2}(-1)^n}$

Simetría

Una función es PAR cuando

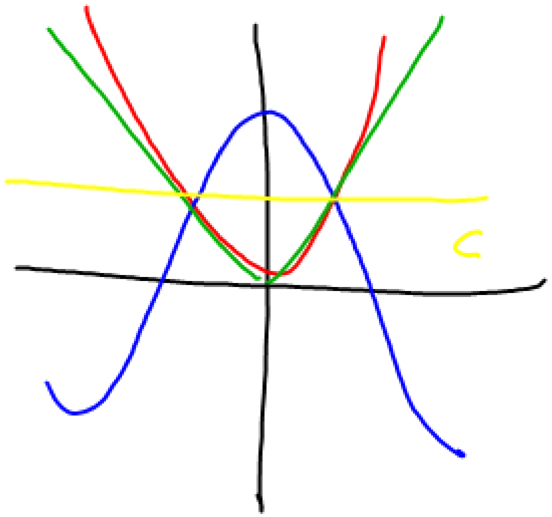
$$f(-x) = f(x) \quad -L < x < L$$

$f = x^2$

$f = \cos(bx)$

$f = |x|$

$f = c \quad c \in \mathbb{R}$



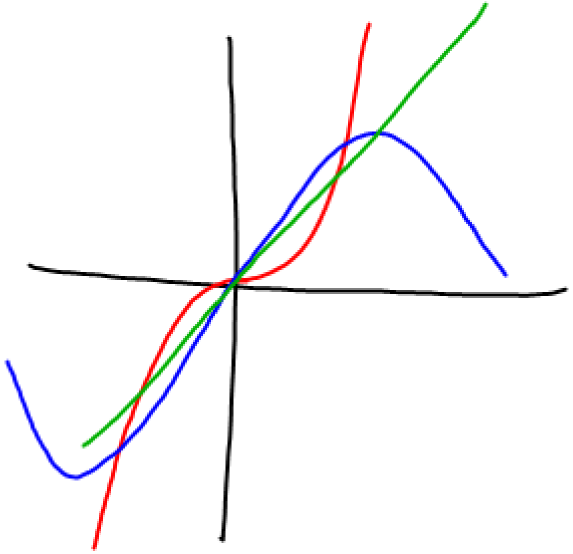
una función IMPAR

$$f(-x) = -f(x) \quad -L < x < L$$

$f = x^3$

$f = \operatorname{sen}(bx)$

$f = x$



propiedades

$$\langle par \rangle * \langle par \rangle = \langle par \rangle$$

$$\langle impar \rangle * \langle impar \rangle = \langle par \rangle$$

$$\langle par \rangle \cdot \langle impar \rangle = \langle impar \rangle$$

$$\int_{-L}^{L} \langle impar \rangle = 0.$$

para una función PAR

$\Updownarrow$

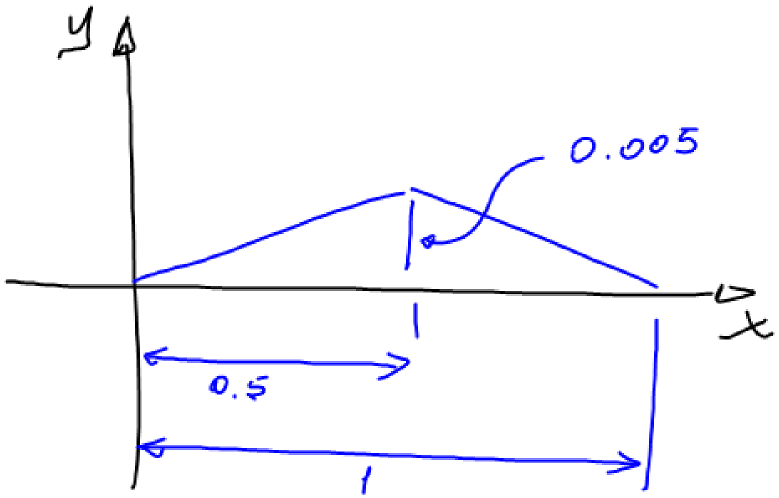
SERIE COSENO

---

para una función IMPAR

$\Updownarrow$

SERIE SENO.



$$\varphi(x,t)$$

$$\left.\begin{array}{l} y(0,t)=0 \\ y(1,t)=0 \end{array}\right\} \text{ cond. frontera}$$

$t=0$

$$\varphi(x,0) = \begin{cases} \dfrac{0.005}{0.5}x & ;\ 0 \le x < 0.5 \\ -\dfrac{0.005}{0.5}x + 0.01 & ;\ 0.5 \le x \le 1 \end{cases}$$

$$\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0.$$

condiciones iniciales en el tiempo.