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> restart
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PROBLEMA DE LA CUERDA DE GUITARRA

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> Ecuacion := diff(y(x, t), t$2) = c·2·diff(y(x, t), x$2)
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$$Ecuacion := \frac{\partial^2}{\partial t^2} y(x, t) = c^2 \left(\frac{\partial^2}{\partial x^2} y(x, t) \right) \quad (1)$$

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> c := 1
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$$c := 1 \quad (2)$$

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> Ecuacion;
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$$\frac{\partial^2}{\partial t^2} y(x, t) = \frac{\partial^2}{\partial x^2} y(x, t) \quad (3)$$

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> CondicionesFrontera := y(0, t) = 0, y(1, t) = 0
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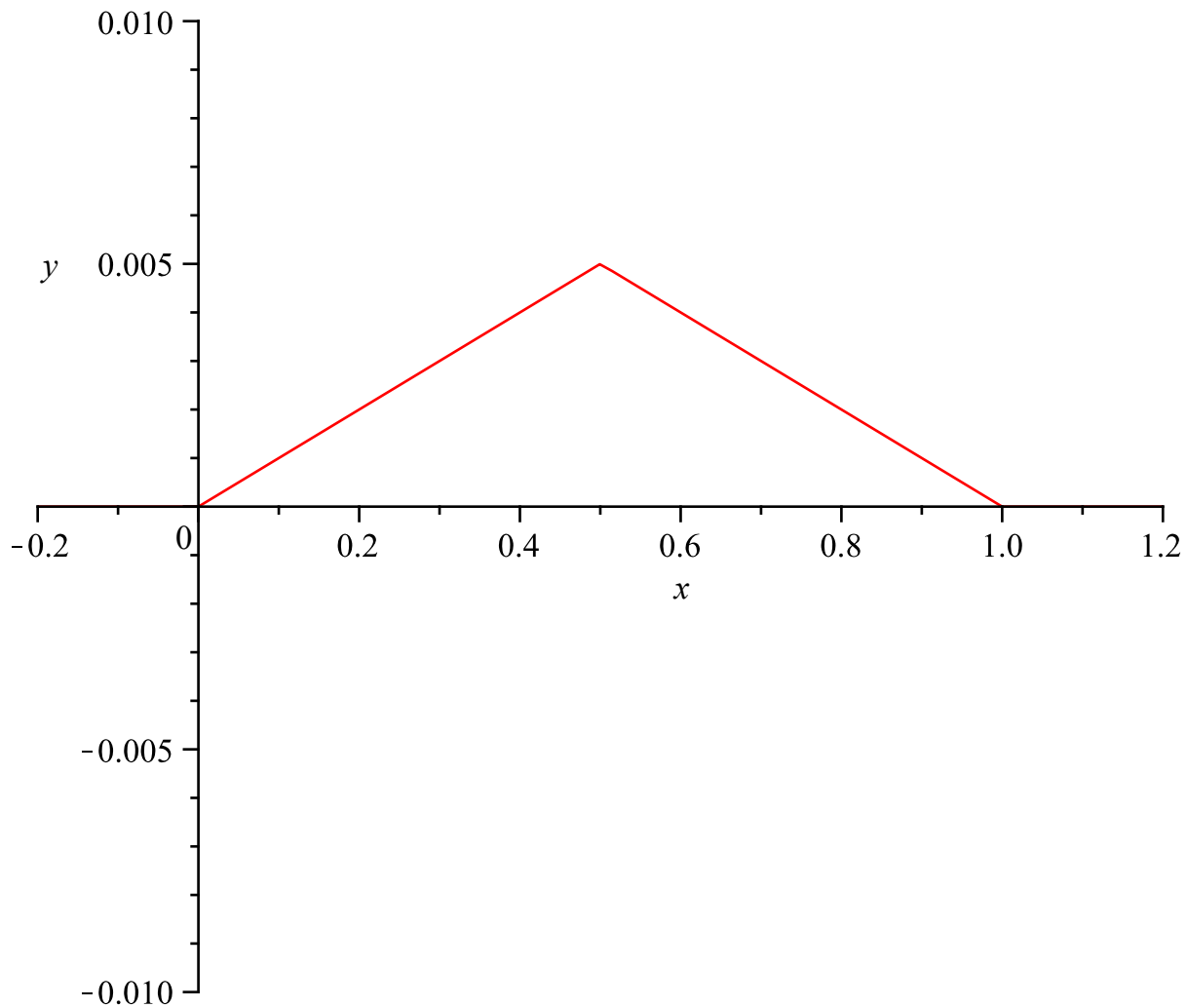
$$CondicionesFrontera := y(0, t) = 0, y(1, t) = 0 \quad (4)$$

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> CondicionInicialTrayectoria := f = \left(\frac{5}{1000}\right) \cdot x \cdot \text{Heaviside}(x) - \frac{2 \cdot \left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot \left(x - \frac{5}{10}\right)
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$$\cdot \text{Heaviside}\left(x - \frac{5}{10}\right) + \frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)} \cdot (x - 1) \cdot \text{Heaviside}(x - 1);$$

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plot(rhs(CondicionInicialTrayectoria), x=-0.2..1.2, y=-0.01..0.01)
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$$CondicionInicialTrayectoria := f = \frac{1}{100} x \text{Heaviside}(x) - \frac{1}{50} \left(x - \frac{1}{2}\right) \text{Heaviside}\left(x - \frac{1}{2}\right) + \frac{1}{100} (x - 1) \text{Heaviside}(x - 1)$$



> *CondicionInicialVelocidad* := *DerYcero* = 0;
CondicionInicialVelocidad := *DerYcero* = 0 (5)

>

MÉTODO DE SEPARACIÓN DE VARIABLES

> *Ecuacion*

$$\frac{\partial^2}{\partial t^2} y(x, t) = \frac{\partial^2}{\partial x^2} y(x, t) \quad (6)$$

> *EcuacionInicial* := *eval*(*subs*(*y*(*x*, *t*) = *F*(*x*) · *G*(*t*), *Ecuacion*))

$$EcuacionInicial := F(x) \left(\frac{d^2}{dt^2} G(t) \right) = \left(\frac{d^2}{dx^2} F(x) \right) G(t) \quad (7)$$

> *EcuacionSeparada* := $\frac{lhs(EcuacionInicial)}{F(x) \cdot G(t)} = \frac{rhs(EcuacionInicial)}{F(x) \cdot G(t)}$

$$EcuacionSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{\frac{d^2}{dx^2} F(x)}{F(x)} \quad (8)$$

> *EcuacionX* := *rhs*(*EcuacionSeparada*) = *alpha*; *EcuacionT* := *lhs*(*EcuacionSeparada*) = *alpha*

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$$

$$EcuacionT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (9)$$

> para alpha=0

$$> SolucionXcero := dsolve(subs(alpha=0, EcuacionX))$$

$$SolucionXcero := F(x) = _C1 x + _C2 \quad (10)$$

$$> CondicionesFronteraX := F(0) = 0, F(1) = 0;$$

$$CondicionesFronteraX := F(0) = 0, F(1) = 0 \quad (11)$$

$$> SolucioParticularX := dsolve(\{subs(alpha=0, EcuacionX), CondicionesFronteraX\})$$

$$SolucioParticularX := F(x) = 0 \quad (12)$$

> para alpha positivo

$$> SolucionXpos := dsolve(subs(alpha=beta \cdot 2, EcuacionX))$$

$$SolucionXpos := F(x) = _C1 e^{\beta x} + _C2 e^{-\beta x} \quad (13)$$

$$> SolucionPositivaX := dsolve(\{subs(alpha=beta \cdot 2, EcuacionX), CondicionesFronteraX\})$$

$$SolucionPositivaX := F(x) = 0 \quad (14)$$

> para alpha negativa

$$> SolucionXneg := dsolve(subs(alpha=-beta \cdot 2, EcuacionX))$$

$$SolucionXneg := F(x) = _C1 \sin(\beta x) + _C2 \cos(\beta x) \quad (15)$$

$$> SolucionNegativaX := F(x) = \sin(n \cdot \text{Pi} \cdot x)$$

$$SolucionNegativaX := F(x) = \sin(n \pi x) \quad (16)$$

$$> SolucionNegativaT := dsolve(subs(alpha=-n \cdot 2 \cdot \text{Pi} \cdot 2, EcuacionT))$$

$$SolucionNegativaT := G(t) = _C1 \sin(n \pi t) + _C2 \cos(n \pi t) \quad (17)$$

$$> SolucionNegativaGeneral := y(x, t) = rhs(SolucionNegativaX) \cdot rhs(SolucionNegativaT)$$

$$SolucionNegativaGeneral := y(x, t) = \sin(n \pi x) (_C1 \sin(n \pi t) + _C2 \cos(n \pi t)) \quad (18)$$

$$> SolucionGeneral := y(x, t) = \text{Sum}(\sin(n \cdot \text{Pi} \cdot x) \cdot (b_n \cdot \cos(n \cdot \text{Pi} \cdot t) + a_n \cdot \sin(n \cdot \text{Pi} \cdot t)), n = 1 \dots \text{infinity})$$

$$SolucionGeneral := y(x, t) = \sum_{n=1}^{\infty} \sin(n \pi x) (b_n \cos(n \pi t) + a_n \sin(n \pi t)) \quad (19)$$

$$> SolucionParticularInicial := eval(subs(t=0, SolucionGeneral))$$

$$SolucionParticularInicial := y(x, 0) = \sum_{n=1}^{\infty} \sin(n \pi x) b_n \quad (20)$$

$$> b_n := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \left(\frac{1}{\left(\frac{5}{10}\right)}\right) \cdot \text{int}\left(\text{rhs}(\text{CondicionInicialTrayectoria}) \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{1}\right), \right.\right.$$

$$x=0..1) \Bigg)$$

$$b_n := \frac{1}{25} \frac{\sin\left(\frac{1}{2} n \pi\right)}{n^2 \pi^2} \quad (21)$$

$$> a_n := 0;$$

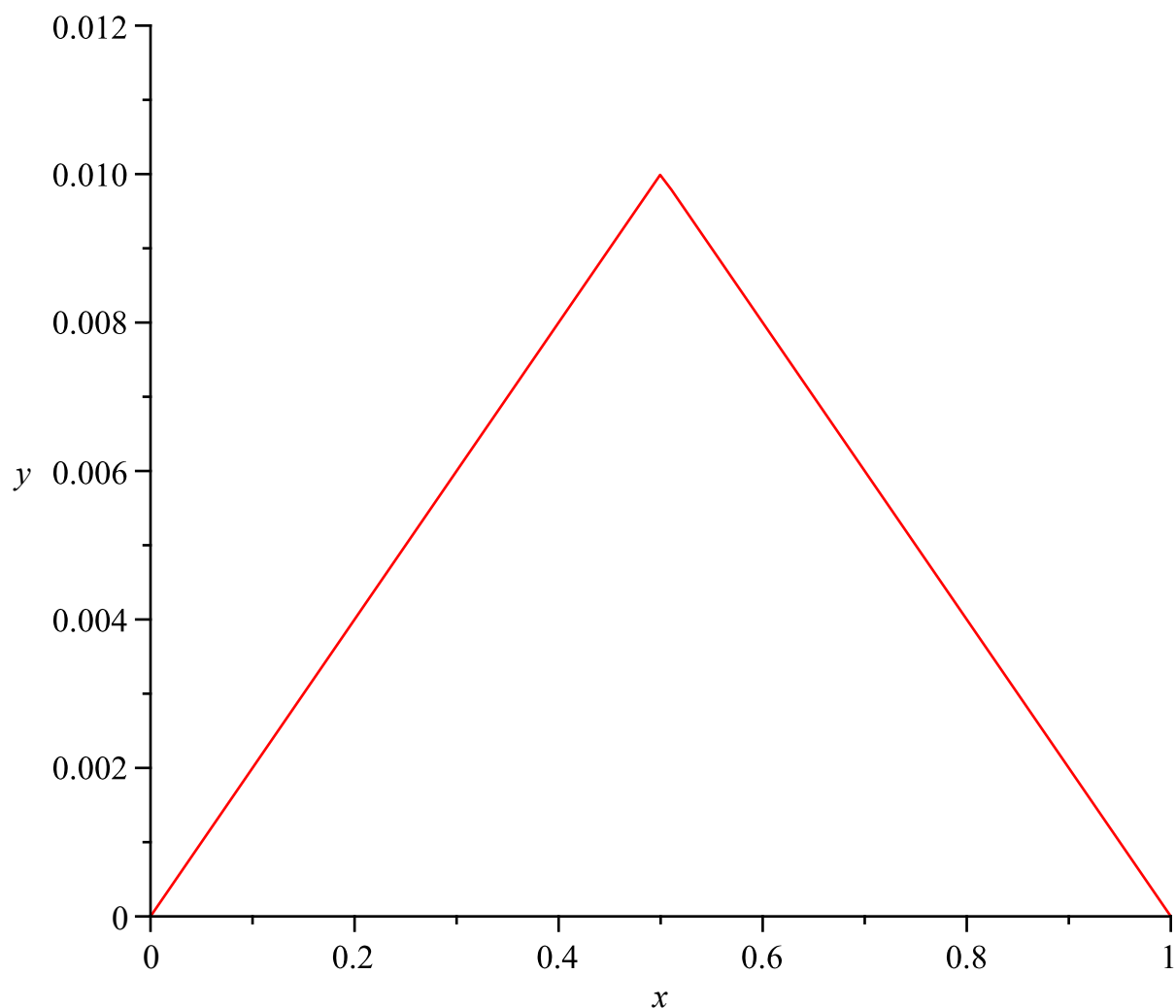
$$a_n := 0 \quad (22)$$

$$> \text{SolucionParticular} := \text{SolucionGeneral};$$

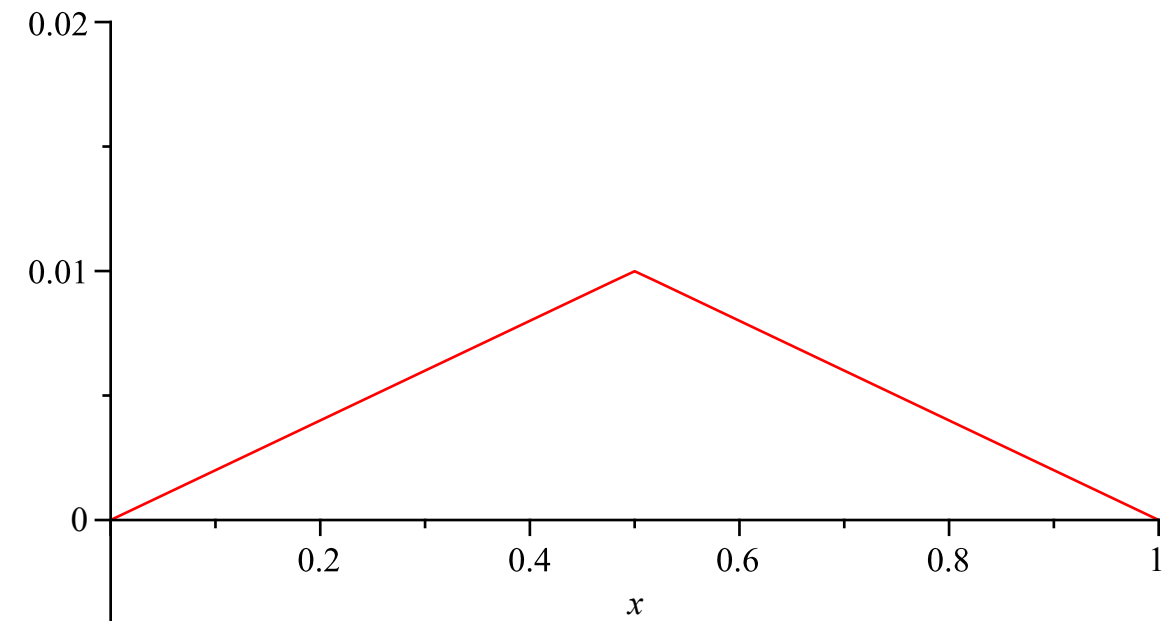
$$\text{SolucionParticular} := y(x, t) = \sum_{n=1}^{\infty} \frac{1}{25} \frac{\sin(n \pi x) \sin\left(\frac{1}{2} n \pi\right) \cos(n \pi t)}{n^2 \pi^2} \quad (23)$$

$$> \text{SolucionParticular}_{500} := \sum_{n=1}^{500} \frac{1}{25} \frac{\sin(n \pi x) \left(-\sin(n \pi) + 2 \sin\left(\frac{1}{2} n \pi\right) \right) \cos(n \pi t)}{n^2 \pi^2} :$$

$$> \text{plot}\left(\text{subs}(t=0, \text{SolucionParticular}_{500}), x=0..1, y=0..0.012\right)$$



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> with(plots) :  
> animate(SolucionParticular500, x = 0 .. 1, t = 0 .. 4, frames = 150, view = [0 .. 1, -0.02 .. 0.02])
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