(1) $$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2$$

(2) $$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2$$

de (1)

$$x_2 = \frac{1}{a_{12}}\left(\frac{dx_1}{dt} - a_{11}x_1\right)$$

$$\frac{d}{dt}\left(\ \right)$$

$$\frac{dx_2}{dt} = \frac{1}{a_{12}}\left(\frac{d^2x_1}{dt^2} - a_{11}\frac{dx_1}{dt}\right)$$

Sust en (2)

$$\frac{1}{a_{12}}\left(\frac{d^2x_1}{dt^2} - a_{11}\frac{dx_1}{dt}\right) = a_{21}x_1 + a_{22}\left(\frac{1}{a_{12}}\left(\frac{dx_1}{dt} - a_{11}x_1\right)\right)$$

$$\frac{d^2x_1}{dt^2} - a_{11}\frac{dx_1}{dt} = a_{21}a_{12}x_1 + a_{22}\frac{dx_1}{dt} - a_{22}a_{11}x_1$$

$$\frac{d^2x_1}{dt^2} + \left(-a_{11} - a_{22}\right)\frac{dx_1}{dt} + \left(a_{22}a_{11} - a_{12}a_{21}\right)x_1 = 0$$

S(2) EDO(1) LCCH $\Longrightarrow$ EDO(2) LCCH.

MÉTODO DE SUSTITUCION

S(n) EDO(1) L $\longrightarrow$ EDO(n) L

$$\frac{d^4y(t)}{dt^4} - 6\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} - 3\frac{dy(t)}{dt} + 11y(t) = 0$$

EDO(4) LCCH $\longrightarrow$ S(4) EDO(1) LCCH.

$$y(t) \Rightarrow y_1(t)$$

$$\frac{dy(t)}{dt} \Rightarrow \frac{dy_1(t)}{dt} = y_2(t)$$

$$\frac{d^2y(t)}{dt^2} \Rightarrow \frac{dy_2(t)}{dt} = y_3(t)$$

$$\frac{d^3y(t)}{dt^3} \Rightarrow \frac{dy_3(t)}{dt} = y_4(t)$$

$$\frac{d^4y(t)}{dt^4} \Rightarrow \frac{dy_4(t)}{dt}$$

$$\frac{dy_4(t)}{dt} - 6y_4(t) + 4y_3(t) - 3y_2(t) + 11y_1(t) = 0$$

$$\frac{dy_4(t)}{dt} = -11y_1(t) + 3y_2(t) - 4y_3(t) + 6y_4(t)$$

$$\frac{dy_1(t)}{dt} = (0)y_1(t) + (1)y_2(t) + (0)y_3(t) + (0)y_4(t)$$

$$\frac{dy_2(t)}{dt} = (0)y_1(t) + (0)y_2(t) + (1)y_3(t) + (0)y_4(t)$$

$$\frac{dy_3(t)}{dt} = (0)y_1(t) + (0)y_2(t) + (0)y_3(t) + (1)y_4(t)$$

$$\frac{dy_4(t)}{dt} = (-11)y_1(t) + (3)y_2(t) + (-4)y_3(t) + (6)y_4$$

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -11 & 3 & -4 & 6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^k}{k!} + \dots$$

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \dots$$

$$= 2.0 + 0.5 + 0.17 \to 2.72\dots$$

$$e^{At} = I + \frac{A}{1!}t + \frac{A^2}{2!}t^2 + \dots + \frac{A^k}{k!}t^k + \dots$$

teorema Hamilton-Cayle

$[A]_{n \times n}$ Satisface a su propia ecuación característica

$$\det(A - \lambda I) = 0$$

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + a_3\lambda^{n-3} + \dots + a_n = 0$$

$$A^n = -a_n I - a_{n-1}A - a_{n-2}A^2 - \dots - a_1 A^{n-1} = \begin{bmatrix} 0 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0$$

$$\det(A - \lambda I) = 0$$

$$(2-\lambda)(4-\lambda) - (1)(3) = 0$$

$$\lambda^2 - 6\lambda + 8 - 3 = 0 \qquad \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix}$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}^2 - 6\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 18 \\ 6 & 19 \end{bmatrix} + \begin{bmatrix} -12 & -18 \\ -6 & -24 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7-12+5 & 18-18+0 \\ 6-6+0 & 19-24+5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A_{n\times n}$

$$e^{At} = \beta_0(t) I + \beta_1(t) A + \dots + \beta_{n-1}(t) A^{n-1}$$

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad e^{At} = \beta_0(t) I + \beta_1(t) A$$

$$\lambda^2 - 6\lambda + 5 = 0 \qquad e^{\lambda_i t} = \beta_0(t)(1) + \beta_1(t)\lambda_i$$

$\lambda_1 = 1$

$\lambda_2 = 5$

$$e^{t} = \beta_0(t)(1) + \beta_1(t)(1)$$

$$e^{5t} = \beta_0(t)(1) + \beta_1(t)(5)$$

$$e^{5t} - e^{t} = 4\beta_1(t)$$

$$\boxed{\beta_1(t) = \frac{1}{4}\left(e^{5t} - e^{t}\right)}$$

$$\beta_0(t) = e^{t} - \beta_1(t)$$

$$\beta_0(t) = e^{t} - \frac{1}{4}\left(e^{5t} - e^{t}\right)$$

$$\boxed{\beta_0(t) = \frac{5e^{t}}{4} - \frac{e^{5t}}{4}}$$

$$e^{At} = \left(\frac{5}{4}e^{t} - \frac{1}{4}e^{5t}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{1}{4}e^{5t} - \frac{1}{4}e^{t}\right)\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} -1+2 & 3 \\ 1 & -1+4 \end{bmatrix}\frac{e^{5t}}{4} + \begin{bmatrix} 5-2 & -3 \\ -1 & 5-4 \end{bmatrix}\frac{e^{t}}{4}$$

$$\boxed{e^{At} = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}\frac{e^{5t}}{4} + \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}\frac{e^{t}}{4}}$$

$$ME := \begin{bmatrix} \frac{3}{4}\,e^{t} + \frac{1}{4}\,e^{5\,t} & \frac{3}{4}\,e^{5\,t} - \frac{3}{4}\,e^{t} \\ \frac{1}{4}\,e^{5\,t} - \frac{1}{4}\,e^{t} & \frac{1}{4}\,e^{t} + \frac{3}{4}\,e^{5\,t} \end{bmatrix}$$

$$\boxed{e^{At} = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} \frac{e^{5t}}{4} + \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \frac{e^{t}}{4}}$$