> *restart*
>

$$\frac{dx_1}{dt} = 3x_1 - 6x_2 + 8e^{2t} \qquad X_1(0) = C_1$$

$$\frac{dx_2}{dt} = -9x_1 + 18x_2 - 24e^{2t} \qquad X_2(0) = C_2$$

> *Sistema := diff $(x_1(t), t) = 3 \cdot x_1(t) - 6 \cdot x_2(t) + 8 \cdot \exp(2 \cdot t)$, diff $(x_2(t), t) = -9 \cdot x_1(t) + 18$*
> *$\cdot x_2(t) - 24 \cdot \exp(2 \cdot t)$: Sistema$_1$; Sistema$_2$*

$$\frac{d}{dt} x_1(t) = 3\,x_1(t) - 6\,x_2(t) + 8\,e^{2\,t}$$

$$\frac{d}{dt} x_2(t) = -9\,x_1(t) + 18\,x_2(t) - 24\,e^{2\,t} \tag{1}$$

> *Condiciones := $x_1(0) = C_1, x_2(0) = C_2$*

$$Condiciones := x_1(0) = C_1, x_2(0) = C_2 \tag{2}$$

> *SolucionGeneral := dsolve({Sistema, Condiciones}) : evalf (expand(SolucionGeneral$_1$), 3);*
> *evalf (expand(SolucionGeneral$_2$), 3)*

$$x_1(t) = -0.421 \left(e^t\right)^2 + 0.421 \left(e^t\right)^{21} - 0.286 \left(e^t\right)^{21} C_2 + 0.143 \left(e^t\right)^{21} C_1 + 0.286 \, C_2$$
$$+ \, 0.857 \, C_1$$

$$x_2(t) = 1.26 \left(e^t\right)^2 - 1.26 \left(e^t\right)^{21} + 0.857 \left(e^t\right)^{21} C_2 - 0.429 \left(e^t\right)^{21} C_1 + 0.143 \, C_2 + 0.429 \, C_1 \tag{3}$$

> *AA := array([[3, -6], [-9, 18]])*

$$AA := \begin{bmatrix} 3 & -6 \\ -9 & 18 \end{bmatrix} \tag{4}$$

> *with(linalg) :*
> *MatExp := exponential(AA, t)*

$$MatExp := \begin{bmatrix} \dfrac{6}{7} + \dfrac{1}{7} \, e^{21\,t} & -\dfrac{2}{7} \, e^{21\,t} + \dfrac{2}{7} \\[2ex] -\dfrac{3}{7} \, e^{21\,t} + \dfrac{3}{7} & \dfrac{1}{7} + \dfrac{6}{7} \, e^{21\,t} \end{bmatrix} \tag{5}$$

> *BB := array([8·exp(2 t), -24·exp(2 t)])*

$$BB := \begin{bmatrix} 8\,e^{2\,t} & -24\,e^{2\,t} \end{bmatrix} \tag{6}$$

> *Xcero := array([C$_1$, C$_2$])*

$$Xcero := \begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{7}$$

> $SolHom := evalm(MatExp \&* Xcero) : SolHom_1; SolHom_2$

$$\left(\frac{6}{7} + \frac{1}{7} e^{21t}\right) C_1 + \left(-\frac{2}{7} e^{21t} + \frac{2}{7}\right) C_2$$

$$\left(-\frac{3}{7} e^{21t} + \frac{3}{7}\right) C_1 + \left(\frac{1}{7} + \frac{6}{7} e^{21t}\right) C_2 \tag{8}$$

> $Comprobacion_1 := map(rcurry(eval, t ='0'), SolHom)$

$$Comprobacion_1 := \begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{9}$$

> $MatExpTau := map(rcurry(eval, t ='t - tau'), MatExp)$

$$MatExpTau := \begin{bmatrix} \frac{6}{7} + \frac{1}{7} e^{21t - 21\tau} & -\frac{2}{7} e^{21t - 21\tau} + \frac{2}{7} \\[2mm] -\frac{3}{7} e^{21t - 21\tau} + \frac{3}{7} & \frac{1}{7} + \frac{6}{7} e^{21t - 21\tau} \end{bmatrix} \tag{10}$$

> $BBtau := map(rcurry(eval, t ='tau'), BB)$

$$BBtau := \begin{bmatrix} 8 e^{2\tau} & -24 e^{2\tau} \end{bmatrix} \tag{11}$$

> $ProdTau := evalm(MatExpTau \&* BBtau) : ProdTau_1; ProdTau_2$

$$8 \left(\frac{6}{7} + \frac{1}{7} e^{21t - 21\tau}\right) e^{2\tau} - 24 \left(-\frac{2}{7} e^{21t - 21\tau} + \frac{2}{7}\right) e^{2\tau}$$

$$8 \left(-\frac{3}{7} e^{21t - 21\tau} + \frac{3}{7}\right) e^{2\tau} - 24 \left(\frac{1}{7} + \frac{6}{7} e^{21t - 21\tau}\right) e^{2\tau} \tag{12}$$

> $IntTau := map(int, ProdTau, tau = 0 ..t) : IntTau_1; IntTau_2$

$$\frac{8}{19} e^{21t} - \frac{8}{19} e^{2t}$$

$$-\frac{24}{19} e^{21t} + \frac{24}{19} e^{2t} \tag{13}$$

> $Comprobacion_2 := map(rcurry(eval, t ='0'), IntTau)$

$$Comprobacion_2 := \begin{bmatrix} 0 & 0 \end{bmatrix} \tag{14}$$

> $SolGral := evalm(SolHom + IntTau) : SolUno := xx_1(t) = SolGral_1; SolDos := xx_2(t) = SolGral_2$

$$SolUno := xx_1(t) = \left(\frac{6}{7} + \frac{1}{7} e^{21t}\right) C_1 + \left(-\frac{2}{7} e^{21t} + \frac{2}{7}\right) C_2 + \frac{8}{19} e^{21t} - \frac{8}{19} e^{2t}$$

$$SolDos := xx_2(t) = \left(-\frac{3}{7} e^{21t} + \frac{3}{7}\right) C_1 + \left(\frac{1}{7} + \frac{6}{7} e^{21t}\right) C_2 - \frac{24}{19} e^{21t} + \frac{24}{19} e^{2t} \tag{15}$$

> $SolucionGeneral_1; SolucionGeneral_2$

$$x_1(t) = -\frac{8}{19} e^{2t} + \frac{1}{21} e^{21t} \left(\frac{168}{19} - 6 C_2 + 3 C_1\right) + \frac{2}{7} C_2 + \frac{6}{7} C_1$$

$$x_2(t) = \frac{24}{19} e^{2t} - \frac{1}{7} e^{21t} \left(\frac{168}{19} - 6 C_2 + 3 C_1\right) + \frac{1}{7} C_2 + \frac{3}{7} C_1 \tag{16}$$

> $SolInic := map\left(rcurry\left(eval, C_1 ='1'\right), SolGral\right)$

$$\tag{17}$$

$$SolInic := \left[\frac{6}{7} + \frac{75}{133} e^{21t} + \left(-\frac{2}{7} e^{21t} + \frac{2}{7} \right) C_2 - \frac{8}{19} e^{2t}, -\frac{225}{133} e^{21t} + \frac{3}{7} + \left(\frac{1}{7} \right.\right.$$ **(17)**

$$\left.\left. + \frac{6}{7} e^{21t} \right) C_2 + \frac{24}{19} e^{2t} \right]$$

> $SolPart := map\left(rcurry(eval, C_2 = '2'), SolInic \right) : SolPart_1; SolPart_2$

$$\frac{10}{7} - \frac{1}{133} e^{21t} - \frac{8}{19} e^{2t}$$

$$\frac{3}{133} e^{21t} + \frac{5}{7} + \frac{24}{19} e^{2t}$$ **(18)**

> $SolucionInicial := map\left(rcurry(eval, C_1 = '1'), SolucionGeneral \right)$

$$SolucionInicial := \left\{ x_1(t) = -\frac{8}{19} e^{2t} + \frac{1}{21} e^{21t} \left(\frac{225}{19} - 6 C_2 \right) + \frac{2}{7} C_2 + \frac{6}{7}, x_2(t) = \frac{24}{19} e^{2t} \right.$$ **(19)**

$$\left. - \frac{1}{7} e^{21t} \left(\frac{225}{19} - 6 C_2 \right) + \frac{1}{7} C_2 + \frac{3}{7} \right\}$$

> $SolucionParticular := map\left(rcurry(eval, C_2 = '2'), SolucionInicial \right) : SolucionParticular_1;$
> $\qquad SolucionParticular_2$

$$x_1(t) = \frac{10}{7} - \frac{1}{133} e^{21t} - \frac{8}{19} e^{2t}$$

$$x_2(t) = \frac{3}{133} e^{21t} + \frac{5}{7} + \frac{24}{19} e^{2t}$$ **(20)**

>