Laplace's Transform Proprierties.

① if $\mathcal{L}\{f\} = F$ and $\mathcal{L}\{g\} = G$.

then for $a, b \in \mathbb{R}$

Linnear $\quad \mathcal{L}\{af + bg\} = aF + bG$

② $\mathcal{L}\{f(at)\} = \dfrac{1}{a} F\left(\dfrac{s}{a}\right)$

$a \in \mathbb{R}$

$$\mathcal{L}\{e^t\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{a}\left(\frac{1}{\frac{s}{a}-1}\right) \Rightarrow \frac{1}{a}\left(\frac{1}{\frac{s-a}{a}}\right)$$

$$= \frac{1}{a}\left(\frac{a}{s-a}\right)$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{\sin(t)\} = \frac{1}{s^2+1^2}$$

$$\mathcal{L}\{\sin(3t)\} = \frac{1}{3}\left(\frac{1}{\left(\frac{s}{3}\right)^2+1^2}\right)$$

$$= \frac{1}{3}\left(\frac{1}{\frac{s^2}{9}+1}\right)$$

$$= \frac{1}{3}\left(\frac{1}{\frac{s^2+9}{9}}\right)$$

$$= \frac{1}{3}\left(\frac{9}{s^2+9}\right)$$

$$= \frac{3}{s^2+(3)^2}$$

$$\mathcal{L}\{\sin(bt)\} = \frac{b}{s^2+b^2}$$

③

$$\mathcal{L}\left\{ f'(t) \right\} = s\mathcal{L}\left\{ f(t) \right\} - f(0)$$

$$\mathcal{L}\left\{ f^{n}(t) \right\} = s^{n}\mathcal{L}\left\{ f(t) \right\} - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left\{ f''(0) \right\} = s^{2}\mathcal{L}\left\{ f(t) \right\} - sf(0) - f'(0)$$

$$\mathcal{L}\left\{ f'''(0) \right\} = s^{3}\mathcal{L}\left\{ f(t) \right\} - s^{2}f(0) - sf'(0) - f''(0)$$

$$\mathcal{L}\left\{ f^{(6)}(0) \right\} = s^{6}\mathcal{L}\left\{ f(t) \right\} - s^{5}f(0) - s^{4}f'(0) - s^{3}f''(0) - s^{2}f'''(0) - sf^{(4)}(0) - f^{(5)}(0)$$

inverse Laplace's Transform.

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

$$F \in \mathbb{R}$$
$$s \in \mathbb{C}$$
$$f, t \in \mathbb{R}$$

$$\boxed{f(t)} = \mathcal{L}^{-1}\{F(s)\} = \lim_{b \to \infty} \int_{a-ib}^{a+ib} e^{st} F(s)\, ds$$

is not unique, always

USA and Japan, Australia

$$\mathcal{L}\{f(t)\} = F(s) \qquad \mathcal{L}^{-1}\{F(s)\} = f(t)$$

Europe

$$\mathcal{L}\{f(t)\} = F(p)$$

④

$$\mathcal{L}^{-1}\left\{ F'(s) \right\} = -t\, f(t)$$

$$\mathcal{L}^{-1}\left\{ F^{(n)}(s) \right\} = (-1)^n\, t^n\, f(t)$$

⑤

$$\mathcal{L}\left\{ \int_0^t f(\alpha)\, d\alpha \right\} = \frac{F(s)}{s}$$

⑥ $$\mathcal{L}^{-1}\left\{ \int_s^\infty F(\alpha)\, d\alpha \right\} = \frac{f(t)}{t}$$

Existence of Laplace's Transform Theorem.

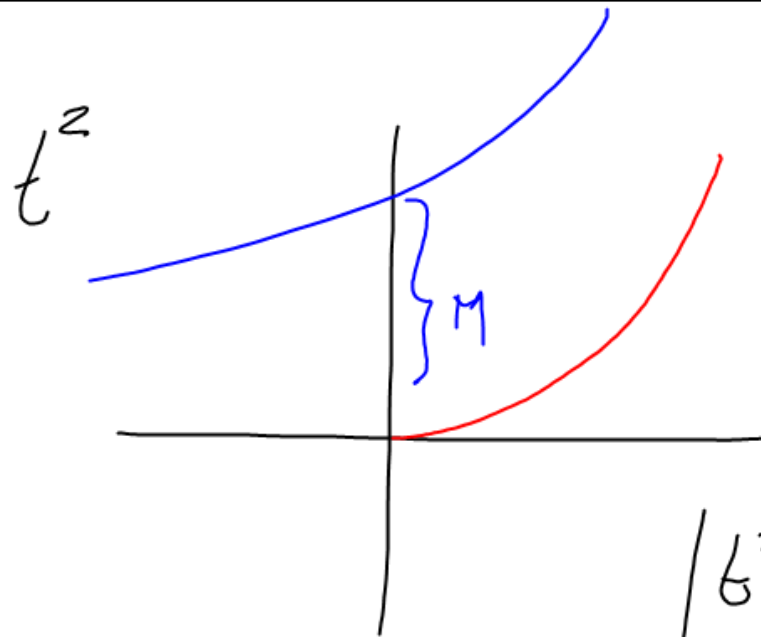If $f(t)$ is a class "A" function

exist $F(s) = L\{f(t)\}$.

A class "A" function is when

1) $f(t)$ is exponential type function

2) $f(t)$ is sectional continous function.

1) exponential type when

$$\left|f(t)\right| \leq Me^{At} \qquad M, A \in \mathbb{R}$$

t^2

$\}M$

$$\left|t^2\right| \le Me^{At}$$

$$\mathcal{L}\left\{f(t)\right\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$\boxed{e^{t^n}} \quad n>1 \qquad \left|e^{t^2}\right| \not\le Me^{At}$$

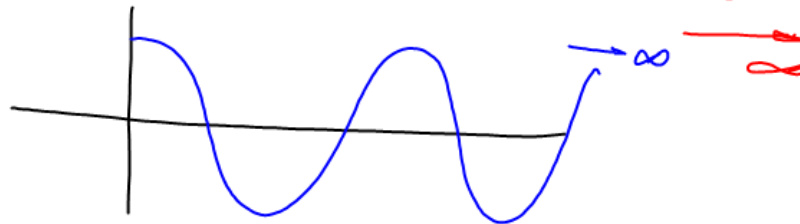2) is sectional continous order
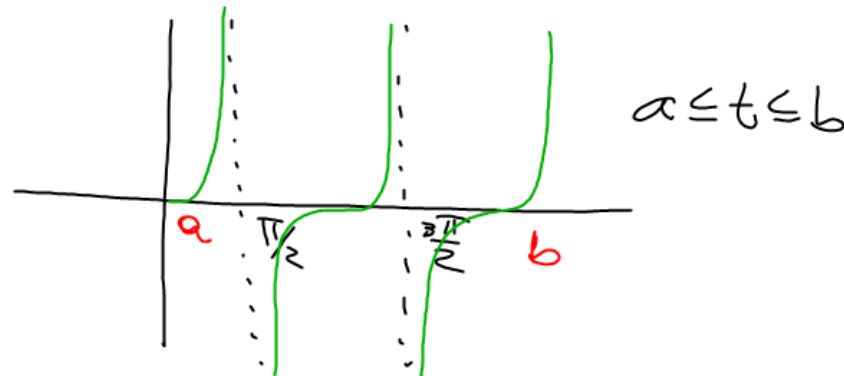
$$\frac{df}{dt}$$

f(t)

you need than

f is totaly continous.

sin(t)

cos(t)

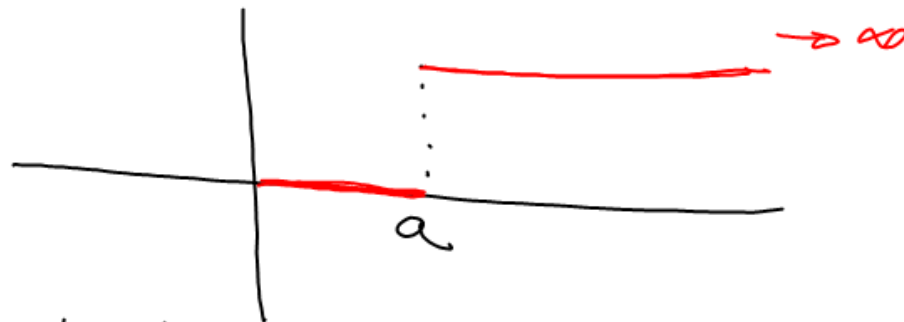
$\rightarrow \infty$ ∞

tan(t)

$a \leq t \leq b$

a $\frac{\pi}{2}$ $\frac{3\pi}{2}$ b
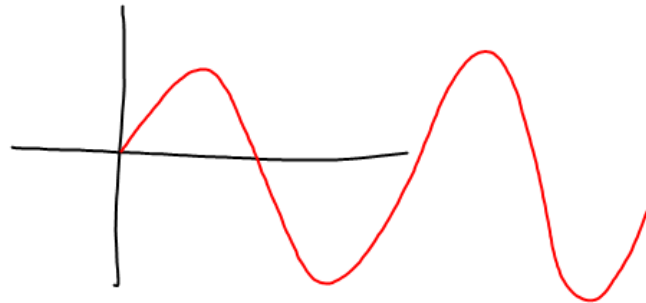
unity step function

$$\mu(t-a) = \begin{cases} 0 & ; \ t < a \\ 1 & ; \ a < t < \infty \end{cases}$$

$$\boxed{\text{Heaviside}(t-a)}$$

$$\mu(t-1) - \mu(t-2) =$$

$$100\,1\,000\,\mu$$

$$\mu(t) - \mu(t-2) + \mu(t-6) - \mu(t-7) + \mu(t-9) - \mu(t-10)$$

CtE

Lamp.
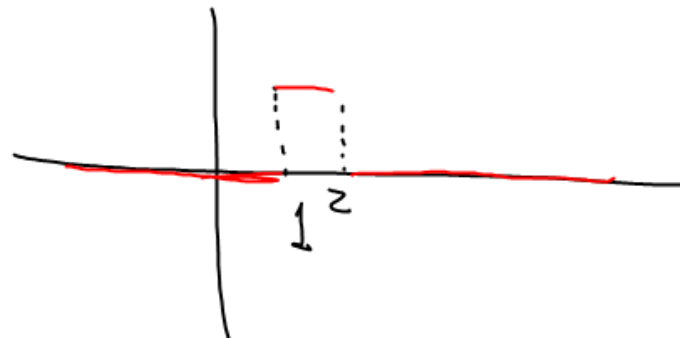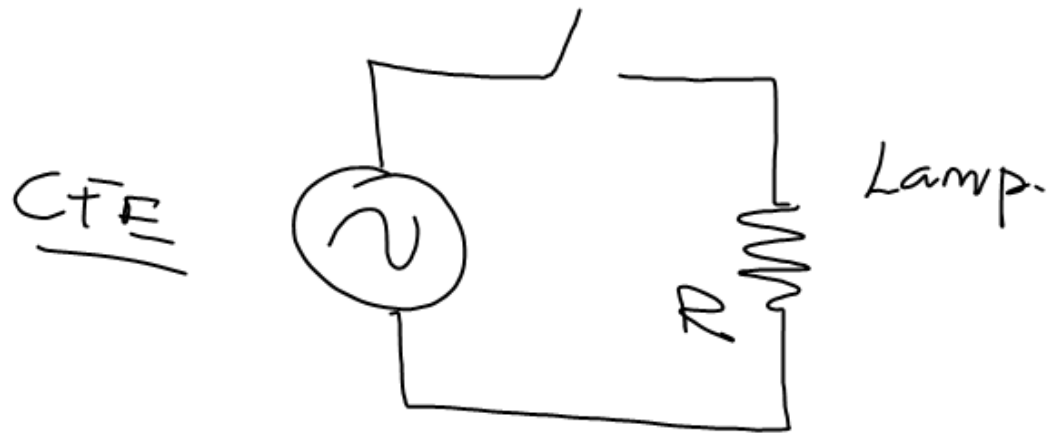
$$RI = \mu(t-a)\,120\,\sin(60(\pi t))$$

slope function

$$r(t-a) = \begin{cases} 0 & : t < a \\ (t-a) & ; t > a \end{cases}$$

$$\boxed{(t-a)\,Heaviside(t-a)}$$

$$\frac{d}{dt} \, r(t-a) = u(t-a)$$

$$\frac{d}{dt} \, u(t-a) = \delta(t-a)$$