```
> restart
>
```

$$F(s) = \frac{e^{-4s}}{(s+3)^5} \qquad G(s) = \frac{1}{s^2-4s+8}$$

```
> F := exp(-4*s)/(s+3)^5; G := 1/(s^2-4*s+8)
```

$$F := \frac{e^{-4s}}{(s+3)^5}$$

$$G := \frac{1}{s^2-4s+8} \tag{1}$$

```
> with(inttrans):
> f := invlaplace(F, s, t)
```

$$f := \frac{1}{24}\,\text{Heaviside}(t-4)\,(t-4)^4\,e^{-3t+12} \tag{2}$$

```
> g := invlaplace(G, s, t)
```

$$g := \frac{1}{2}\,e^{2t}\sin(2t) \tag{3}$$

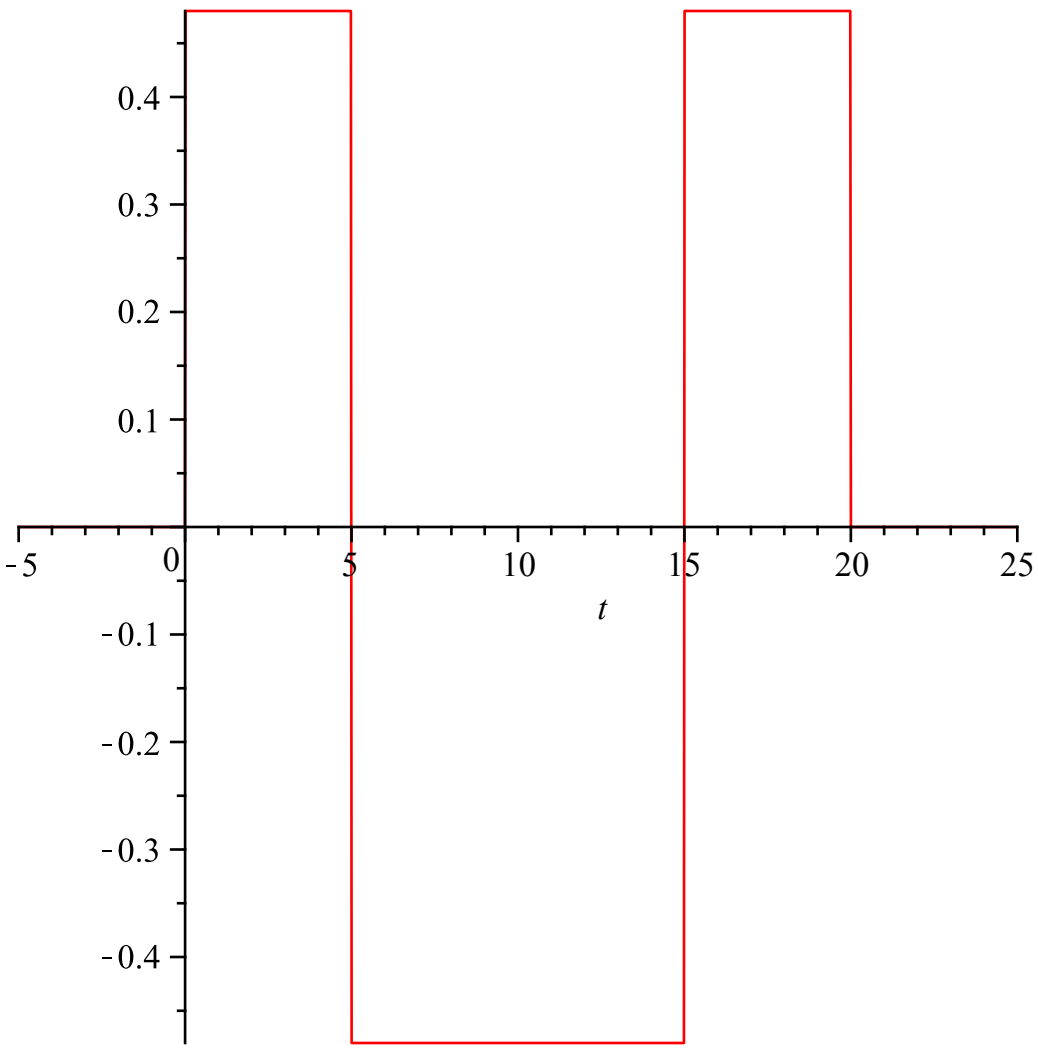
```
> restart
> Ecuacion := diff(y(t), t$3) = s(t)
```

$$Ecuacion := \frac{d^3}{dt^3}\,y(t) = s(t) \tag{4}$$

```
> s(t) := 48/100*Heaviside(t) - 2*48/100*Heaviside(t-a) + 2*48/100*Heaviside(t-3*a) - 48/100
      *Heaviside(t-4*a)
```

$$s(t) := \frac{12}{25}\,\text{Heaviside}(t) - \frac{24}{25}\,\text{Heaviside}(t-a) + \frac{24}{25}\,\text{Heaviside}(t-3a) - \frac{12}{25}\,\text{Heaviside}(t-4a) \tag{5}$$

```
> plot(subs(a = 5, s(t)), t = -5..25)
```



> *Ecuacion*

$$\frac{d^3}{dt^3} y(t) = \frac{12}{25}\,\text{Heaviside}(t) - \frac{24}{25}\,\text{Heaviside}(t-a) + \frac{24}{25}\,\text{Heaviside}(t-3\,a) - \frac{12}{25}\,\text{Heaviside}(t-4\,a) \quad \textbf{(6)}$$

> $CondIniciales := y(0)=0, D(y)(0)=0, D(D(y))(0)=0$

$$CondIniciales := y(0)=0, D(y)(0)=0, D^{(2)}(y)(0)=0 \quad \textbf{(7)}$$

> $with(inttrans)$:

> $TransLapEcuacion := subs(CondIniciales, laplace(Ecuacion, t, s))$

$$TransLapEcuacion := s^3\,laplace(y(t),t,s) = \frac{12}{25\,s} - \frac{24}{25}\,laplace(\text{Heaviside}(t-a),t,s) + \frac{24}{25}\,laplace(\text{Heaviside}(t-3\,a),t,s) - \frac{12}{25}\,laplace(\text{Heaviside}(t-4\,a),t,s) \quad \textbf{(8)}$$

> $TransLapSolucion := isolate(TransLapEcuacion, laplace(y(t),t,s))$

$$TransLapSolucion := laplace(y(t),t,s) = \frac{1}{s^3}\left(\frac{12}{25\,s} - \frac{24}{25}\,laplace(\text{Heaviside}(t-a),t,s)\right. \quad \textbf{(9)}$$

$$+\frac{24}{25}\,laplace(\text{Heaviside}(t-3\,a),t,s)-\frac{12}{25}\,laplace(\text{Heaviside}(t-4\,a),t,s)\Big)$$

```
> SolucionParticular := invlaplace(TransLapSolucion, s, t)
```

$$SolucionParticular := y(t)=\frac{2}{25}\,t^3-\frac{24}{25}\,\text{Heaviside}(-a)\,a^3-\frac{2}{25}\,\text{Heaviside}(t-4\,a)\,(t-4\,a)^3+\frac{4}{25}\,\text{Heaviside}(t-3\,a)\,(t-3\,a)^3-\frac{4}{25}\,\text{Heaviside}(t-a)\,(t-a)^3 \quad (10)$$

```
> EcuaAlg := subs(t = 4·a, 2/25 t^3 - 2/25 (t - 4 a)^3 + 4/25 (t - 3 a)^3 - 4/25 (t - a)^3 = 225)
```

$$EcuaAlg := \frac{24}{25}\,a^3=225 \quad (11)$$

```
> Tiempo := solve(EcuaAlg, a) : evalf(%)
```

$$6.165530185,\ -3.082765092+5.339505769\,I,\ -3.082765092-5.339505769\,I \quad (12)$$

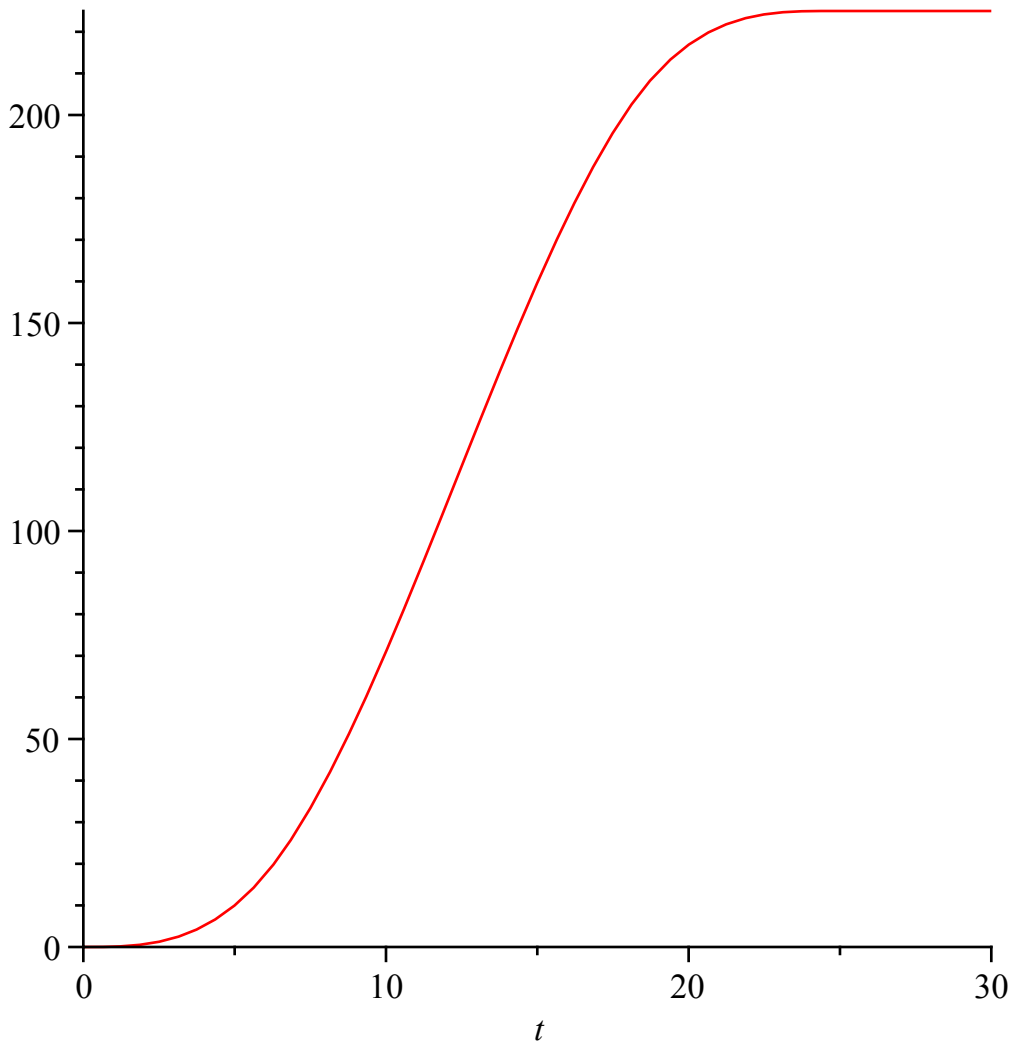
```
> TiempoFinal := 4·Tiempo[1] : evalf(%, 4)
```

$$24.66 \quad (13)$$

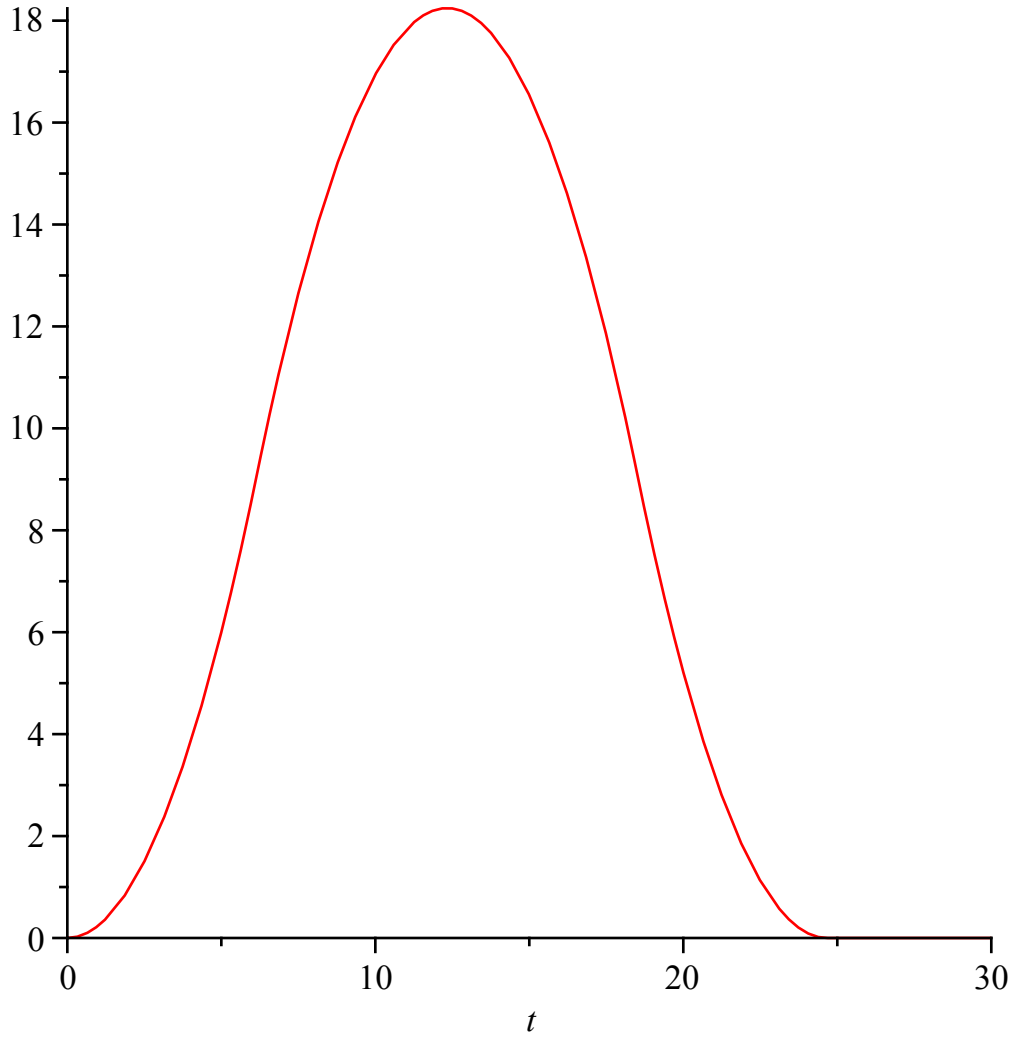
```
> Posicion := subs(a = Tiempo[1], SolucionParticular)
```

$$Posicion := y(t)=\frac{2}{25}\,t^3-225\,\text{Heaviside}\left(-\frac{5}{2}\,15^{1/3}\right)-\frac{2}{25}\,\text{Heaviside}\left(t-10\,15^{1/3}\right)\left(t-10\,15^{1/3}\right)^3+\frac{4}{25}\,\text{Heaviside}\left(t-\frac{15}{2}\,15^{1/3}\right)\left(t-\frac{15}{2}\,15^{1/3}\right)^3-\frac{4}{25}\,\text{Heaviside}\left(t-\frac{5}{2}\,15^{1/3}\right)\left(t-\frac{5}{2}\,15^{1/3}\right)^3 \quad (14)$$

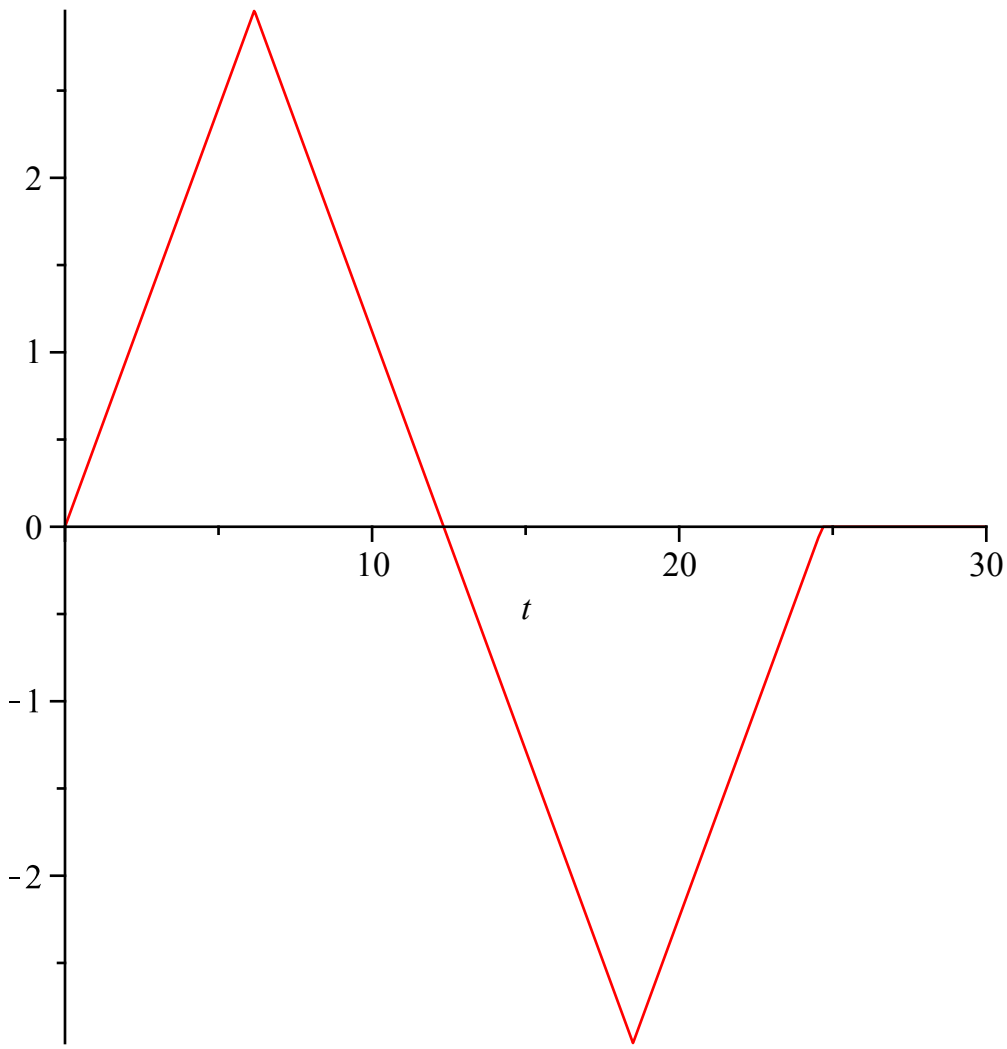
```
> plot(rhs(Posicion), t = 0..30)
```



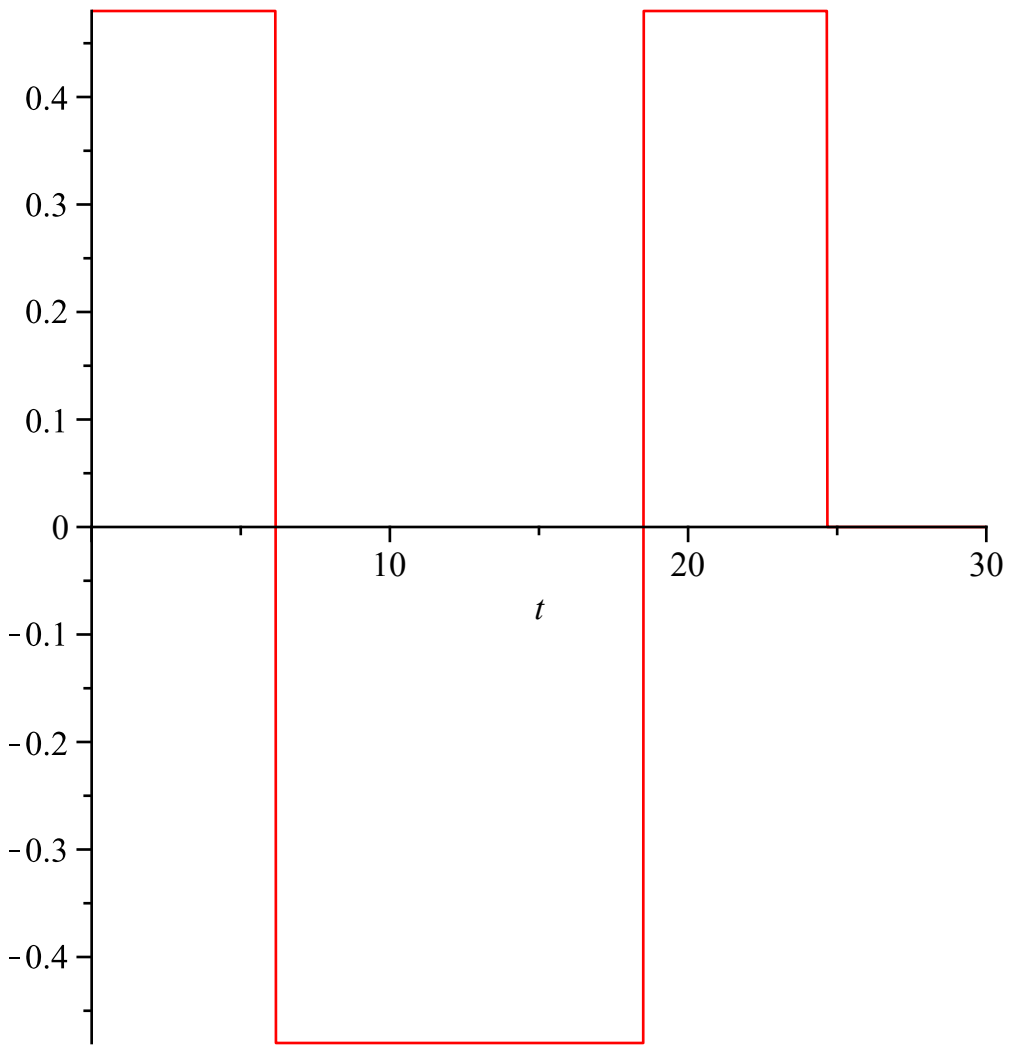

```
> plot(rhs(diff(Posicion, t)), t = 0..30)
```




```
> plot(rhs(diff(Posicion, t$2)), t = 0..30)
```




```
> plot(rhs(diff(Posicion, t$3)), t = 0..30)
```



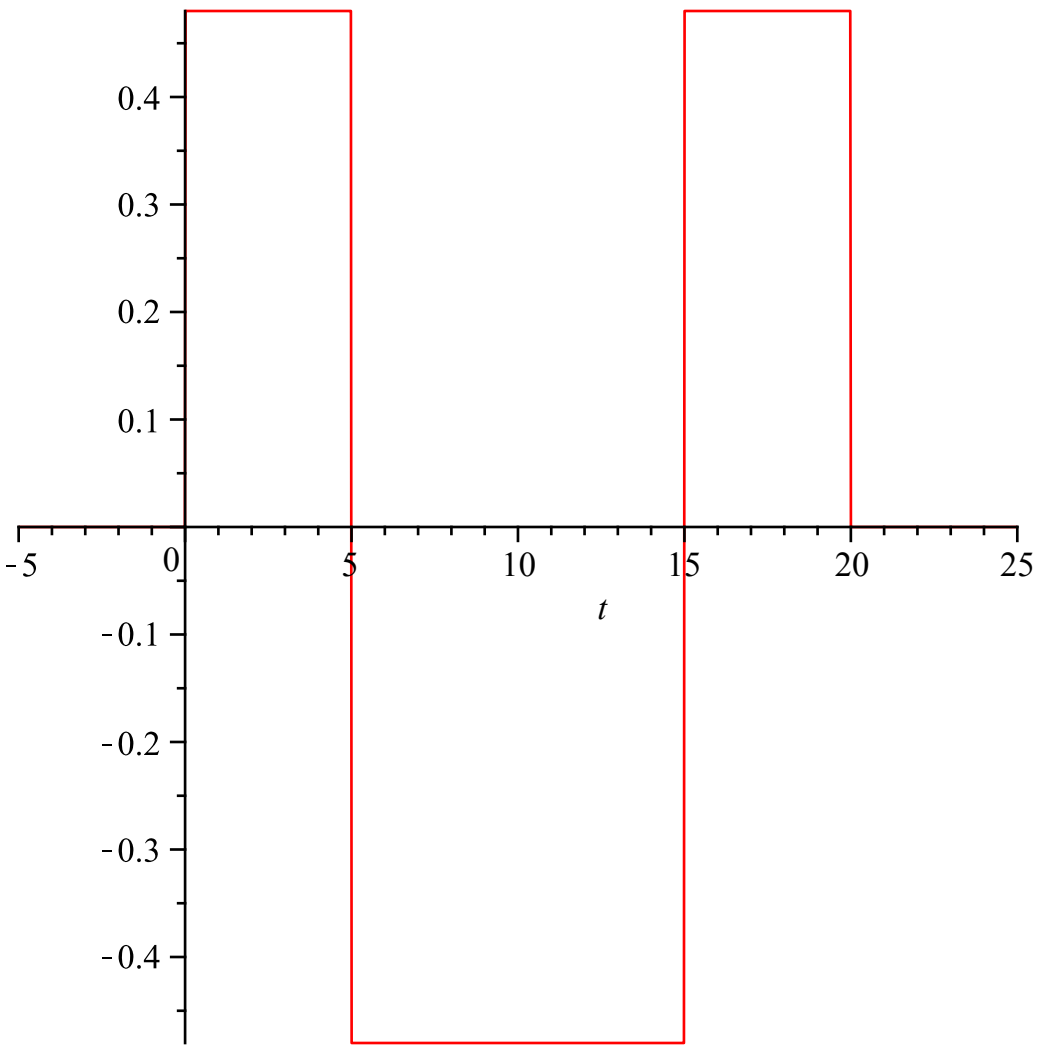

```
> restart
> restart
> Ecuacion := diff(y(t), t$3) = s(t)
```

$$Ecuacion := \frac{d^3}{dt^3} y(t) = s(t) \tag{15}$$

```
> s(t) := 48/100 · Heaviside(t) - 2·48/100 ·Heaviside(t - a) + 2·48/100 ·Heaviside(t - 3·a) - 48/100
      ·Heaviside(t - 4·a)
```

$$s(t) := \frac{12}{25}\,\text{Heaviside}(t) - \frac{24}{25}\,\text{Heaviside}(t-a) + \frac{24}{25}\,\text{Heaviside}(t-3\,a) - \frac{12}{25}\,\text{Heaviside}(t - 4\,a) \tag{16}$$

```
> plot(subs(a = 5, s(t)), t = -5..25)
```



> *Ecuacion*

$$\frac{d^3}{dt^3} y(t) = \frac{12}{25}\,\text{Heaviside}(t) - \frac{24}{25}\,\text{Heaviside}(t-a) + \frac{24}{25}\,\text{Heaviside}(t-3\,a) - \frac{12}{25}\,\text{Heaviside}(t-4\,a) \quad (17)$$

> $CondIniciales := y(0)=0, D(y)(0)=0, D(D(y))(0)=0$

$$CondIniciales := y(0)=0, D(y)(0)=0, D^{(2)}(y)(0)=0 \quad (18)$$

> $with(inttrans)$:

> $TransLapEcuacion := subs(CondIniciales, laplace(Ecuacion, t, s))$

$$TransLapEcuacion := s^3\,laplace(y(t),t,s) = \frac{12}{25\,s} - \frac{24}{25}\,laplace(\text{Heaviside}(t-a),t,s) + \frac{24}{25}\,laplace(\text{Heaviside}(t-3\,a),t,s) - \frac{12}{25}\,laplace(\text{Heaviside}(t-4\,a),t,s) \quad (19)$$

> $TransLapSolucion := isolate(TransLapEcuacion, laplace(y(t),t,s))$

$$TransLapSolucion := laplace(y(t),t,s) = \frac{1}{s^3}\left(\frac{12}{25\,s} - \frac{24}{25}\,laplace(\text{Heaviside}(t-a),t,s)\right) \quad (20)$$

$$+\frac{24}{25}\,laplace(\text{Heaviside}(t-3\,a),t,s)-\frac{12}{25}\,laplace(\text{Heaviside}(t-4\,a),t,s)\Big)$$

```
> SolucionParticular := invlaplace(TransLapSolucion, s, t)
```

$$SolucionParticular := y(t)=\frac{2}{25}\,t^3-\frac{24}{25}\,\text{Heaviside}(-a)\,a^3-\frac{2}{25}\,\text{Heaviside}(t-4\,a)\,(t-4\,a)^3+\frac{4}{25}\,\text{Heaviside}(t-3\,a)\,(t-3\,a)^3-\frac{4}{25}\,\text{Heaviside}(t-a)\,(t-a)^3 \quad (21)$$

```
> EcuaAlg := subs(t = 4·a, 2/25 t^3 - 2/25 (t - 4 a)^3 + 4/25 (t - 3 a)^3 - 4/25 (t - a)^3 = 150)
```

$$EcuaAlg := \frac{24}{25}\,a^3=150 \quad (22)$$

```
> Tiempo := solve(EcuaAlg, a) : evalf(%)
```

$$5.386086725,\ -2.693043362+4.664487932\,I,\ -2.693043362-4.664487932\,I \quad (23)$$

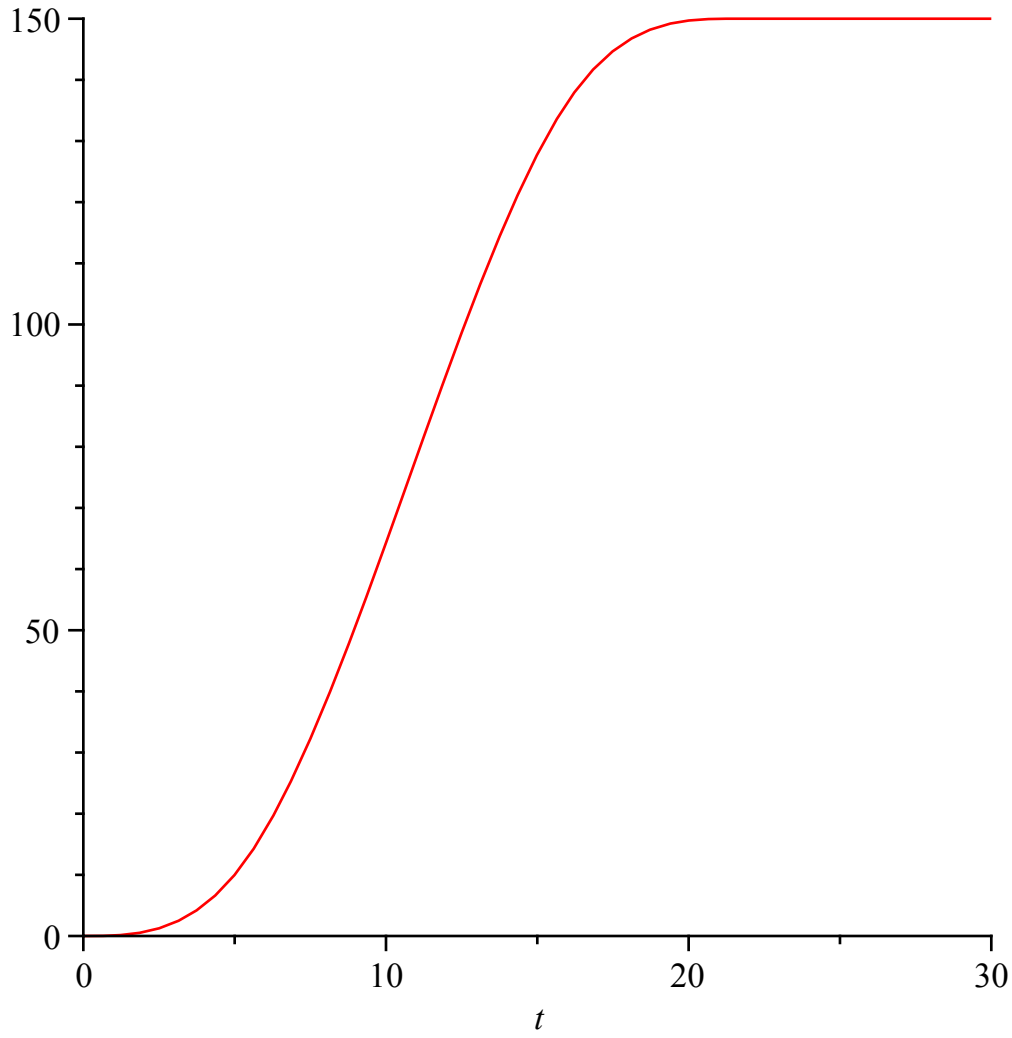
```
> TiempoFinal := 4·Tiempo[1] : evalf(%, 4)
```

$$21.54 \quad (24)$$

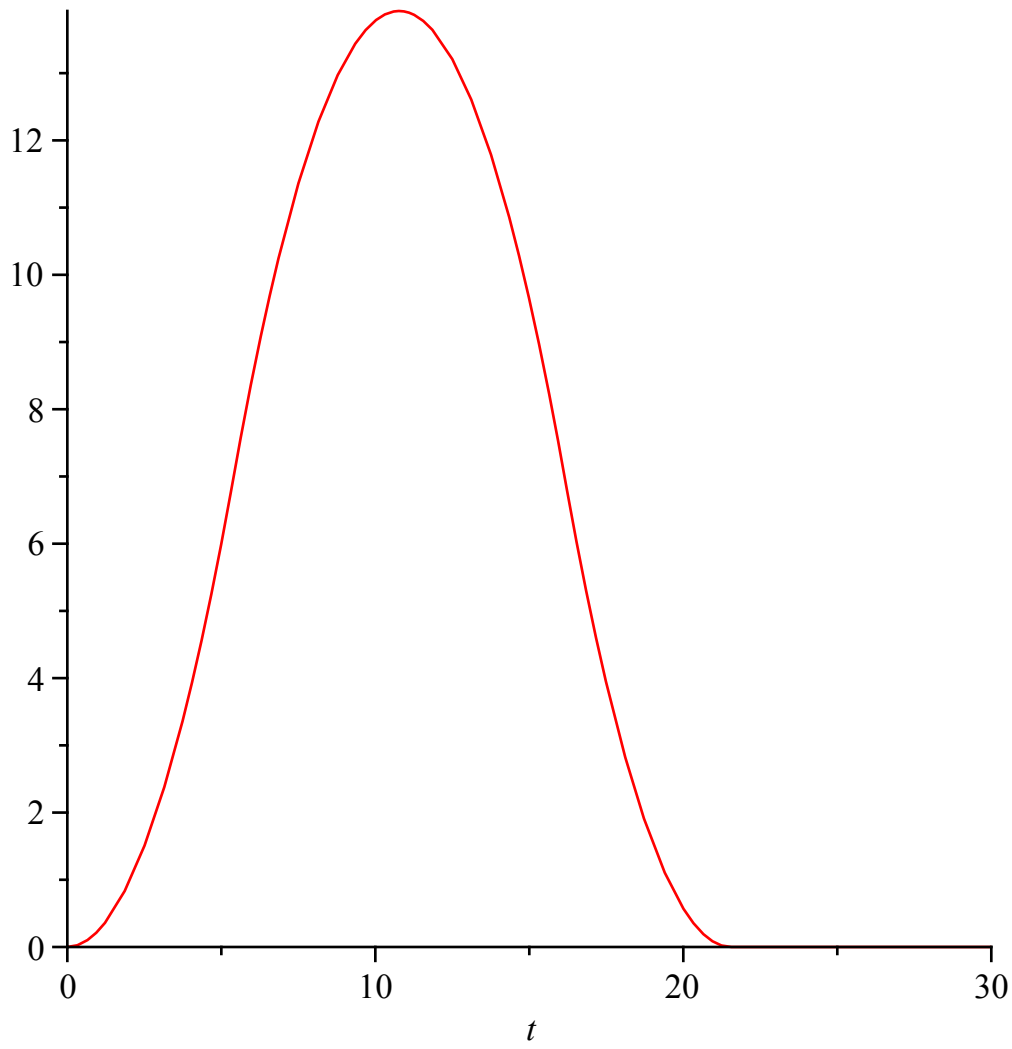
```
> Posicion := subs(a = Tiempo[1], SolucionParticular)
```

$$Posicion := y(t)=\frac{2}{25}\,t^3-150\,\text{Heaviside}\left(-\frac{5}{2}\,10^{1/3}\right)-\frac{2}{25}\,\text{Heaviside}\left(t-10\,10^{1/3}\right)\left(t-10\,10^{1/3}\right)^3+\frac{4}{25}\,\text{Heaviside}\left(t-\frac{15}{2}\,10^{1/3}\right)\left(t-\frac{15}{2}\,10^{1/3}\right)^3-\frac{4}{25}\,\text{Heaviside}\left(t-\frac{5}{2}\,10^{1/3}\right)\left(t-\frac{5}{2}\,10^{1/3}\right)^3 \quad (25)$$

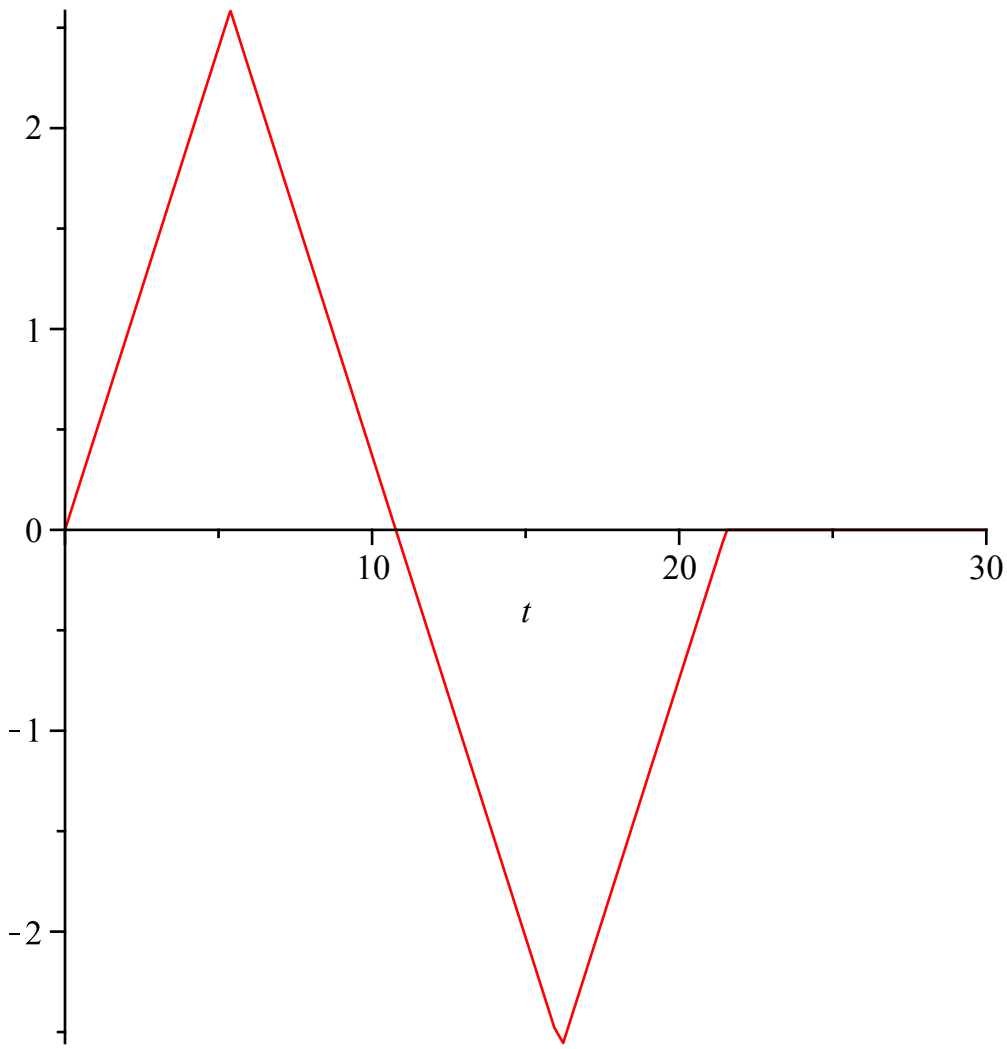
```
> plot(rhs(Posicion), t = 0..30)
```



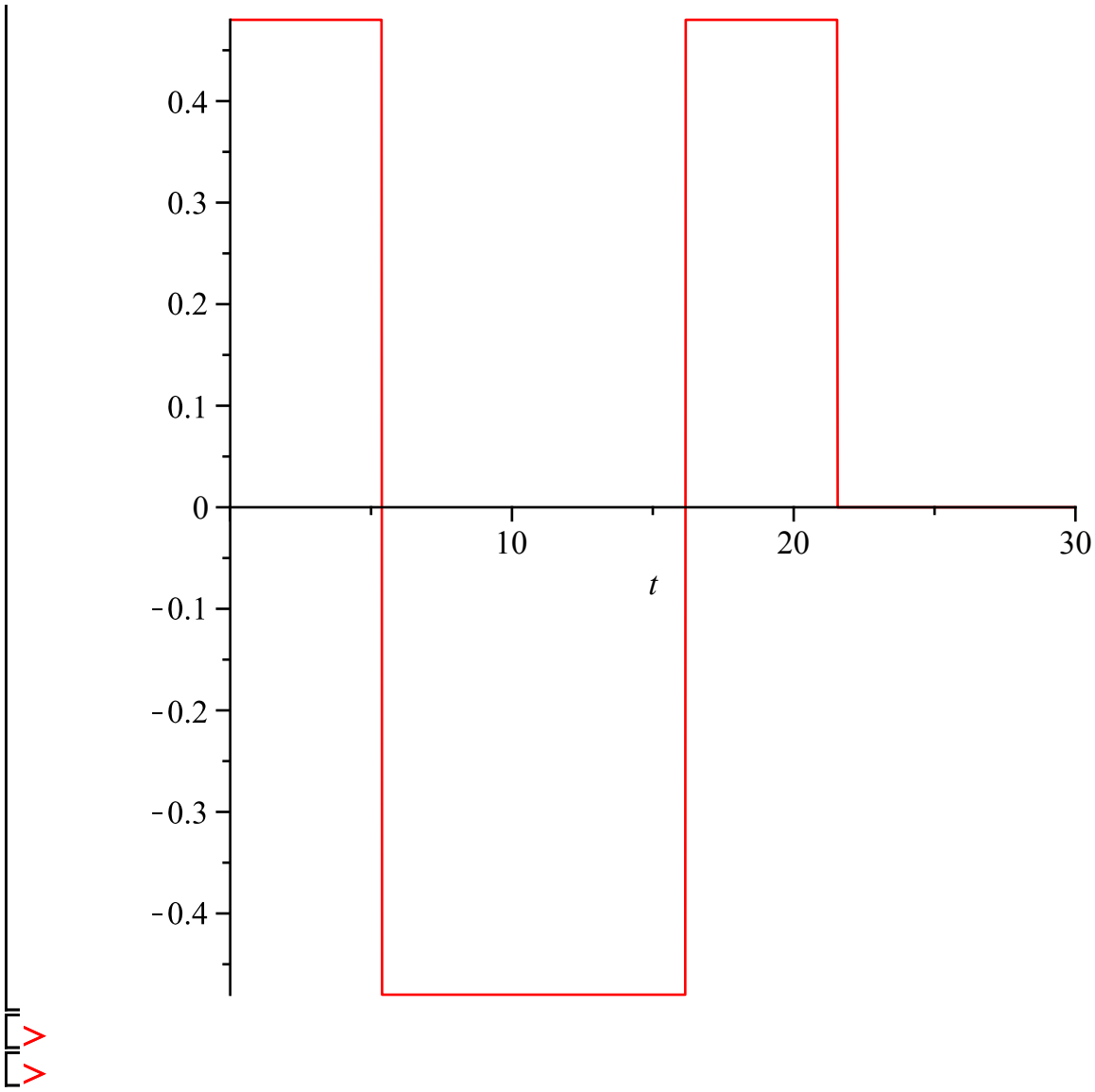

```
> plot(rhs(diff(Posicion, t)), t = 0..30)
```




```
> plot(rhs(diff(Posicion, t$2)), t = 0..30)
```




```
> plot(rhs(diff(Posicion, t$3)), t = 0..30)
```


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
10
20
30
t