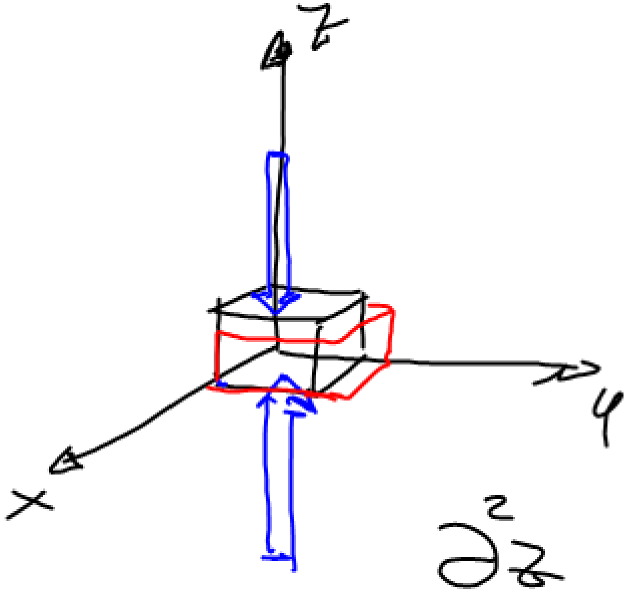
Orden: está dado por la derivada de mayor orden

Ecuaciones Diferenciales en Derivadas Parciales

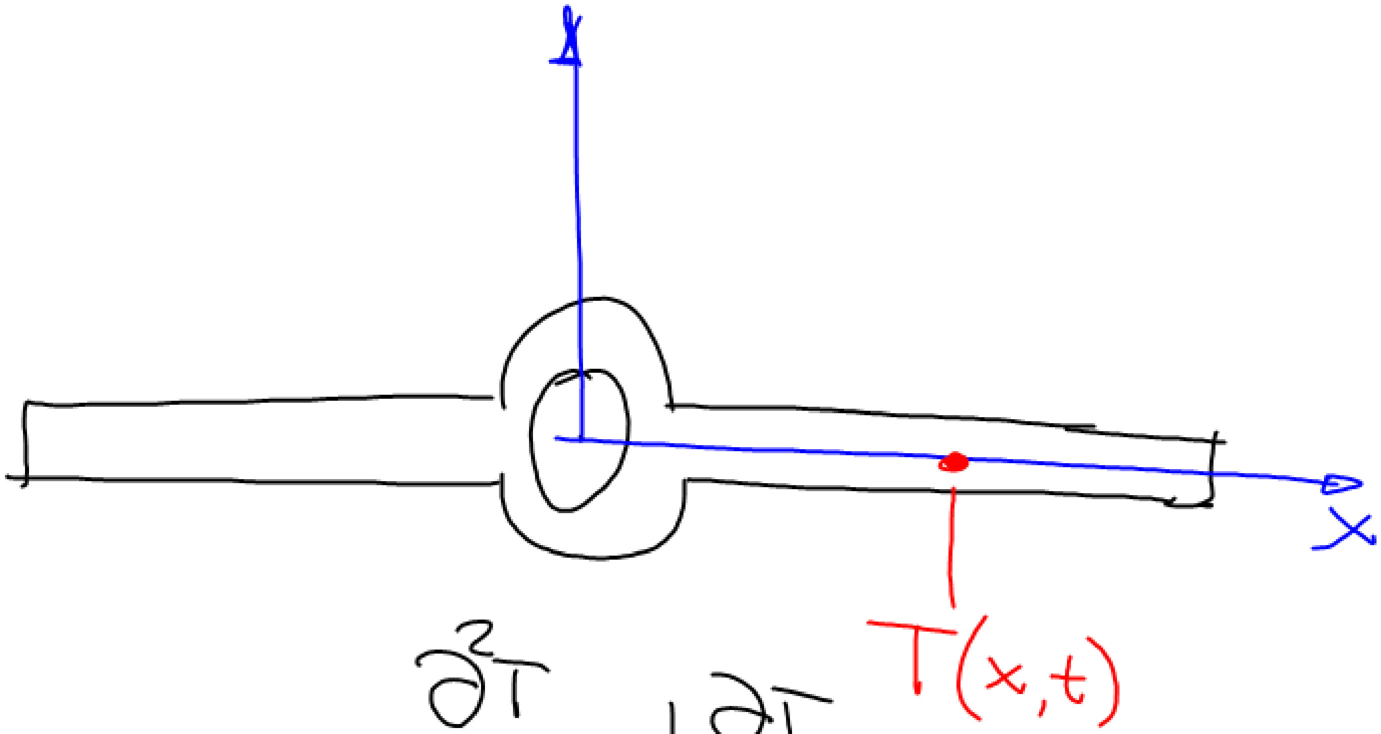
$z(x, y, t)$ $\quad$ $y(x, t)$ $\quad$ $y, x, t \in \mathbb{R}$

$\longrightarrow$

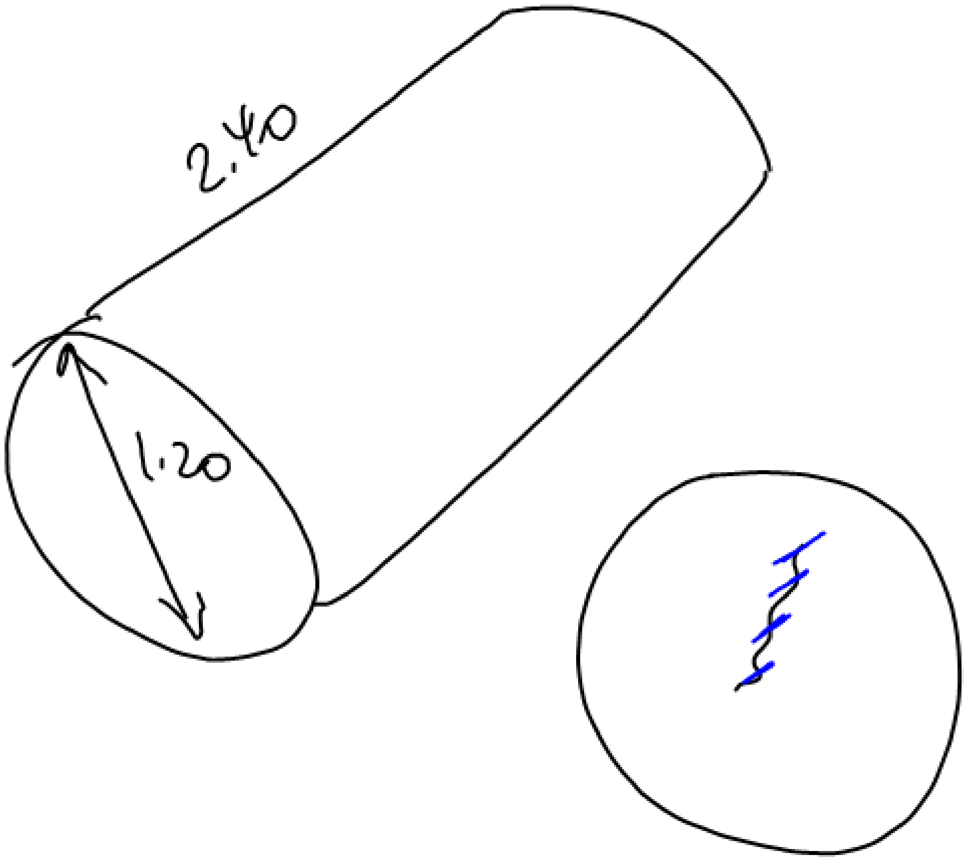
$z(x_1, x_2, x_3, x_4, \ldots, x_n)$



$$\frac{\partial^2 z}{\partial x^2} + k\frac{\partial^2 z}{\partial y^2} = 0$$



$$\frac{\partial^2 T}{\partial t^2} = k \frac{\partial T}{\partial x}$$


2.40
1.20

$$\frac{\partial^2 z}{\partial x^2} + a_1 \frac{\partial^2 z}{\partial x \partial y} + a_2 \frac{\partial^2 z}{\partial y^2} = 0$$

orden = 2

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = z^n \qquad n > 1$$

✓✓ Lineales

× cuasi-lineales

× No Lineales

$$z \frac{\partial z}{\partial x} + \left(\frac{\partial^2 z}{\partial y^2}\right)^3 = 0$$

$$z(x,y) = F(y + mx) \qquad m \in \mathbb{R}$$

$$\frac{\partial z}{\partial x} = F' \cdot (m) \qquad \frac{\partial z}{\partial y} = F' \cdot (1)$$

$$\frac{\partial^2 z}{\partial x^2} = F'' \cdot (m) \cdot (m) = m^2 F''$$

$$\frac{\partial^2 z}{\partial x \partial y} = m F''$$

$$\frac{\partial^2 z}{\partial y^2} = F''(1)(1) = F''$$

$$(m^2 F'') + a_1 (m F'') + a_2 F'' = 0$$

$$(m^2 + a_1 m + a_2) F'' = 0 \qquad F(y+mx)$$

$$m^2 + a_1 m + a_2 = 0$$

ecuación característica

Solución Trivial $F'' = 0 \quad F' = k_1$

$$\boxed{F(y+mx) = k_1 (y+mx) + k_2}$$

m_1, m_2 } Solución EC

$$F_1(y + m_1 x)$$

$$F_2(y + m_2 x)$$

$$z(x,y) = F_1(y + m_1 x) + F_2(y + m_2 x)$$

SOLUCIÓN GENERAL EDenDP(2)L.

$$\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 6\frac{\partial^2 z}{\partial y^2} = 0$$

$$z(x,y) = F(y+mx)$$

$$m^2 - 5m + 6 = 0 \qquad m_1 = 3 \quad m_2 = 2$$

$$z = e^{(y+3x)} + \cos(y+2x)$$

$$\boxed{z(x,y) = F_1(y+3x) + F_2(y+2x)}$$

SG

$$z(x,y) = (y+3x)^3 + (y+2x)^2$$

$$z(x,y) = y^3 + 9xy^2 + 27x^2y + 27x^3 + y^2 + 4xy + 4x^2$$

$$\frac{\partial z}{\partial x} = (0) + 9y^2 + 54xy + 81x^2 + (0) + 4y + 8x$$

$$\frac{\partial^2 z}{\partial x^2} = 54y + 162x + 8$$

$$\frac{\partial^2 z}{\partial y \partial x} = 18y + 54x + 4$$

$$\frac{\partial z}{\partial y} = 3y^2 + 18xy + 27x^2 + 2y + 4x$$

$$\frac{\partial^2 z}{\partial y^2} = 6y + 18x + 2$$

$$[54y + 162x + 8] - 5[18y + 54x + 4] + 6[6y + 18x + 2] = 0$$

$$(54 - 90 + 36)y + (162 - 270 + 108)x + (8 - 20 + 12) = 0$$

$$(0)y + (0)x + (0) = 0$$

$$0 \equiv 0$$

$$0$$

En EDenDP la Solución General puede no ser única

$$\frac{\partial^2 y}{\partial x^2} + 9\frac{\partial^2 y}{\partial t^2} = 0 \qquad y(x,t)$$

$$y(x,t) = F(t+mx)$$

$$\frac{\partial y}{\partial x} = mF' \quad \frac{\partial y}{\partial t} = F' \qquad (m^2+9)F'' = 0$$

$$\frac{\partial^2 y}{\partial x^2} = m^2 F'' \quad \frac{\partial^2 y}{\partial t^2} = F'' \qquad m^2+9=0 \quad m=\pm 3i$$

$$\boxed{y(x,t) = F_1(t+3ix) + F_2(t-3ix)}$$

$$\frac{\partial^2 y}{\partial t^2} - 2\frac{\partial^2 y}{\partial x \partial t} + \frac{\partial^2 y}{\partial x^2} = 0$$

$$y(x,t) = F(x+mt)$$

$$\frac{\partial y}{\partial x} = F' \quad \frac{\partial y}{\partial t} = mF'$$

$$\frac{\partial^2 y}{\partial x^2} = F'' \quad \frac{\partial^2 y}{\partial x \partial t} = mF'' \quad \frac{\partial^2 y}{\partial t^2} = m^2 F''$$

$$[F''] - 2[mF''] + [m^2F''] = 0$$

$$m^2 - 2m + 1 = 0 \qquad (m-1)^2 = 0 \qquad \left.\begin{array}{l} m_1 = 1 \\ m_2 = 1 \end{array}\right\} \text{raíces iguales}$$

$$\boxed{y(x,t) = F_1(x+t) + tF_2(x+t)}$$

$$\boxed{y(x,y) = F_1(x+t) + xF_2(x+t)}$$

$$y(x,t) = e^{(x+t)} + te^{(x+t)}$$

$$\frac{\partial y}{\partial x} = e^{(x+t)} + te^{(x+t)}$$

$$\frac{\partial y}{\partial t} = e^{(x+t)} + te^{(x+t)} + e^{(x+t)}$$

$$= 2e^{(x+t)} + te^{(x+t)}$$

$$\frac{\partial^2 y}{\partial x^2} = e^{(x+t)} + te^{(x+t)}$$

$$\frac{\partial^2 y}{\partial t^2} = 2e^{(x+t)} + te^{(x+t)} + e^{(x+t)}$$

$$= 3e^{(x+t)} + te^{(x+t)}$$

$$\frac{\partial^2 y}{\partial x \partial t} = 2e^{(x+t)} + te^{(x+t)}$$

$$\left[3e^{(x+t)} + te^{(x+t)}\right] - 2\left[2e^{(x+t)} + te^{(x+t)}\right] + \left[e^{(x+t)} + te^{(x+t)}\right] = 0$$

$$(0)e^{(x+t)} + (0)te^{(x+t)} = 0$$

$$\underline{\underline{0 = 0.}}$$

$$y(x,t) = \cos(x+t) - x\,\text{sen}(x+t)$$

$$\frac{\partial y}{\partial t} = -\text{sen}(x+t) - x\cos(x+t)$$

$$\frac{\partial y}{\partial x} = -\text{sen}(x+t) - x\cos(x+t) - \text{sen}(x+t)$$

$$= -2\,\text{sen}(x+t) - x\cos(x+t)$$

$$\frac{\partial^2 y}{\partial t^2} = -\cos(x+t) + x\,\text{sen}(x+t)$$

$$\frac{\partial^2 y}{\partial x^2} = -2\cos(x+t) + x\,\text{sen}(x+t) - \cos(x+t)$$

$$= -3\cos(x+t) + x\,\text{sen}(x+t).$$

$$\frac{\partial^2 y}{\partial x \partial t} = -2\cos(x+t) + x\,\text{sen}(x+t)$$

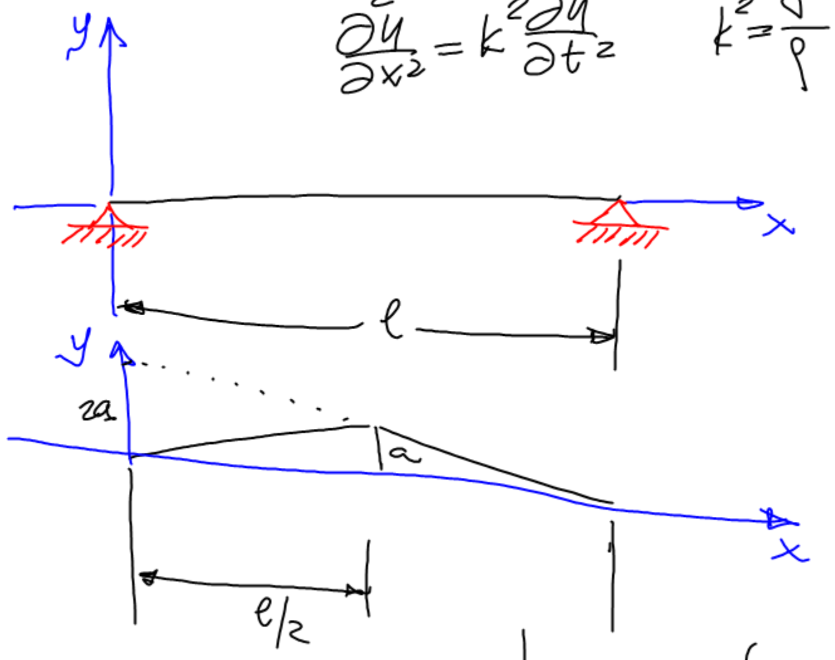
$$\left[-\cos(x+t) + x\,\text{sen}(x+t)\right] - 2\left[-2\cos(x+t) + x\,\text{sen}(x+t)\right] +$$

$$+ \left[-3\cos(x+t) + x\,\text{sen}(x+t)\right] = 0$$

$$(0)\cos(x+t) + (0)\,x\,\text{sen}(x+t) = 0$$

$$0 \equiv 0$$

$$\frac{\partial^2 y}{\partial x^2} = k^2 \frac{\partial^2 y}{\partial t^2} \qquad k^2 = \frac{\sigma}{\rho}$$



$$\left.\begin{aligned} y(0,t) &= 0 \\ y(\ell,t) &= 0 \end{aligned}\right\} \text{ condiciones en la frontera}$$

$$y(x,0) = \begin{cases} \frac{2a}{\ell}x & ; 0 < x < \ell/2 \\ 2a - \frac{2a}{\ell}x & ; \ell/2 < x < \ell \end{cases}$$

$$y'(x,0) = 0$$

condiciones iniciales $t=0$

SERIE TRIGONOMÉTRICA DE FOURIER

$$f(x) = C + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \operatorname{sen}\left(\frac{n\pi}{L}x\right) \right)$$

$$-L \leq x \leq L$$