

LINEAL PRIMER ORDEN

$$\frac{dy}{dx} + a_1 y = 0 \longrightarrow y = C_1 e^{-a_1 x}$$

$$\frac{dy}{dx} + p(x)y = 0 \longrightarrow y = C_1 e^{-\int p(x)dx}$$

$$\frac{dy}{dx} + p(x)y = q(x) \longrightarrow y = C e^{-\int p(x)dx} + C \int e^{\int p(x)dx} q(x) dx$$

$$\frac{dy}{dx} + a_1 y = q(x) \longrightarrow y_g = C e^{-a_1 x} + C \int e^{a_1 x} q(x) dx.$$

MPV. $y_g = C y_1 \rightarrow y_{g/h} = A(x) y_1$

LIN AL 2º ORDEN OC. HOMOGENEA.

$$\frac{dy^2}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0.$$

coeficiente derivada de mayor orden
debe ser obligadamente la unidad.

$$y = C_1 y_1 + C_2 y_2$$

Hipótesis

$$y_1 = e^{mx}$$

$m \Rightarrow$ parámetro. Solución particular
fundamental

$$\frac{dy_1}{dx} = me^{mx}$$

$$\frac{d^2y_1}{dx^2} = m^2 e^{mx}$$

$$\begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix} \neq 0$$

 y_1, y_2 linealmente
independientes.

$$m^2 e^{mx} + a_1 [me^{mx}] + a_2 [e^{mx}] = 0$$

$$\text{ecuación algebraica } (m^2 + a_1 m + a_2) e^{mx} = 0$$

$$e^{mx} = 0$$

$$y_1 = 0 \text{ trivial}$$

$$\text{ECUACIÓN CARACTÉRISTICA } m^2 + a_1 m + a_2 = 0$$

independ.
lineal

$$\begin{cases} y_1 = e^{m_1 x} \\ y_2 = e^{m_2 x} \end{cases}$$

$$\left. \begin{array}{l} m_1 \\ m_2 \end{array} \right\} \text{Raíces}$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$\frac{dy^2}{dx^2} - 5 \frac{dy}{dx} + 6y = 0 \quad EDO(2) \text{ LCCF.}$$

$$m^2 - 5m + 6 = 0 \quad \begin{matrix} \text{ECUACIÓN} \\ \text{CARACTERÍSTICA} \end{matrix}$$

$$(m-m_1)(m-m_2)=0$$

$$(m-2)(m-3)=0$$

$$y = e^{mx} \quad m_1=2 \quad m_2=3. \quad \text{RAÍCES}$$

$$y_1 = e^{2x} \quad y_2 = e^{3x} \quad \text{SPF.}$$

$$\begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix} \neq 0 \quad e^{2x}(3e^{3x}) - e^{3x}(2e^{2x}) \neq 0$$

$$(3-2)e^{5x} \neq 0 \quad e^{5x} \neq 0.$$

$$\boxed{y_g = C_1 e^{2x} + C_2 e^{3x}} \quad \begin{matrix} \text{Sol- GRAL} \\ EDO(2) \text{ LCCF.} \end{matrix}$$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad y_p = e^{mx}$$

$$m^2 + a_1 m + a_2 = 0 \quad m_1, \quad m_2$$

$$y_1 = e^{m_1 x} \quad y_2 = e^{m_2 x}$$

$$\begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0 \quad m_2 e^{m_1 x} - m_1 e^{m_2 x} \neq 0$$

$$(m_2 - m_1) e^{(m_1 - m_2)x} \neq 0$$

$$(m_2 - m_1) \neq 0 \Rightarrow \begin{cases} e^{m_1 x} \neq 0 \\ e^{m_2 x} \neq 0 \end{cases}$$

$$m_2 \neq m_1$$

$$\frac{\partial^2 y}{\partial x^2} + a_1 \frac{\partial y}{\partial x} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \quad \left. \begin{array}{l} m_1 \\ m_2 \end{array} \right\}$$

$$y_1 = e^{(a+bi)x} \Rightarrow e^{\alpha x} e^{(bx)i}$$

$$y_2 = e^{(a-bi)x} \Rightarrow e^{\alpha x} e^{(-bx)i}$$

$$y = C_1 e^{\alpha x} e^{bx i} + C_2 e^{\alpha x} e^{-bx i}$$

Caso I: $m_1 \neq m_2 \in \mathbb{R}$

Caso II: $m_1 = m_2 \in \mathbb{R}$

Caso III: $m_1, m_2 \in \mathbb{C}$

$$m_1 = a+bi$$

$$m_2 = a-bi$$

$$m_1 \neq m_2$$

$$\forall x \in \mathbb{R} \Rightarrow \forall y \in \mathbb{R} \quad C_1 \in \mathbb{C}$$

$$C_2 \in \mathbb{C}$$

Euler

$$\text{Definición}$$

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

Diagram illustrating Euler's formula in the complex plane. A vector r is shown originating from the origin, making an angle θ with the positive real axis. The vector is decomposed into its horizontal component a (real part) and vertical component b (imaginary part). The complex number $a+bi$ is labeled with a circled 1. The complex conjugate $a-bi$ is labeled with a circled 2. The magnitude r is given by $r = \sqrt{a^2 + b^2}$, and the angle θ is given by $\theta = \arg \tan\left(\frac{b}{a}\right)$. The exponential form is $e^{i\theta} = r e^{i\theta}$, where $r e^{i\theta} = a+bi$ and $r e^{-i\theta} = a-bi$.

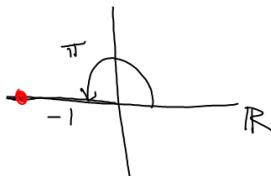
$$a = r \cos(\theta) = r \cos(-\theta) \quad \cos(\theta) = \cos(-\theta)$$

$$\begin{aligned} b &= r \sin(\theta) \\ -b &= r \sin(-\theta) \end{aligned} \quad \left. \begin{aligned} b \\ -b \end{aligned} \right\} \Rightarrow \sin(-\theta) = -\sin(\theta)$$

$$\begin{aligned} r e^{i\theta} &= a+bi \\ r e^{-i\theta} &= a-bi \end{aligned} \quad \left. \begin{aligned} r e^{i\theta} &= r \cos(\theta) + r \sin(\theta) i \\ r e^{-i\theta} &= r \cos(\theta) - r \sin(\theta) i \end{aligned} \right.$$

$$\begin{aligned} e^{i\theta} &= \cos(\theta) + \sin(\theta) i \\ e^{-i\theta} &= \cos(\theta) - \sin(\theta) i \end{aligned} \quad \left. \begin{aligned} e^{i\theta} \\ e^{-i\theta} \end{aligned} \right\}$$

$$\theta \rightarrow \pi \text{ rad.}$$



$$\begin{aligned} e^{i\pi} &= \cos(\pi) + \sin(\pi) i \\ e^{-i\pi} &= \cos(\pi) - \sin(\pi) i \end{aligned}$$

$$e^{i\pi} = -1$$

$$\boxed{e^{i\pi} + 1 = 0}$$

$$y = C_1 e^{\alpha x} e^{bx i} + C_2 e^{\alpha x} e^{-bx i}$$

$$y_g = C_1 e^{\alpha x} \left(\cos(bx) + \operatorname{sen}(bx)i \right) + C_2 e^{\alpha x} \left(\cos(bx) - \operatorname{sen}(bx)i \right)$$

$$y = (C_1 + C_2) e^{\alpha x} \cos(bx) + (C_1 i - C_2 i) e^{\alpha x} \operatorname{sen}(bx)$$

$$\boxed{y_g = C_1 e^{\alpha x} \cos(bx) + C_2 e^{\alpha x} \operatorname{sen}(bx)}$$

$$m_1 = a + bi$$

$$m_2 = a - bi$$

$$a \in \mathbb{R}$$

$$b \in \mathbb{R}^+$$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$\begin{aligned} m^2 + a_1 m + a_2 &= 0 & \left. \begin{array}{l} m_1 \\ m_2 \end{array} \right\} m_1 = m_2 \\ (m - m_1)^2 &= 0 \\ \frac{d}{dm} \rightarrow 2m + a_1 &= 0 \\ z(m - m_1) &= 0 \end{aligned}$$

$\xrightarrow[m=m_1]{e^{mx}}$

$$\begin{aligned} y_1 &= e^{m_1 x} \\ y_2 &= e^{m_1 x} \\ x e^{m_1 x} &\xrightarrow[m=m_1]{\rightarrow} x e^{m_1 x} \end{aligned}$$

$$\begin{aligned} \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y &= 0 \\ \left[m_1^2 e^{m_1 x} + 2m_1 e^{m_1 x} \right] + a_1 \left[m_1 x e^{m_1 x} + e^{m_1 x} \right] + \\ + a_2 \left[x e^{m_1 x} \right] &= 0 \end{aligned}$$

$\left. \begin{array}{l} y = x e^{m_1 x} \\ \frac{dy}{dx} = m_1 x e^{m_1 x} + e^{m_1 x} \\ \frac{d^2y}{dx^2} = m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x} \end{array} \right\}$

$$\begin{aligned} (m_1^2 + a_1 m_1 + a_2) x e^{m_1 x} + (z m_1 + a_1) e^{m_1 x} &= 0 \\ (0) x e^{m_1 x} + (0) e^{m_1 x} &\stackrel{x \neq 0}{=} 0 \end{aligned}$$

$\boxed{y = C_1 e^{m_1 x} + C_2 x e^{m_1 x}}$

$$m_1 = m_2 \in \mathbb{R}$$

$$\frac{d^2y}{dt^2} = 0 \quad \frac{d}{dt} \left(\frac{dy}{dt} \right) = 0 \quad \frac{dy}{dt} = C_1$$

$$\int dy = C_1 dt \quad y = C_1 t + C_2$$

$$\begin{aligned} m^2 &= 0 & m_1 = m_2 = 0 & y_1 = e^{(0)t} & y_2 = t e^{(0)t} \\ y_1 &= 1 & y_2 &= t \end{aligned}$$