LINEAL PRIMER ORDEN

$$\frac{dy}{dx} + a_1 y = 0 \longrightarrow y_g = C_1 e^{-a_1 x}$$

$$\frac{dy}{dx} + p(x) y = 0 \longrightarrow y_g = C_1 e^{-\int p(x)dx}$$

$$\frac{dy}{dx} + p(x) y = q(x) \longrightarrow y_g = C_1 e^{-\int p(x)dx} + e^{-\int p(x)dx} \int e^{\int p(x)dx} q(x)\,dx$$

$$\frac{dy}{dx} + a_1 y = q(x) \longrightarrow y_g = C_1 e^{-a_1 x} + e^{-a_1 x} \int e^{a_1 x} q(x)\,dx.$$

MPV. $y_{g/h} = C_1 y_1 \longrightarrow y_{g/n-h} = A(x) y_1$

LIN DE 2º ORDEN CC. HOMOGENEA.

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0.$$

coeficiente derivada de mayor orden debe ser obligadamente la unidad.

$$y_g = C_1 y_1 + C_2 y_2 \qquad \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

y_1, y_2 Linealmente independientes.

Hipótesis

$$y_1 = e^{mx}$$

$m \Rightarrow$ parámetro. Solución particular fundamental

$$\frac{dy_1}{dx} = me^{mx}$$

$$\frac{d^2y_1}{dx^2} = m^2 e^{mx}$$

$$m^2 e^{mx} + a_1\left[me^{mx}\right] + a_2\left[e^{mx}\right] = 0$$

ecuación algebraica $(m^2 + a_1 m + a_2)e^{mx} = 0$ $\qquad e^{mx} = 0$

ECUACIÓN CARACTERÍSTICA $m^2 + a_1 m + a_2 = 0$ $\qquad y_1 = 0$ trivial

$\left.\begin{matrix} m_1 \\ m_2 \end{matrix}\right\}$ Raíces

independ. lineal $\begin{cases} y_1 = e^{m_1 x} \\ y_2 = e^{m_2 x} \end{cases}$

$$\boxed{y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x}}$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$ EDO(2)LccH.

ECUACIÓN CARACTERÍSTICA

$$m^2 - 5m + 6 = 0$$

$$(m - m_1)(m - m_2) = 0$$

$$y = e^{mx}$$

$$(m-2)(m-3) = 0$$

$m_1 = 2$ $m_2 = 3.$ RAÍCES

$y_1 = e^{2x}$ $y_2 = e^{3x}$ SPF

$$\begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix} \neq 0$$

$$e^{2x}(3e^{3x}) - e^{3x}(2e^{2x}) \neq 0$$

$$(3-2)e^{5x} \neq 0 \quad e^{5x} \neq 0.$$

$$\boxed{y_g = C_1 e^{2x} + C_2 e^{3x}}$$ SOL- GRAL EDO(2)LccH.

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0 \qquad y_p = e^{mx}$$

$$m^2 + a_1 m + a_2 = 0 \qquad m_1 \quad m_2$$

$$y_1 = e^{m_1x} \qquad y_2 = e^{m_2x}$$

$$\begin{vmatrix} e^{m_1x} & e^{m_2x} \\ m_1e^{m_1x} & m_2e^{m_2x} \end{vmatrix} \neq 0 \qquad m_2e^{m_1x}e^{m_2x} - m_1e^{m_2x}e^{m_1x} \neq 0$$

$$(m_2 - m_1)e^{m_1x}e^{m_2x} \neq 0$$

$$e^{m_1x} \neq 0 \qquad e^{m_2x} \neq 0$$

$$(m_2 - m_1) \neq 0 \Rightarrow m_2 \neq m_1$$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \quad \left.\begin{matrix} m_1 \\ m_2 \end{matrix}\right\}$$

$$\begin{cases} \text{Caso I: } m_1 \neq m_2 \in \mathbb{R} \\ \boxed{\text{Caso II: } m_1 = m_2 \in \mathbb{R}} \\ \text{Caso III: } m_1, m_2 \in \mathbb{C} \end{cases}$$

$m_1 = a + bi$
$m_2 = a - bi$

$m_1 \neq m_2$

$$y_1 = e^{(a+bi)x} \Rightarrow e^{ax} \cdot e^{(bx)i}$$
$$y_2 = e^{(a-bi)x} \Rightarrow e^{ax} \cdot e^{(-bx)i}$$

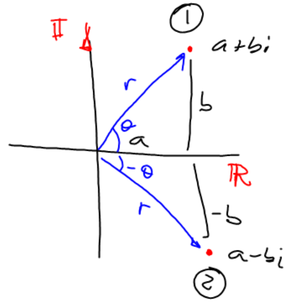
$$\boxed{y_g = C_1 e^{ax} e^{bxi} + C_2 e^{ax} e^{-bxi}}$$

$$\forall x \in \mathbb{R} \Rightarrow \forall y \in \mathbb{R} \qquad C_1 \in \mathbb{C}$$
$$C_2 \in \mathbb{C}$$

Euler

$$e^{\pi i}+1=0$$

$$\left\{\begin{array}{l} e = 2.72\ldots \\ \pi = 3.1415\ldots \\ i=\sqrt{-1} \\ 1 \end{array}\right.$$



$r=\sqrt{a^2+b^2}$

$\theta = \text{arg tan}\left(\frac{b}{a}\right)$

① $re^{\theta i}$

② $re^{-\theta i}$

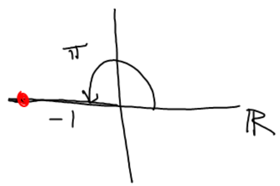
$a = r\cos(\theta) = r\cos(-\theta) \qquad \cos(\theta)=\cos(-\theta)$

$$\left.\begin{array}{l} b = r\,\text{sen}(\theta) \\ -b = r\,\text{sen}(-\theta) \end{array}\right\} \rightarrow \text{sen}(-\theta) = -\text{sen}(\theta)$$

$re^{\theta i} = a+bi$	$re^{\theta i} = r\cos(\theta)+r\,\text{sen}(\theta)\,i$
$re^{-\theta i} = a-bi$	$re^{-\theta i} = r\cos(\theta)-r\,\text{sen}(\theta)\,i$

$$\left.\begin{array}{l} e^{\theta i} = \cos(\theta)+\text{sen}(\theta)\,i \\ e^{-\theta i} = \cos(\theta)-\text{sen}(\theta)\,i \end{array}\right\}$$

$\theta = \pi$ rad.



$e^{\pi i} = \cos(\pi)+\text{sen}(\pi)\,i$

$e^{\pi i} = -1 + (0)\,i$

$e^{\pi i} = -1$

$$\boxed{e^{\pi i}+1=0}$$

$$y = C_1 e^{ax} e^{(bx)i} + C_2 e^{ax} e^{(-bx)i}$$

$$y_g = C_1 e^{ax}\left(\cos(bx) + \operatorname{sen}(bx)i\right) + C_2 e^{ax}\left(\cos(bx) - \operatorname{sen}(bx)i\right)$$

$$y = (C_1 + C_2)\, e^{ax}\cos(bx) + (C_1 i - C_2 i)\, e^{ax}\operatorname{sen}(bx)$$

$$\boxed{y_g = C_{10} e^{ax}\cos(bx) + C_{20} e^{ax}\operatorname{sen}(bx)}$$

$m_1 = a + bi$

$m_2 = a - bi$

$a \in \mathbb{R}$

$b \in \mathbb{R}^+$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \qquad \left.\begin{matrix} m_1 \\ m_2 \end{matrix}\right\} m_1 = m_2$$

$$(m - m_1)^2 = 0$$

$$\frac{d}{dm} \rightarrow 2m + a_1 = 0$$

$$2(m - m_1) = 0$$

$$\left.\begin{matrix} \boxed{y_1 = e^{m_1 x}} \\ y_2 = \cancel{e}^{m_1 x} \end{matrix}\right\} x$$

$$e^{mx} \xrightarrow{m=m_1} e^{m_1 x}$$

$$\downarrow$$

$$xe^{mx} \xrightarrow{m=m_1} xe^{m_1 x}$$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$[m_1^2 \cancel{x} e^{m_1 x} + 2\cancel{m_1} e^{m_1 x}] + a_1[\cancel{m_1} x e^{m_1 x} + e^{m_1 x}] +$$

$$+ a_2[\cancel{x} e^{m_1 x}] = 0$$

$$y_p = xe^{m_1 x}$$

$$\frac{dy}{dx} = m_1 x e^{m_1 x} + e^{m_1 x}$$

$$\frac{d^2y}{dx^2} = m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x}$$

$$(m_1^2 + a_1 m_1 + a_2)xe^{m_1 x} + (2m_1 + a_1)e^{m_1 x} = 0$$

$$(0)xe^{m_1 x} + (0)e^{m_1 x} = 0 \qquad 0 \equiv 0$$

$$\boxed{y_g = c_1 e^{m_1 x} + c_2 x e^{m_1 x}} \qquad m_1 = m_2 \in \mathbb{R}$$

$$\frac{d^2y}{dt^2} = 0 \qquad \frac{d}{dt}\left(\frac{dy}{dt}\right) = 0 \qquad \frac{dy}{dt} = c_1$$

$$\int dy = c_1 \int dt \qquad y = c_1 t + c_2$$

$$m^2 = 0 \qquad m_1 = m_2 = 0 \qquad y_1 = e^{(0)t} \qquad y_2 = te^{(0)t}$$

$$y_1 = 1 \qquad y_2 = t$$