Capítulo III.- Sistemas de Ecuaciones Lineales.

$$\begin{aligned} &(1)\rightarrow \quad \frac{dx}{dt} = 3x + 4y \\ &(2)\rightarrow \quad \frac{dy}{dt} = 2x + 5y \end{aligned}$$

S(2) EDO(1) LccH

$x(t)$ $y(t)$

Método por sustitución 2x2

De (2)

$$2x = \frac{dy}{dt} - 5y$$

$$x = \frac{1}{2}\frac{dy}{dt} - \frac{5}{2}y$$

$$\frac{dx}{dt} = \frac{1}{2}\frac{d^2y}{dt^2} - \frac{5}{2}\frac{dy}{dt}$$

En (1)

$$\left[\frac{1}{2}\frac{d^2y}{dt^2} - \frac{5}{2}\frac{dy}{dt}\right] = 3\left[\frac{1}{2}\frac{dy}{dt} - \frac{5y}{2}\right] + 4y$$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} = 3\left(\frac{dy}{dt} - 5y\right) + 8y$$

$$\boxed{\frac{d^2y}{dt^2} - 8\frac{dy}{dt} + 7y = 0} \rightarrow \text{EDO(2) LccH.}$$

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = 2x + 5y$$

S(2) EDO(1)

$m^2 - 8m + 7 = 0$ $\quad (m-1)(m-7) = 0$

$y(t) = c_1 e^{t} + c_2 e^{7t}$ $\quad m_1 = 1 \quad m_2 = 7 \quad m_1 \neq m_2 \in \mathbb{R}$

$$\frac{dy}{dt} = c_1 e^{t} + 7c_2 e^{7t}$$

$$x(t) = \frac{1}{2}\left(c_1 e^{t} + 7c_2 e^{7t}\right) - \frac{5}{2}\left(c_1 e^{t} + c_2 e^{7t}\right)$$

$$x(t) = -2c_1 e^{t} + c_2 e^{7t} \rightarrow \frac{dx}{dt} = -2c_1 e^{t} + 7c_2 e^{7t}$$
$$y(t) = c_1 e^{t} + c_2 e^{7t} \rightarrow \frac{dy}{dt} = c_1 e^{t} + 7c_2 e^{7t}$$

$$\frac{dx}{dt} = 3x + 4y$$
$$\frac{dy}{dt} = 2x + 5y$$

$$-2c_1 e^{t} + 7c_2 e^{7t} = -6c_1 e^{t} + 3c_2 e^{7t} + 4c_1 e^{t} + 4c_2 e^{7t}$$
$$-2c_1 e^{t} + 7c_2 e^{7t} = -2c_1 e^{t} + 7c_2 e^{7t}$$
$$(-2c_1 + 2c_1)e^{t} + (7c_2 - 7c_2)e^{7t} = 0$$
$$(0)e^{t} + (0)e^{7t} = 0$$
$$0 \equiv 0$$

$$\frac{dx}{dt} = 3x + 4y \qquad \bar{x} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \qquad \frac{d}{dt}\bar{x} = \begin{bmatrix} \frac{d}{dt}x(t) \\ \frac{d}{dt}y(t) \end{bmatrix}$$

$$\frac{dy}{dt} = 2x + 5y$$

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

Matriz Exponencial

$$\boxed{\frac{d}{dt}\bar{x} = A \cdot \bar{x}} \longrightarrow \bar{x} = \left[e^{At}\right]\bar{x}(0)$$

$$\begin{aligned} x(t) &= \_C1\, e^{7t} + \_C2\, e^{t} \\ y(t) &= \_C1\, e^{7t} - \frac{1}{2}\, \_C2\, e^{t} \end{aligned} \quad \left\} \begin{aligned} x(t) &= C_1 e^{7t} - 2C_2 e^{t} \\ y(t) &= C_1 e^{7t} + C_2 e^{t} \end{aligned} \right.$$

$$x(t) = \frac{2}{3} C_1 e^{t} + \frac{1}{3} C_1 e^{7t} + \frac{2}{3} C_2 e^{7t} - \frac{2}{3} C_2 e^{t}$$

$$y(t) = \frac{1}{3} C_1 e^{7t} - \frac{1}{3} C_1 e^{t} + \frac{1}{3} C_2 e^{t} + \frac{2}{3} C_2 e^{7t}$$

$$x(t) = \left(\frac{2}{3}C_1 - \frac{2}{3}C_2\right)e^{t} + \left(\frac{1}{3}C_1 + \frac{2}{3}C_2\right)e^{7t}$$

$$y(t) = \left(-\frac{1}{3}C_1 + \frac{1}{3}C_2\right)e^{t} + \left(\frac{1}{3}C_1 + \frac{2}{3}C_2\right)e^{7t}$$

$$C_{10} = -\frac{1}{3}C_1 + \frac{1}{3}C_2 \qquad C_{20} = \frac{1}{3}C_1 + \frac{2}{3}C_2$$

$$\begin{aligned} x(t) &= -2C_{10} e^{t} + C_{20} e^{7t} \\ y(t) &= C_{10} e^{t} + C_{20} e^{7t} \end{aligned}$$

$$\frac{d}{dx} e^{ax} = a e^{ax}$$

$$e^{ax}\Big|_{x=0} = 1$$

$$\frac{d}{dt}\left[e^{At}\right] = A \left[e^{At}\right]$$

$$\left[e^{At}\right]_{t=0} = I.$$

$$e^{-ax} = \left[e^{ax}\right]^{-1}$$

$$\left[e^{At}\right]^{-1} = e^{A(-t)}$$

$$e^{t} = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots + \frac{t^k}{k!}$$

$$e^{1} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \ldots\ldots$$

$$= 2 + 0.5 + 0.166 + 0.0416$$

$$= 2.70766$$

$$e^{at} = 1 + \frac{at}{1!} + \frac{a^2t^2}{2!} + \frac{a^3t^3}{3!} + \ldots + \frac{a^kt^k}{k!}$$

$$e^{at} = 1 + \frac{a}{1!}t + \frac{a^2}{2!}t^2 + \frac{a^3}{3!}t^3 + \ldots + \frac{a^k}{k!}t^k +$$

$$e^{At} = I + \frac{A}{1!}t + \frac{A^2}{2!}t^2 + \frac{A^3}{3!}t^3 + \ldots + \frac{A^k}{k!}t^k + .$$

Hamilton- Cayle

todo valor propio satisface su matriz A.

$$\det(A - \lambda I) = 0 \quad \lambda_1\ \lambda_2\ \lambda_3 \ldots \lambda_n$$

$$\lambda^n + a_1\lambda^{n-1} \ldots\ldots + a_{n-1}\lambda + a_n = 0$$

$$A^n + a_1 A^{n-1} + \ldots + a_{n-1} A + a_n I = [0]$$

$$A^n = -a_1 A^{n-1} - \ldots - a_{n-1}A - a_n I.$$

$n \times n$

$$e^{At} = \beta_0(t)\, I + \beta_1(t) A + \dots + \beta_{n-1}(t) A^{n-1}$$

$$e^{\lambda_1 t} = \beta_0(t)\,(1) + \beta_1(t)\,\lambda_1 + \dots + \beta_{n-1}(t)\,\lambda_1^{n-1}$$

$$e^{\lambda_2 t} = \beta_0(t)\,(1) + \beta_2(t)\,\lambda_2 + \dots + \beta_{n-1}(t)\,\lambda_2^{n-1}$$

$$\vdots$$

$$e^{\lambda_n t} = \underline{\qquad\qquad}$$

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 3-\lambda & 4 \\ 2 & 5-\lambda \end{vmatrix} = 0 \qquad (3-\lambda)(5-\lambda) - 8 = 0$$

$$\lambda^2 - 8\lambda + 15 - 8 = 0$$

$$\lambda^2 - 8\lambda + 7 = 0 \qquad \lambda_1 = 1$$
$$\lambda_2 = 7$$

$$e^{At} = B_0(t) I + B_1(t) A.$$

$$e^{t} = B_0(t) + B_1(t)$$
$$e^{7t} = B_0(t) + 7B_1(t)$$

$$e^{7t} - e^{t} = 6B_1(t) \longrightarrow B_1(t) = \frac{1}{6}\left(e^{7t} - e^{t}\right)$$

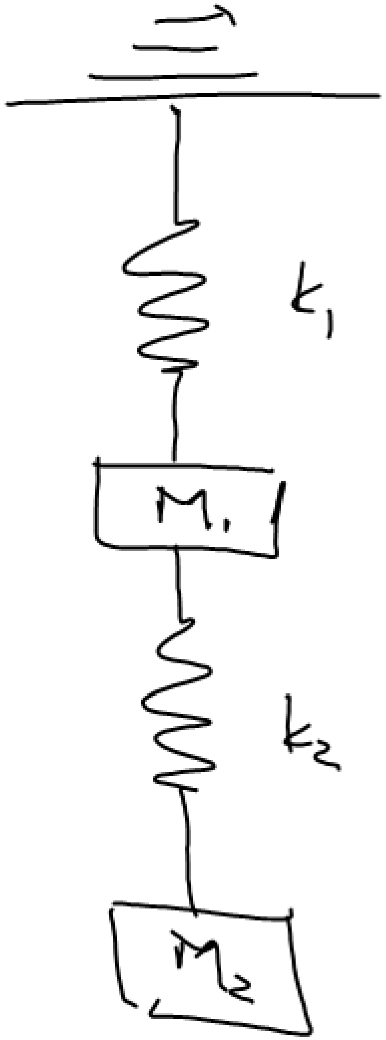
$$B_0(t) = e^{t} - B_1(t)$$
$$B_0(t) = e^{t} - \frac{1}{6}e^{7t} + \frac{1}{6}e^{t}$$
$$B_0(t) = -\frac{1}{6}e^{7t} + \frac{7}{6}e^{t}$$

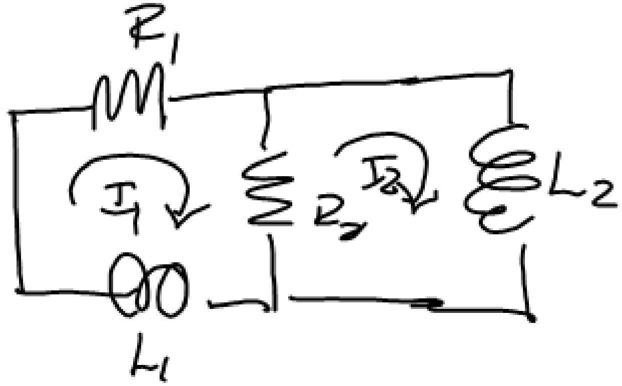
$$e^{At} = \left(-\frac{1}{6}e^{7t} + \frac{7}{6}e^{t}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{1}{6}e^{7t} - \frac{1}{6}e^{t}\right)\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} -\frac{1}{6}+\frac{3}{6} & \frac{4}{6} \\ \frac{2}{6} & -\frac{1}{6}+\frac{5}{6} \end{bmatrix} e^{7t} + \begin{bmatrix} \frac{7}{6}-\frac{3}{6} & -\frac{4}{6} \\ -\frac{2}{6} & \frac{7}{6}-\frac{5}{6} \end{bmatrix} e^{t}$$

$$e^{At} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & +\frac{2}{3} \end{bmatrix} e^{7t} + \begin{bmatrix} \frac{2}{3} & -\frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} e^{t}$$

$$e^{At}\Big|_{t=0} = \begin{bmatrix} \frac{3}{3} & 0 \\ 0 & \frac{3}{3} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$


k1
M1
k2
M2


R1
I1
R2
I2
L2
L1