Ejemplo 4. Dada la familia de curvas integrales

$$y^2-(x+C)^3=0$$

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}\cdot\frac{dy}{dx}=0$$

$$\left[(0)-3(x+C)^2\right]+\left[2y+(0)\right]\frac{dy}{dx}=0$$

$$y^2=(x+C)^3 \qquad y^{2/3}=x+C$$

$$C=y^{2/3}-x$$

$$-3\left(\cancel{x}+y^{2/3}-\cancel{x}\right)^2+2y\frac{dy}{dx}=0$$

$$-3\left(y^{2/3}\right)^2+2y\frac{dy}{dx}=0$$

$$-3y^{4/3}+2y\frac{dy}{dx}=0$$

313. $(xy' + y)^2 = y^2 y'$; $\quad y(C - x) = C^2$.

solución singular $\quad y(x) = 4x$

149. $y' = \dfrac{2xy}{3x^2 - y^2}$.

$(3x^2 - y^2)\, y' = 2xy$

$-2xy + (3x^2 - y^2)\, y' = 0$

$$\boxed{-\frac{2}{y^3}x + \left(\frac{3x^2}{y^4} - \frac{1}{y^2}\right)\frac{dy}{dx} = 0}$$

$\dfrac{\partial M}{\partial y} = -2x \qquad \dfrac{\partial N}{\partial x} = 6x \qquad$ NO EXACTA

$\mu(y)$

$\dfrac{d\mu}{\mu} = \left(\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\right) dy$

$\dfrac{d\mu}{\mu} = \left(\dfrac{6x + 2x}{-2xy}\right) dy$

$\dfrac{d\mu}{\mu} = \left(-\dfrac{8x}{2xy}\right) dy$

$\dfrac{d\mu}{\mu} = -\dfrac{4}{y}\, dy$

$\displaystyle\int \frac{d\mu}{\mu} = -4 \int \frac{dy}{y}$

$L\mu = -4\, Ly$

$L\mu = L\, y^{-4}$

$$\boxed{\mu = \frac{1}{y^4}}$$

$$\left\{ \begin{array}{l} \dfrac{\partial M}{\partial y} = -2x\left(\dfrac{-3}{y^4}\right) \Rightarrow \dfrac{6x}{y^4} \\ \dfrac{\partial N}{\partial x} = \dfrac{6x}{y^4} + (0) \Rightarrow \dfrac{6x}{y^4} \end{array} \right.$$

EXACTA

$\displaystyle\int M dx = -\frac{1}{y^3}\int 2x\, dx$

$= \dfrac{-x^2}{y^3}$

$\dfrac{\partial}{\partial y}\displaystyle\int M dx = \frac{3x^2}{y^4}$

$N - \dfrac{\partial}{\partial y}\displaystyle\int M dx = \left(\cancel{\frac{3x^2}{y^4}} - \frac{1}{y^2}\right) - \cancel{\frac{3x^2}{y^4}}$

SOLGRAL =

$\displaystyle\int M dx + \int\left(N - \frac{\partial}{\partial y}\int M dx\right) dy = C_1$

SOLGRAL $\Rightarrow \; -\dfrac{x^2}{y^3} + \displaystyle\int -\frac{dy}{y^2} = C_1$

$$\boxed{-\frac{x^2}{y^3} + \frac{1}{y} = C_1}$$

149. $y' = \frac{2xy}{3x^2 - y^2}.$

$$-2xy + (3x^2 - y^2)y' = 0$$

$$-2(\lambda x)(\lambda y) = \lambda^2(-2xy) \quad m = 2$$

$$3(\lambda x)^2 - (\lambda y)^2 = 3\lambda^2 x^2 - \lambda^2 y^2 = \lambda^2(3x^2 - y^2) \quad n = 2$$

$m = n$

COEFICIENTES HOMOGENEOS.

$$y = vx \qquad \frac{dy}{dx} = x\frac{dv}{dx} + v$$

$$-2x(vx) + (3x^2 - (vx)^2)\left(x\frac{dv}{dx} + v\right) = 0$$

$$-2vx^2 + 3x^3\frac{dv}{dx} + 3vx^2 - v^2x^3\frac{dv}{dx} - v^3x^2 = 0$$

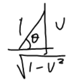
$$(vx^2 - v^3x^2) + (3x^3 - v^2x^3)\frac{dv}{dx} = 0$$

$$x^2(v - v^3) + x^3(3 - v^2)\frac{dv}{dx} = 0$$

$$\frac{1}{x} + \left(\frac{3 - v^2}{v - v^3}\right)\frac{dv}{dx} = 0$$

$$\frac{dx}{x} + \left(\frac{3 - v^2}{v - v^3}\right)dv = 0$$

$$u = v - v^3 \qquad du = (-3v^2 + 1)dv$$



$$\int\frac{dx}{x} + \frac{1}{3}\int\frac{-3v^2 + 9}{v - v^3}dv = C_1$$

$$\frac{v}{1} = \operatorname{sen}(\theta)$$

$$dv = \cos(\theta)\,d\theta$$

$$\int\frac{dx}{x} + \frac{1}{3}\int\frac{-3v^2 + 1}{v - v^3}dv + \frac{8}{3}\int\frac{dv}{v - v^3} = C_1$$

$$Lx + \frac{1}{3}L(v - v^3) + \frac{8}{3}\int\frac{dv}{v(1 - v^2)} = C_1$$

$$\sqrt{1 - v^2} = \cos(\theta)$$

$$Lx + \frac{1}{3}L(v - v^3) + \frac{8}{3}\int\frac{\cos(\theta)\,d\theta}{\operatorname{sen}(\theta)\cdot\cos^2(\theta)} = C_1$$

$$Lx + \frac{1}{3}L(v - v^3) + \frac{8}{3}\int\frac{d\theta}{\operatorname{sen}(\theta)\cos(\theta)} = C_1$$

$$+\frac{8}{3}\int\frac{2\,d\theta}{\operatorname{sen}(2\theta)} = C_1$$

$$+\frac{8}{3}\int\csc(2\theta)\,2\,d\theta = C_1$$

$$Lx + \frac{1}{3}L(v - v^3) + \frac{8}{3}L(\csc(2\theta) - \cot(2\theta)) = C_1$$

$$Lx + \frac{1}{3}L(v - v^3) + \frac{8}{3}L\left(\frac{1}{v} - \frac{\sqrt{1 - v^2}}{v}\right) = C_1$$

$$Lx + \frac{1}{3}L\left(\frac{y}{x} - \left(\frac{y}{x}\right)^3\right) + \frac{8}{3}L\left(\frac{x}{y} - \frac{x}{y}\sqrt{1 - \left(\frac{y}{x}\right)^2}\right) = C_1$$

$$y = vx \qquad v = \frac{y}{x}$$