URSUL SOLANES JUAN

FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SEMESTRE 2014-2
SEGUNDO EXAMEN PARCIAL

ABRIL 21 DE 2014

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> restart
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1) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL LINEAL (sin usar dsolve o relativos)

$$2x^2y(x)+2y(x)+5+\left(2x^3+2x\right)\left(\frac{d}{dx}y(x)\right)=0 \quad (1)$$

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> restart
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RESPUESTA 1)

```
> Ecuacion := 2*x^2*y(x)+2*y(x)+5+(2*x^3+2*x)*(d/dx y(x))=0
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$$Ecuacion := 2x^2y(x)+2y(x)+5+\left(2x^3+2x\right)\left(\frac{d}{dx}y(x)\right)=0 \quad (2)$$

```
> with(DEtools):
> odeadvisor(Ecuacion)
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$$[\_linear] \quad (3)$$

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> FactInt := intfactor(Ecuacion)
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$$FactInt := \frac{1}{x^2+1} \quad (4)$$

```
> M := 2*x^2*y+2*y+5
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$$M := 2x^2y+2y+5 \quad (5)$$

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> N := 2*x^3+2*x
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$$N := 2x^3+2x \quad (6)$$

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> MM := M*FactInt
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$$MM := \frac{2x^2y+2y+5}{x^2+1} \quad (7)$$

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> NN := N*FactInt
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$$NN := \frac{2x^3+2x}{x^2+1} \quad (8)$$

```
> Comprobacion := simplify(diff(MM,y)-diff(NN,x))=0
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$$Comprobacion := 0=0 \quad (9)$$

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> IntMMx := int(MM,x)
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$$IntMMx := 2xy+5\arctan(x) \quad (10)$$

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> Solucion := simplify(IntMMx+int((NN-diff(IntMMx,y)),y))=C[1]
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$$Solucion := 2xy+5\arctan(x)=C_1 \quad (11)$$

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> Comprobar := isolate(dsolve(Ecuacion),_C1)
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$$Comprobar := \_C1=y(x)\,x+\frac{5}{2}\arctan(x) \quad (12)$$

FIN RESPUESTA 1)

> *restart*

2) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$y(x) + \sqrt{y(x)^2 - x^2} - x\left(\frac{d}{dx}y(x)\right) = 0 \quad (13)$$

> *restart*

RESPUESTA 2)

> $Ecuacion := y(x) + \sqrt{y(x)^2 - x^2} - x\left(\frac{d}{dx}y(x)\right) = 0$

$$Ecuacion := y(x) + \sqrt{y(x)^2 - x^2} - x\left(\frac{d}{dx}y(x)\right) = 0 \quad (14)$$

> *with(DEtools) :*

> *odeadvisor(Ecuacion)*

$$[[\_homogeneous, class\ A], \_rational, \_dAlembert] \quad (15)$$

> $EcuacionDos := simplify(isolate(eval(subs(y(x) = v(x)\cdot x, Ecuacion)), diff(v(x), x)))$

$$EcuacionDos := \frac{d}{dx}v(x) = \frac{\sqrt{x^2\left(v(x)^2 - 1\right)}}{x^2} \quad (16)$$

> $P := \frac{1}{x}; Q := \sqrt{(v^2 - 1)}$

$$P := \frac{1}{x}$$

$$Q := \sqrt{v^2 - 1} \quad (17)$$

> $P\cdot Q$

$$\frac{\sqrt{v^2 - 1}}{x} \quad (18)$$

> $Solucion := isolate\left(int\left(\frac{1}{Q}, v\right) - int(P, x) = \log(C_1), C_1\right)$

$$Solucion := C_1 = \frac{v + \sqrt{v^2 - 1}}{x} \quad (19)$$

> $SolucionDos := simplify\left(subs\left(v = \frac{y}{x}, Solucion\right)\right)$

$$SolucionDos := C_1 = \frac{y + \sqrt{-\frac{-y^2 + x^2}{x^2}}\,x}{x^2} \quad (20)$$

> $SolucionTres := \left(lhs(SolucionDos) - \frac{y}{x..2}\right)..2 = simplify\left(\left(rhs(SolucionDos) - \frac{y}{x..2}\right)\cdot\right.$
> $\left.\cdot 2\right)$

$$SolucionTres := \left(C_1 - \frac{y}{x^2}\right)^2 = -\frac{-y^2 + x^2}{x^4} \quad (21)$$

> $SolucionGeneral := isolate(SolucionTres, C_1)$

$$SolucionGeneral := C_1 = \sqrt{-\frac{-y^2+x^2}{x^4}} + \frac{y}{x^2} \quad \textbf{(22)}$$

> $comprobar := dsolve(Ecuacion)$

$$comprobar := \frac{y(x)}{x^2} + \frac{\sqrt{y(x)^2-x^2}}{x^2} - \_C1 = 0 \quad \textbf{(23)}$$

>

FIN RESPUESTA 2)

> *restart*

3) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$1 - x^2 y(x) + x^2 (y(x) - x)\left(\frac{d}{dx} y(x)\right) = 0 \quad \textbf{(24)}$$

> *restart*

RESPUESTA 3)

> $Ecuacion := 1 - x^2 y(x) + x^2 (y(x) - x)\left(\frac{d}{dx} y(x)\right) = 0$

$$Ecuacion := 1 - x^2 y(x) + x^2 (y(x) - x)\left(\frac{d}{dx} y(x)\right) = 0 \quad \textbf{(25)}$$

> *with(DEtools) :*

> *odeadvisor(Ecuacion)*

[_rational, [_1st_order, _with_symmetry_[F(x),G(x)]], [_Abel, 2nd type, class B]] **(26)**

> $FactInt := intfactor(Ecuacion)$

$$FactInt := \frac{1}{x^2} \quad \textbf{(27)}$$

> $M := 1 - x^2 y;\ N := x^2 (y - x)$

$$M := 1 - x^2 y$$
$$N := x^2 (y - x) \quad \textbf{(28)}$$

> $MM := expand(M \cdot FactInt);\ NN := N \cdot FactInt$

$$MM := \frac{1}{x^2} - y$$
$$NN := y - x \quad \textbf{(29)}$$

> $Comprobacion := simplify(diff(MM, y) - diff(NN, x)) = 0$

$$Comprobacion := 0 = 0 \quad \textbf{(30)}$$

> $IntMMx := int(MM, x)$

$$IntMMx := -\frac{1}{x} - y x \quad \textbf{(31)}$$

> $SolucionGeneral := IntMMx + int((NN - diff(IntMMx, y)), y) = C_1$

$$SolucionGeneral := -\frac{1}{x} - y x + \frac{1}{2} y^2 = C_1 \quad \textbf{(32)}$$

>

FIN RESPUESTA 3)

> *restart*

4) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$\left(1+y(x)^2\right)\left(e^{2x}-e^{y(x)}\left(\frac{d}{dx}y(x)\right)\right)-(1+y(x))\left(\frac{d}{dx}y(x)\right)=0 \quad (33)$$

> *restart*

RESPUESTA 4)

> $Ecuacion := \left(1+y(x)^2\right)\left(e^{2x}-e^{y(x)}\left(\frac{d}{dx}y(x)\right)\right)-(1+y(x))\left(\frac{d}{dx}y(x)\right)=0$

$$Ecuacion := \left(1+y(x)^2\right)\left(e^{2x}-e^{y(x)}\left(\frac{d}{dx}y(x)\right)\right)-(1+y(x))\left(\frac{d}{dx}y(x)\right)=0 \quad (34)$$

> *with*(*DEtools*) :

> *odeadvisor*(*Ecuacion*)

$$[\_separable] \quad (35)$$

> *EcuacionDos* := *simplify*(*isolate*(*Ecuacion*, *diff*(*y*(*x*), *x*)))

$$EcuacionDos := \frac{d}{dx}y(x)=\frac{\left(1+y(x)^2\right)e^{2x}}{e^{y(x)}+y(x)^2e^{y(x)}+1+y(x)} \quad (36)$$

> $M := -\left(\left(1+y^2\right)e^{2x}\right)$

$$M := -\left(1+y^2\right)e^{2x} \quad (37)$$

> $N := e^y+y^2e^y+1+y$

$$N := e^y+e^y y^2+1+y \quad (38)$$

> $P := \exp(2x); Q := -(1+y\cdot\cdot 2); R := 1; S := N$

$$P := e^{2x}$$

$$Q := -1-y^2$$

$$R := 1$$

$$S := e^y+e^y y^2+1+y \quad (39)$$

> $SolucionGeneral := int\left(\frac{P}{R},x\right)+int\left(\frac{S}{Q},y\right)=C_1$

$$SolucionGeneral := \frac{1}{2}e^{2x}-e^y-\arctan(y)-\frac{1}{2}\ln\left(1+y^2\right)=C_1 \quad (40)$$

>

FIN RESPUESTA 4)

> *restart*

5) (20/100 puntos) OBTENER LA SOLUCIÓN GENERAL DE LA SIGUIENTE ECUACIÓN DIFERENCIAL NO LINEAL (sin usar dsolve o relativos)

$$2x+\frac{x^2+y(x)^2}{x^2y(x)}-\frac{\left(x^2+y(x)^2\right)\left(\frac{d}{dx}y(x)\right)}{x\,y(x)^2}=0 \quad (41)$$

> *restart*

RESPUESTA 5)

> $Ecuacion := 2\,x + \frac{x^2 + y(x)^2}{x^2\,y(x)} - \frac{\left(x^2 + y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right)}{x\,y(x)^2} = 0$

$$Ecuacion := 2\,x + \frac{x^2 + y(x)^2}{x^2\,y(x)} - \frac{\left(x^2 + y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right)}{x\,y(x)^2} = 0 \quad (42)$$

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[[\_homogeneous,\ class\ D],\ \_exact,\ \_rational] \quad (43)$$

> $M := 2\,x + \frac{x^2 + y^2}{x^2\,y}$

$$M := 2\,x + \frac{x^2 + y^2}{x^2\,y} \quad (44)$$

> $N := -\frac{\left(x^2 + y^2\right)}{x\,y^2}$

$$N := -\frac{x^2 + y^2}{x\,y^2} \quad (45)$$

> $Comprobacion := simplify(diff(M, y) - diff(N, x)) = 0$

$$Comprobacion := 0 = 0 \quad (46)$$

> $IntMx := int(M, x)$

$$IntMx := x^2 + \frac{x - \frac{y^2}{x}}{y} \quad (47)$$

> $SolucionGeneral := simplify(IntMx + int((N - diff(IntMx, y)), y)) = C_1$

$$SolucionGeneral := \frac{x^3 y + x^2 - y^2}{x\,y} = C_1 \quad (48)$$

>

>

> $restart$

> $Ecuacion := (x\cdot\cdot 2 + y(x)\cdot\cdot 2) + (x\cdot\cdot 2 - x\cdot y(x))\cdot diff(y(x), x) = 0$

$$Ecuacion := x^2 + y(x)^2 + \left(x^2 - x\,y(x)\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \quad (49)$$

> $with(DEtools):$

> $odeadvisor(Ecuacion)$

$$[[\_homogeneous,\ class\ A],\ \_rational,\ [\_Abel,\ 2nd\ type,\ class\ B]] \quad (50)$$

> $EcuacionDos := simplify(isolate(eval(subs(y(x) = v(x)\cdot x, Ecuacion)), diff(v(x), x)))$

$$EcuacionDos := \frac{d}{dx}\,v(x) = \frac{1 + v(x)}{x\,(-1 + v(x))} \quad (51)$$

> $P := \frac{1}{x};\ Q := \frac{1 + v}{(-1 + v)}$

$$P := \frac{1}{x}$$

$$Q := \frac{1+v}{-1+v} \tag{52}$$

> $P \cdot Q$

$$\frac{1+v}{x\,(-1+v)} \tag{53}$$

> $Solucion := simplify\left(int(P, x) - int\left(\frac{1}{Q}, v\right) = C_1\right)$

$$Solucion := \ln(x) - v + 2\ln(1+v) = C_1 \tag{54}$$

> $SolucionDos := isolate\left(subs\left(v = \frac{y}{x}, Solucion\right), C_1\right)$

$$SolucionDos := C_1 = \ln(x) - \frac{y}{x} + 2\ln\left(1 + \frac{y}{x}\right) \tag{55}$$

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FIN DEL EXAMEN