$\mathcal{L}\{f(t)\} = F(s)$ es única

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$\mathcal{L}^{-1}\{F(s)\} = f(t)$ <u>no es única</u>

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} F(s)\,ds.$$

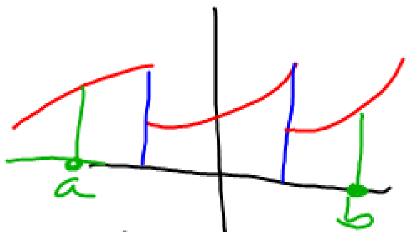
Teorema de existencia de la TL.

Si tenemos $f(t)$ existirá su $F(s)$ cuando:

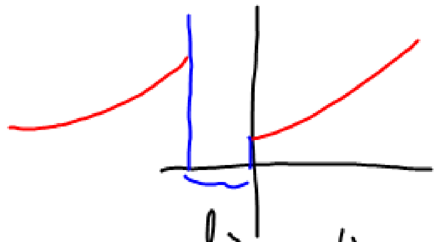
a) $f(t)$ sea de orden exponencial

$$|f(t)| \le Me^{At} \quad M, A \in \mathbb{R}$$

b) sea seccionalmente continua

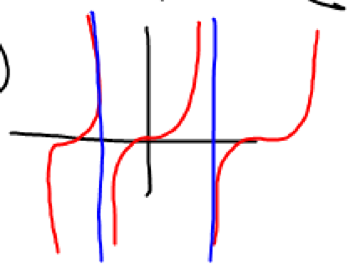


seccionalmente continua



discontinua

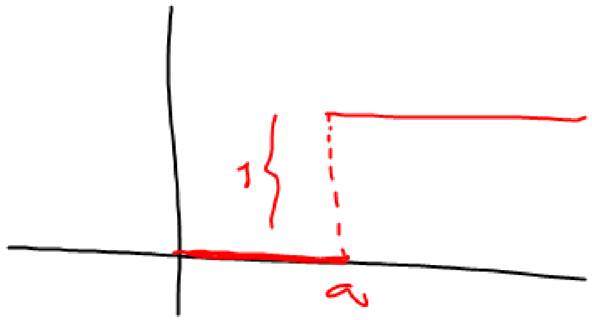
tan(x)



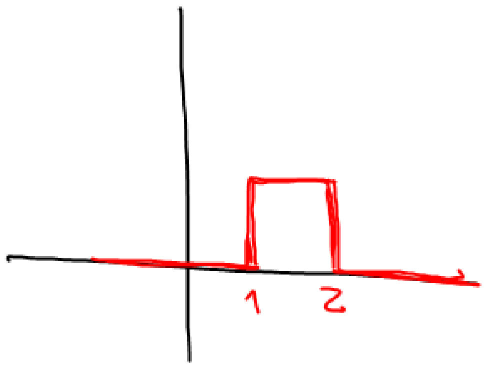
ESCALÓN UNITARIO

$$u(t)=\begin{cases} 0\ ; & t<a \\ 1\ ; & t\geq a \end{cases}$$

Heaviside$(t-a)$



$$\text{Pulso}(t)=\mu(t-1)-\mu(t-2)$$



$$X \quad e^{x^2} \quad e^{x^3} \quad e^{x^4} \quad e^{x^n} \quad n\neq 1.$$

$$\mathcal{L}\{u(t-a)\} = \int_0^\infty e^{-st} u(t-a)\,dt$$

$$= \cancelto{0}{\int_0^a e^{-st}\cdot(0)\cdot dt} + \int_a^\infty e^{-st}(1)\,dt$$

$$\mathcal{L}\{u(t-a)\} = \left(\int e^{-st}\,dt\right)_a^\infty \Rightarrow \left(\frac{e^{-st}}{-s}\right)_a^\infty$$

$$= -\frac{1}{s}\left(\cancelto{0}{\lim_{b\to\infty} e^{-sb}} - e^{-as}\right)$$

$$\boxed{\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}}$$

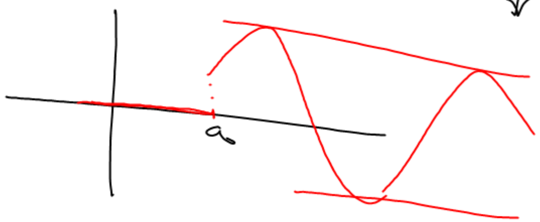
$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\cdot u(t-a)$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-as}}{s^2}\right\} = (t-a)\cdot u(t-a)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-as}}{s^2+1}\right\} = \text{sen}(t-a)\cdot u(t-a)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \text{sen}(t)$$



$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$$\mathcal{L}\{e^{at}\cos(bt)\} = \frac{(s-a)}{(s-a)^2 + b^2}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\boxed{\mathcal{L}\{y\} = \frac{\frac{1}{s-1} + 4s - 23}{(s^2 - 5s + 6)}}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$k\mathcal{L}\{e^{at}\} = \frac{k}{s-a}$$

$$\mathcal{L}\{y\} = \frac{1 + (4s-23)(s-1)}{(s-1)(s^2-5s+6)}$$

$$\mathcal{L}\{y\} = \frac{4s^2 - 27s + 24}{(s-1)(s-2)(s-3)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{D}{s-3}$$

$$\Rightarrow 4s^2 - 27s + 24 = A(s-2)(s-3) + B(s-1)(s-3) + D(s-1)(s-2)$$

si $s = 1$

$$4(1)^2 - 27(1) + 24 = A(-1)(-2) + (0) + (0)$$

$$4 - 27 + 24 = 2A$$

$$1 = 2A \qquad \boxed{A = \frac{1}{2}}$$

si $s = 2$

$$4(2)^2 - 27(2) + 24 = B(1)(-1)$$

$$16 - 54 + 24 = -B$$

$$-14 = -B \qquad \boxed{B = 14}$$

si $s = 3$

$$4(3)^2 - 27(3) + 24 = D(2)(1)$$

$$36 - 81 + 24 = 2D$$

$$-21 = 2D \qquad \boxed{D = -\frac{21}{2}}$$

$$\mathcal{L}\{y\} = \frac{\frac{1}{2}}{s-1} + \frac{14}{s-2} - \frac{\frac{21}{2}}{s-3}$$

$$y = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + 14\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{21}{2}\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

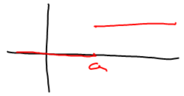
$$\boxed{y(t) = \frac{1}{2}e^{t} + 14e^{2t} - \frac{21}{2}e^{3t}}$$

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = e^{t} \qquad \begin{matrix} y(0) = 4 \\ y'(0) = -3 \end{matrix}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} = s^2\mathcal{L}\{y\} - s\cdot y(0) - y'(0)$$

escalón unitario

$$u(t-a) = \begin{cases} 0 \; ; & t<a \\ 1 \; : & t \geq a \end{cases}$$



rampa unitaria

$$r(t-a) = \begin{cases} 0 \; ; & t<a \\ (t-a) \; : & t \geq a \end{cases}$$

$$\mathcal{L}\{r(t-a)\} = \int_0^\infty e^{-st} r(t-a)\,dt$$

$$= \int_0^a e^{-st}\cdot(0)\cdot dt + \int_a^\infty e^{-st}(t-a)\,dt$$

$$= (0) + \int_a^\infty e^{-st}\,t\,dt - a\int_a^\infty e^{-st}\,dt$$

$$= \frac{e^{-as}}{s^2} - \frac{a e^{-as}}{s}$$

$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = s\mathcal{L}\{r(t-a)\} - \cancel{r(0-a)}^{\,0}$$

$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = s\left(\frac{e^{-as}}{s^2} - \frac{a e^{-as}}{s}\right)$$

$$= \frac{e^{-as}}{s} - \cancel{a e^{-as}}^{\,0}$$

$$= \frac{e^{-as}}{s}$$

$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = \mathcal{L}\{u(t-a)\}.$$

$$\frac{d}{dt} r(t-a) = u(t-a).$$