Propiedades de la Transformada de Laplace

① $\boxed{\mathcal{L}\{af(t)+bg(t)\} = aF(s)+bG(s)}$ $\quad q, a, b \in \mathbb{R}$, $s \in \mathbb{C}$.

$\mathcal{L}\{f(t)\} = F(s)$

② $\boxed{\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)}$

$$\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1}$$

$$\mathcal{L}\{\cos(4t)\} = \frac{1}{4}\left(\frac{\frac{s}{4}}{\left(\frac{s}{4}\right)^2+1}\right)$$

$$= \frac{\frac{s}{16}}{\frac{s^2}{16}+1}$$

$$\mathcal{L}\{\cos(4t)\} = \frac{\frac{s}{16}}{\frac{s^2+16}{16}} \Rightarrow \frac{s}{s^2+16}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

3 $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} \cdot f(0) - s^{n-2} \cdot f'(0) - \ldots - f^{(n-1)}(0)$$

4

$$\mathcal{L}^{-1}\{F'(s)\} = -t\,f(t)$$

5

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

6

$$\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \frac{f(t)}{t}$$

7 $\mathcal{L}^{-1}\left\{ e^{-as} F(s) \right\} = f(t-a) \cdot u(t-a)$

8 $\mathcal{L}\left\{ e^{at} f(t) \right\} = F(s-a)$

9 $\mathcal{L}^{-1}\left\{ F(s) \cdot G(s) \right\} = f(t) * g(t)$

convolución

$$f(t) * g(t) = \int_0^t f(z) \cdot g(t-z)\, dz.$$

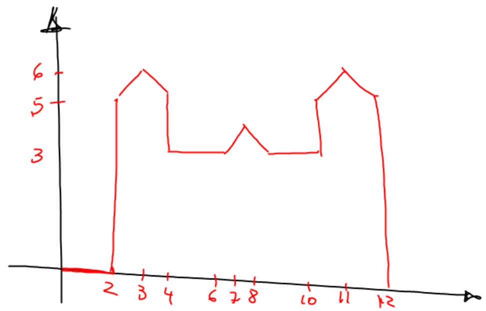
$$\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{(s-5)^2}\right\} = f(t-6)\cdot u(t-6)$$

$$\mathcal{L}^{-1}\left\{e^{-as}F(s)\right\} = f(t-a)\cdot u(t-a)$$

$$\rightarrow \mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-5)^2}\right\} = e^{5t}\cdot t \qquad \boxed{\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{(s-5)^2}\right\} = (t-6)e^{5(t-6)}\cdot u(t-6)$$

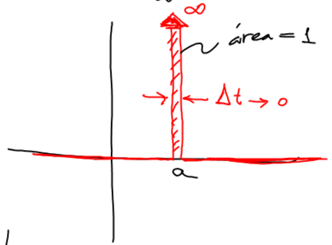


$$u(t-a) \Rightarrow \text{Heaviside}(t-a)$$

$$r(t-a) \Rightarrow (t-a)\cdot\text{Heaviside}(t-a)$$

$$\delta(t-a) \Rightarrow \text{Dirac}(t-a)$$

$$\delta(t-a) = \begin{cases} 0 \; ; \; t \neq a \\ \int_{-\infty}^{\infty} \delta(t-a)\,dt = 1 \end{cases}$$



$$\mathcal{L}\{\delta(t-a)\} = e^{-as} \qquad \delta(t-a)\Big|_{t=0} = 0$$

$$\mathcal{L}\left\{\frac{d}{dt}u(t-a)\right\} = \cancel{s}\,\mathcal{L}\left\{\frac{e^{-as}}{\cancel{s}}\right\} - \cancel{u(t-a)}^{\,0}\Big|_{t=0}$$

$$\mathcal{L}\left\{\frac{d}{dt}u(t-a)\right\} = \mathcal{L}\{\delta(t-a)\}$$

$$\boxed{\frac{d}{dt}u(t-a) = \delta(t-a)}$$

$$\frac{d}{dt}r(t-a) = u(t-a)$$

$$F(s)=\frac{4s}{(s^2+16)^2} \Rightarrow \left(\frac{s}{s^2+16}\right)\cdot\left(\frac{4}{s^2+16}\right)$$

$$\mathcal{L}^{-1}\{F(s)\}=\mathcal{L}^{-1}\left\{\left(\frac{s}{s^2+16}\right)\cdot\left(\frac{4}{s^2+16}\right)\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+16}\right\}=\cos(4t)$$

$$\mathcal{L}^{-1}\left\{\frac{4}{s^2+16}\right\}=\operatorname{sen}(4t)$$

$$\mathcal{L}^{-1}\{F(s)\}=\cos(4t)*\operatorname{sen}(4t)$$

$$\cos(4t)*\operatorname{sen}(4t)=\int_0^t\cos(4\tau)\cdot\operatorname{sen}(4(t-\tau))\,d\tau.$$

$$=\int_0^t\cos(4\tau)\left[\operatorname{sen}(4t)\cos(4\tau)-\cos(4t)\cdot\operatorname{sen}(4\tau)\right]d\tau.$$

$$=\left[\operatorname{sen}(4t)\int\cos^2(4\tau)\,d\tau-\cos(4t)\int\cos(4\tau)\operatorname{sen}(4\tau)\,d\tau\right]_0^t$$

$$=\left[\operatorname{sen}(4t)\int\left(\frac{1}{2}+\frac{1}{2}\cos(8\tau)\right)d\tau-\frac{\cos(4t)}{4}\int\operatorname{sen}(4\tau)\cdot 4\cos(4\tau)\,d\tau\right]_0^t$$

$$=\left[\frac{\operatorname{sen}(4t)}{2}\int d\tau+\frac{\operatorname{sen}(4t)}{16}\int 8\cos(8\tau)\,d\tau-\frac{\cos(4t)}{8}\operatorname{sen}^2(4\tau)\right]_0^t$$

$$=\left[\frac{\operatorname{sen}(4t)}{2}(\tau)+\frac{\operatorname{sen}(4t)}{16}\cdot\operatorname{sen}(8\tau)-\frac{\cos(4t)}{8}\operatorname{sen}^2(4\tau)\right]_0^t$$

$$=\left[\frac{\operatorname{sen}(4t)}{2}(\tau)+\frac{\operatorname{sen}(4t)}{16}\left(2\operatorname{sen}(4\tau)\cos(4\tau)\right)-\frac{\cos(4t)}{8}\operatorname{sen}^2(4\tau)\right]_0^t$$

$$=\frac{\operatorname{sen}(4t)}{2}(t-0)+\frac{\operatorname{sen}(4t)}{8}\left(\operatorname{sen}(4t)\cos(4t)-0\right)-\frac{\cos(4t)}{8}\left(\operatorname{sen}^2(4t)-0\right)$$

$$=\frac{t\operatorname{sen}(4t)}{2}+\cancel{\frac{\operatorname{sen}^2(4t)\cos(4t)}{8}}-\cancel{\frac{\cos(4t)\operatorname{sen}^2(4t)}{8}}$$

$$\boxed{\mathcal{L}^{-1}\{F(s)\}=\frac{t\operatorname{sen}(4t)}{2}}$$

$$\mathcal{L}\left\{\frac{4s}{(s^2+16)^2}\right\}=\frac{t\operatorname{sen}(4t)}{2}$$