Convolución

$$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(x) * g(x)$$

$$f(x) * g(x) = \int_0^x f(\sigma)\cdot g(x-\sigma)\,d\sigma$$

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$$

$$= aF(s) + bG(s)$$

$f(t)$

$\mathcal{L}\{f(t)\} = F(s)$

$\mathcal{L}^{-1}\{F(s)\} = f(t)$

$$f(t) = 4t^2 - e^{-t} - \int_0^t f(\tau)\, e^{t-\tau}\, d\tau$$

$$\mathcal{L}\{f(t)\} = 4\mathcal{L}\{t^2\} - \mathcal{L}\{e^{-t}\} - F(s)\mathcal{L}\{e^{t}\}$$

$$F(s) = 4\left(\frac{2!}{s^3}\right) - \frac{1}{s+1} - \frac{F(s)}{s-1}$$

$$F(s) = \frac{8}{s^3} - \frac{1}{s+1} - \frac{F(s)}{s-1}$$

$$F(s) + \frac{F(s)}{s-1} = \frac{8(s+1) - s^3}{s^3(s+1)}$$

$$\frac{F(s)(s-1) + F(s)}{(s-1)} = \frac{-s^3 + 8s + 8}{s^3(s+1)}$$

$$F(s)\,s - \cancel{F(s)} + \cancel{F(s)} = \frac{(-s^3 + 8s + 8)(s-1)}{s^3(s+1)}$$

$$F(s) = \frac{-s^4 + 8s^2 + \cancel{8s} + s^3 - \cancel{8s} - 8}{s^4(s+1)}$$

$$\boxed{F(s) = \frac{-s^4 + s^3 + 8s^2 - 8}{s^4(s+1)}}$$

$$\frac{-s^4 + s^3 + 8s^2 - 8}{s^4(s+1)} = \frac{A}{s+1} + \frac{B}{s^4} + \frac{C}{s^3} + \frac{D}{s^2} + \frac{E}{s}$$

$$-s^4 + s^3 + 8s^2 - 8 = As^4 + B(s+1) + Cs(s+1) + D(s+1)s^2 + Es^3(s+1)$$

si $s = -1$

$$Cs^2 + Cs + Ds^3 + Ds^2 + Es^4 + Es^3$$

$$-(-1)^4 + (-1)^3 + 8(-1)^2 - 8 = A(-1)^4$$

$$-1 - 1 + 8 - 8 = A$$

$$\boxed{A = -2}$$

$s = 0$

$$-8 = B(1) \qquad \boxed{B = -8}$$

$$-4s^3 + 3s^2 + 16s = 4As^3 + B + C(2s+1) + D(3s^2 + 2s) + E(4s^3 + 3s^2)$$

$s = 0$

$$0 = -8 + C \qquad \boxed{C = 8}$$

$$-12s^2 + 6s + 16 = 12As^2 + 2C + D(6s+2) + E(12s^2 + 6s)$$

$$16 = 2(8) + 2D \qquad \boxed{D = 0}$$

$$-24s + 6 = 24As + 6D + E(24s + 6)$$

$$\boxed{s = 0}$$

$$6 = 6E \qquad \boxed{E = 1}$$

$$F(s) = \frac{-2}{s+1} + \frac{-8}{s^4} + \frac{8}{s^3} + \frac{0}{s^2} + \frac{1}{s}$$

$$\mathcal{L}\{F(s)\} = -2e^{-t} - \frac{8}{4!}\mathcal{L}\left\{\frac{4!}{s^4}\right\} + \mathcal{L}\left\{\frac{1}{s}\right\}$$

$$f(t) = -2e^{-t} - \frac{8}{24}t^3 + t$$

$$f(t) = -2e^{-t} - \frac{1}{3}t^3 + t$$

Método de Variables Separables

$z(x,y) \qquad \frac{\partial^2 z}{\partial x^2} - 6\frac{\partial z}{\partial y} = 0$

"prueba y error"

Hipótesis inicial (H_0)

$H_0: z(x,y) = F(x)\cdot G(y)$

$\frac{\partial z}{\partial x} = F(x)\cdot \frac{d}{dx}G(y) + G(y)\cdot\frac{d}{dx}F(x)$

$\boxed{\frac{\partial z}{\partial x} = F'(x)\cdot G(y)}$

$\frac{\partial z}{\partial y} = \frac{d}{dy}F(x)\cdot G(y) + F(x)\frac{d}{dy}G(y)$

$\boxed{\frac{\partial z}{\partial y} = F(x)\, G'(y)}$

$\frac{\partial^2 z}{\partial x^2} = \frac{d}{dx}F'(x)\cdot G(y) + \frac{d}{dx}G(y)\cdot F'(x)$

$\boxed{\frac{\partial^2 z}{\partial x^2} = F''(x)\cdot G(y)}$

$F''(x)\cdot G(y) - 6\,F(x)\cdot G'(y) = 0$

$F''(x)\cdot G(y) = 6\,F(x)\,G'(y)$

$\boxed{\text{éxito}}$

$\frac{F''(x)}{6F(x)} = \frac{G'(y)}{G(y)}$

$$\frac{F''(x)}{6F(x)} = \frac{G'(y)}{G(y)}$$

$$\frac{F''(x)}{6F(x)} = \alpha \qquad \frac{G'(y)}{G(y)} = \alpha$$

$$F''(x) = 6\alpha F(x) \qquad G'(y) = \alpha G(y)$$

$$\boxed{F''(x) - 6\alpha F(x) = 0} \qquad \boxed{G'(y) - \alpha G(y) = 0}$$

EDO(2) L cc H. EDO(1) L cc H.

α # abstracto $\quad \alpha = 0 \quad \alpha > 0 \quad \alpha < 0$

para $\boxed{\alpha = 0}$

$$F''(x) = 0 \qquad G'(y) = 0$$

$$F'(x) = C_1 \qquad \boxed{G(y) = K_1}$$

$$\boxed{F(x) = C_1 x + C_2}$$

$$Z(x,y)_{\alpha=0} = (C_1 x + C_2) K_1$$

$$\boxed{Z(x,y)_{\alpha=0} = C_{10} x + C_{20}}$$

$$\frac{\partial z}{\partial x} = C_{10} \qquad \frac{\partial^2 z}{\partial x^2} = 0 \qquad \frac{\partial z}{\partial y} = 0$$

$$\alpha > 0 \longrightarrow \alpha = \beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$$

$$F''(x) - 6\beta^2 F(x) = 0 \qquad G'(y) - \beta^2 G(y) = 0$$

$$m^2 - 6\beta^2 = 0 \qquad m - \beta^2 = 0$$

$$(m - \sqrt{6}\beta)(m + \sqrt{6}\beta) = 0 \qquad \boxed{G(y) = k_1 e^{\beta^2 y}}$$

$$m_1 = \sqrt{6}\beta \qquad m_2 = -\sqrt{6}\beta$$

$$\boxed{F(x) = C_1 e^{\sqrt{6}\beta x} + C_2 e^{-\sqrt{6}\beta x}}$$

$$Z(x,y)_{\alpha>0} = \left(C_1 e^{\sqrt{6}\beta x} + C_2 e^{-\sqrt{6}\beta x}\right) k_1 e^{\beta^2 y}$$

$$\boxed{Z(x,y)_{\alpha>0} = C_{10} e^{\sqrt{6}\beta x} e^{\beta^2 y} + C_{20} e^{-\sqrt{6}\beta x} e^{\beta^2 y}}$$

para $\alpha < 0$ $\quad \alpha = -\beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$F''(x) + 6\beta^2 F(x) = 0 \qquad G'(y) + \beta^2 G(y) = 0$$

$$m^2 + 6\beta^2 = 0 \qquad \boxed{G(y) = k_1 e^{-\beta^2 y}}$$

$$m = \pm\sqrt{6}\,\beta\, i$$

$$\boxed{F(x) = C_1 \cos(\sqrt{6}\beta x) + C_2 \operatorname{sen}(\sqrt{6}\beta x)}$$

$$Z(x,y)_{\alpha<0} = \left(C_1 \cos(\sqrt{6}\beta x) + C_2 \operatorname{sen}(\sqrt{6}\beta x)\right) k_1 e^{-\beta^2 y}$$

$$\boxed{Z(x,y)_{\alpha<0} = C_{10} e^{-\beta^2 y} \cos(\sqrt{6}\beta x) + C_{20} e^{-\beta^2 y} \operatorname{sen}(\sqrt{6}\beta y)}$$