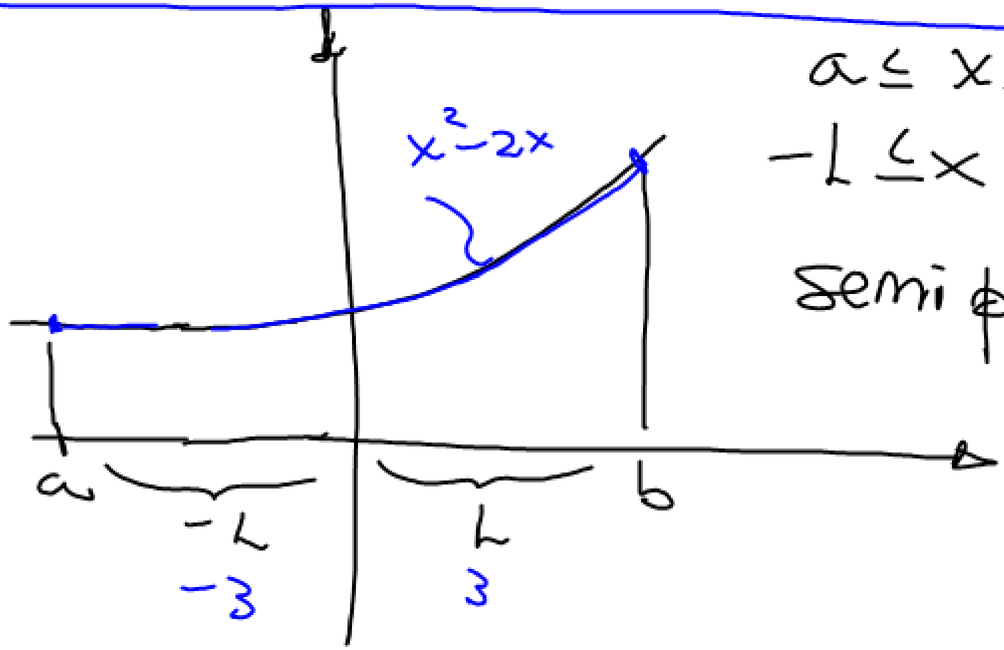
SERIE TRIGONOMÉTRICA DE FOURIER

$$\boxed{f(x) = C + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \operatorname{sen}\left(\frac{n\pi}{L}x\right)\right)}$$

$C = ?$

$a_n = ?$

$b_n = ?$



$a \leq x \leq b$ $\quad L = \frac{a+b}{2}$

$-L \leq x \leq L$

semi periodo

$$\boxed{\text{EDenDP}} \longrightarrow \boxed{\text{M}} \longrightarrow \boxed{\text{SG}} \longrightarrow \boxed{\text{STF}} \longrightarrow \boxed{\text{SP}}$$

STF: $\cos\left(\frac{n\pi}{L}x\right)$, $\operatorname{sen}\left(\frac{n\pi}{L}x\right)$.

SP: aproximada (precisa).

$$C = \frac{a_0}{2} \qquad a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\,\mathrm{sen}\left(\frac{n\pi}{L}x\right)dx$$

$$f(x) = C + \sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{n\pi}{L}x\right) + b_n\,\mathrm{sen}\left(\frac{n\pi}{L}x\right)\right)$$

$f(x) = x^2 - 2x \qquad L = 3$

$C = \frac{a_0}{2} \qquad a_0 = \frac{1}{3}\int_{-3}^{3} (x^2 - 2x)\,dx$

$a_0 = \frac{1}{3}\left[\int x^2 dx - 2\int x\,dx\right]_{-3}^{3}$

$a_0 = \frac{1}{3}\left[\frac{x^3}{3} - x^2\right]_{-3}^{3}$

$a_0 = \frac{1}{3}\left(\frac{27}{3} - \left(\frac{-27}{3}\right) - (9-9)\right)$

$a_0 = \frac{1}{3}\left(\frac{54}{3} - (0)\right) \Rightarrow \frac{54}{9}$

$\boxed{C = \frac{27}{9} \Rightarrow 3.}$

$a_n = \frac{1}{3}\int_{-3}^{3} (x^2 - 2x)\cos\left(\frac{n\pi}{3}x\right)dx$

$= \frac{1}{3}\left[\int x^2 \cos\left(\frac{n\pi}{3}x\right)dx - 2\int x\cos\left(\frac{n\pi}{3}x\right)dx\right]^{3}$

$= \frac{1}{3}\left[\frac{3}{n\pi}x^2 \operatorname{sen}\left(\frac{n\pi}{3}x\right) - \frac{6}{n\pi}\left(-\frac{3}{n\pi}x\cos\left(\frac{n\pi}{3}x\right)\right)^{3} + \frac{3}{n\pi}\left(\frac{3}{n\pi}\operatorname{sen}\left(\frac{n\pi}{3}x\right)\right)\right]$

$\int x^2 \cos\left(\frac{n\pi}{3}x\right)dx = \frac{3}{n\pi}x^2 \operatorname{sen}\left(\frac{n\pi}{3}x\right) - \int \frac{6}{n\pi}x\operatorname{sen}\left(\frac{n\pi}{3}x\right)dx$

$u = x^2 \quad du = 2x\,dx$

$dV = \cos\left(\frac{n\pi}{3}x\right)dx \quad V = \frac{3}{n\pi}\left(\operatorname{sen}\left(\frac{n\pi}{3}x\right)\right)$

$\int x\operatorname{sen}\left(\frac{n\pi}{3}x\right)dx = -\frac{3}{n\pi}x\cos\left(\frac{n\pi}{3}x\right) + \int \frac{3}{n\pi}\cos\left(\frac{n\pi}{3}x\right)dx$

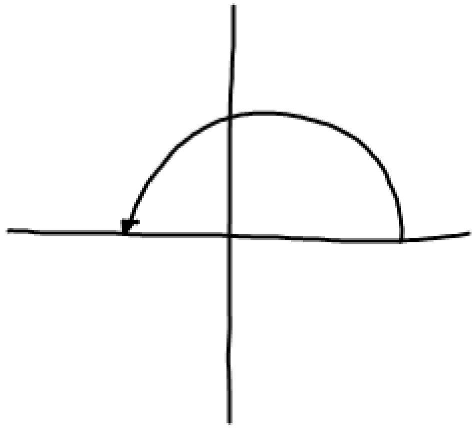
$u = x \quad du = dx$

$dV = \operatorname{sen}\left(\frac{n\pi}{3}x\right)dx \quad V = \frac{3}{n\pi}\left(-\cos\left(\frac{n\pi}{3}x\right)\right)$

$\int \cos\left(\frac{n\pi}{3}x\right)dx = \frac{3}{n\pi}\left(\operatorname{sen}\left(\frac{n\pi}{3}x\right)\right)$

$\operatorname{sen}(n\pi) = 0 \quad n \in \mathbb{Z}$

$\cos(n\pi) = (-1)^n \quad n \in \mathbb{Z}$



$f(x) = x^2 - 2x \qquad L = 3.$

$c = 3$

$a_n = \dfrac{36(-1)^n}{n^2\pi^2}$

$b_n = \dfrac{12(-1)^n}{n\pi}$

$$(x^2 - 2x) = 3 + \sum_{n=1}^{\infty}\left(\frac{36(-1)^n}{n^2\pi^2}\cos\left(\frac{n\pi}{3}x\right) + \frac{12(-1)^n}{n\pi}\operatorname{sen}\left(\frac{n\pi}{3}x\right)\right)$$

$-3 \leq x \leq 3$

función escalón.

$$(t-1)^2 \mu(t-1)$$

$$(t-1)^2 \cdot \text{Heaviside}(t-1)$$

$r(t-a)$ rampa

$u(t-a)$ escalón

$\delta(t-a)$ delta.

$$(t-a)u(t-a)$$

$$u(t-a)$$

$$\delta(t-a)$$

$$\frac{\partial y(x,t)}{\partial x} + \frac{\partial^2 y(x,t)}{\partial t^2} = y(x,t)$$

$$y(x,t) = F(x)\cdot G(t)$$

$$F'G + FG'' = FG$$

$$F'G = FG - FG''$$

$$\frac{F'}{F} = \frac{G - G''}{G}$$

$\alpha = 0$ $\alpha < 0$ $\alpha > 0$

$$\frac{F'}{F} = \alpha \qquad \frac{G - G''}{G} = \alpha$$

$$F'G - FG = -FG''$$

$$\frac{F' - F}{F} = -\frac{G''}{G}$$

$\alpha = 0$ $\alpha < 0$ $\alpha > 0$

$$\frac{F' - F}{F} = \alpha \qquad -\frac{G''}{G} = \alpha$$