```
> restart
> f := x··2 − 2·x
```

$$f := x^2 - 2\,x \tag{1}$$

```
> L := 3
```

$$L := 3 \tag{2}$$

```
> a0 := 1/3 · int( f, x = -L..L)
```

$$a_0 := 6 \tag{3}$$

```
> C := a0/2
```

$$C := 3 \tag{4}$$

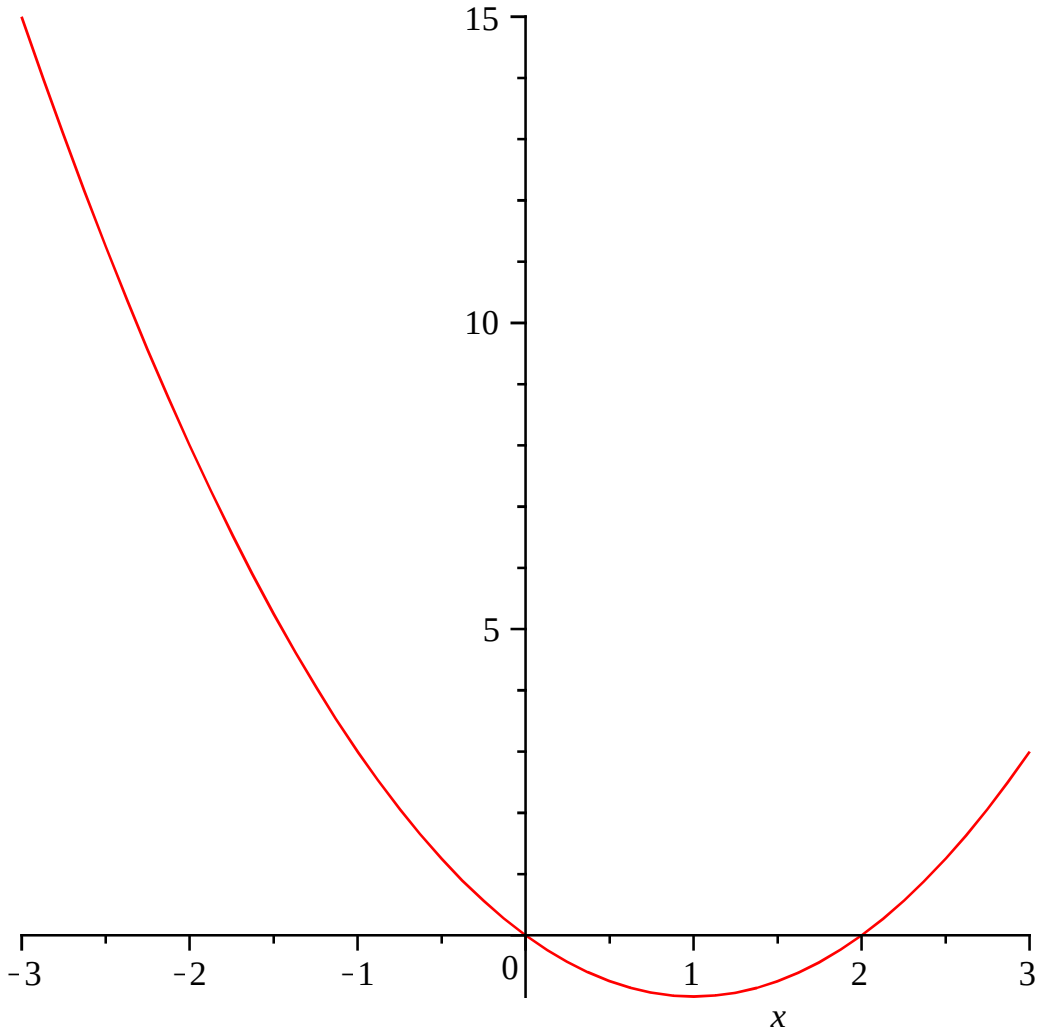
```
> an := subs( sin(n·Pi) = 0, cos(n·Pi) = (-1)··n, 1/3 · int( f·cos( n·Pi/L ·x ), x = -L..L ) )
```

$$a_n := \frac{36\,(-1)^n}{n^2\,\pi^2} \tag{5}$$

```
> bn := subs( sin(n·Pi) = 0, cos(n·Pi) = (-1)··n, 1/3 · int( f·sin( n·Pi/L ·x ), x = -L..L ) )
```

$$b_n := \frac{12\,(-1)^n}{n\,\pi} \tag{6}$$

```
> plot( f, x = -L..L)
```

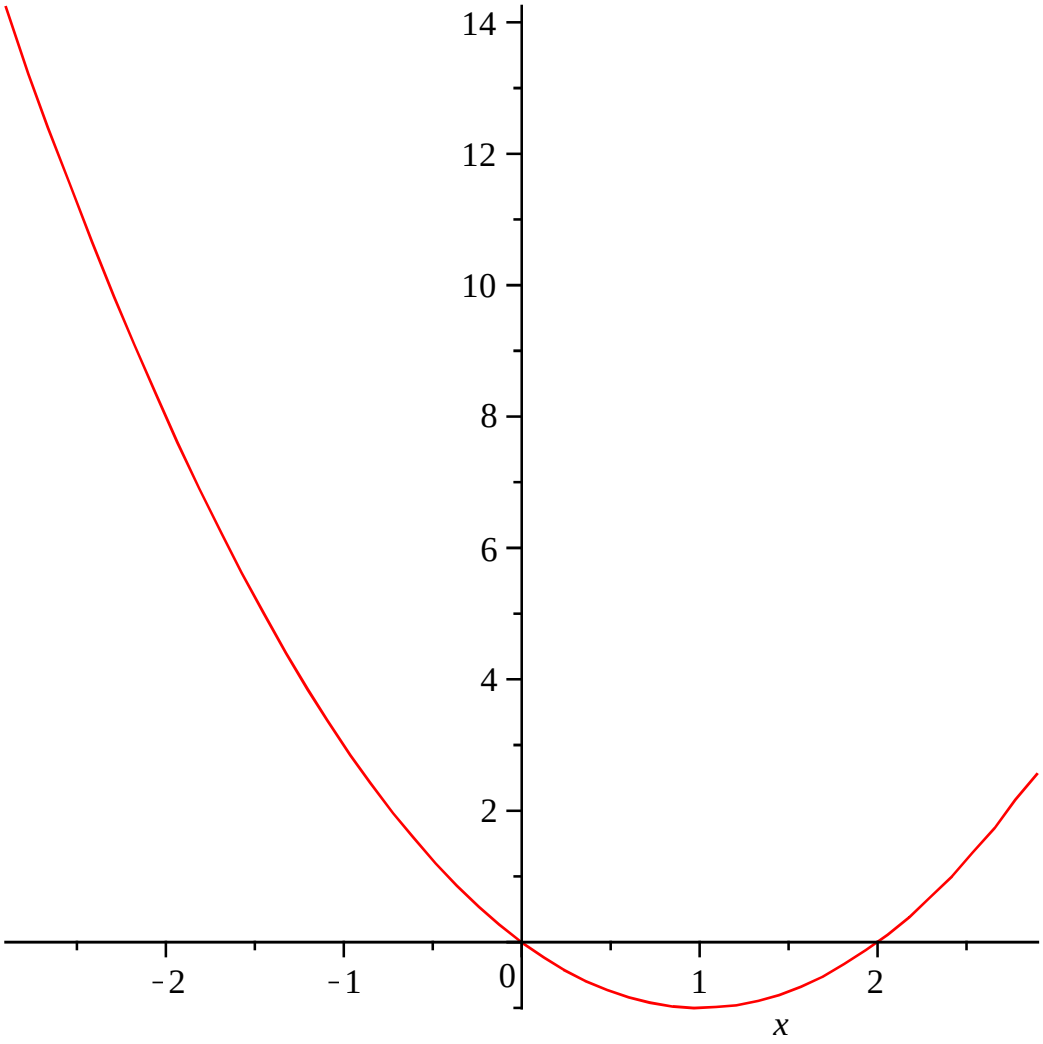


> $STF := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1 \,..\, infinity\right)$

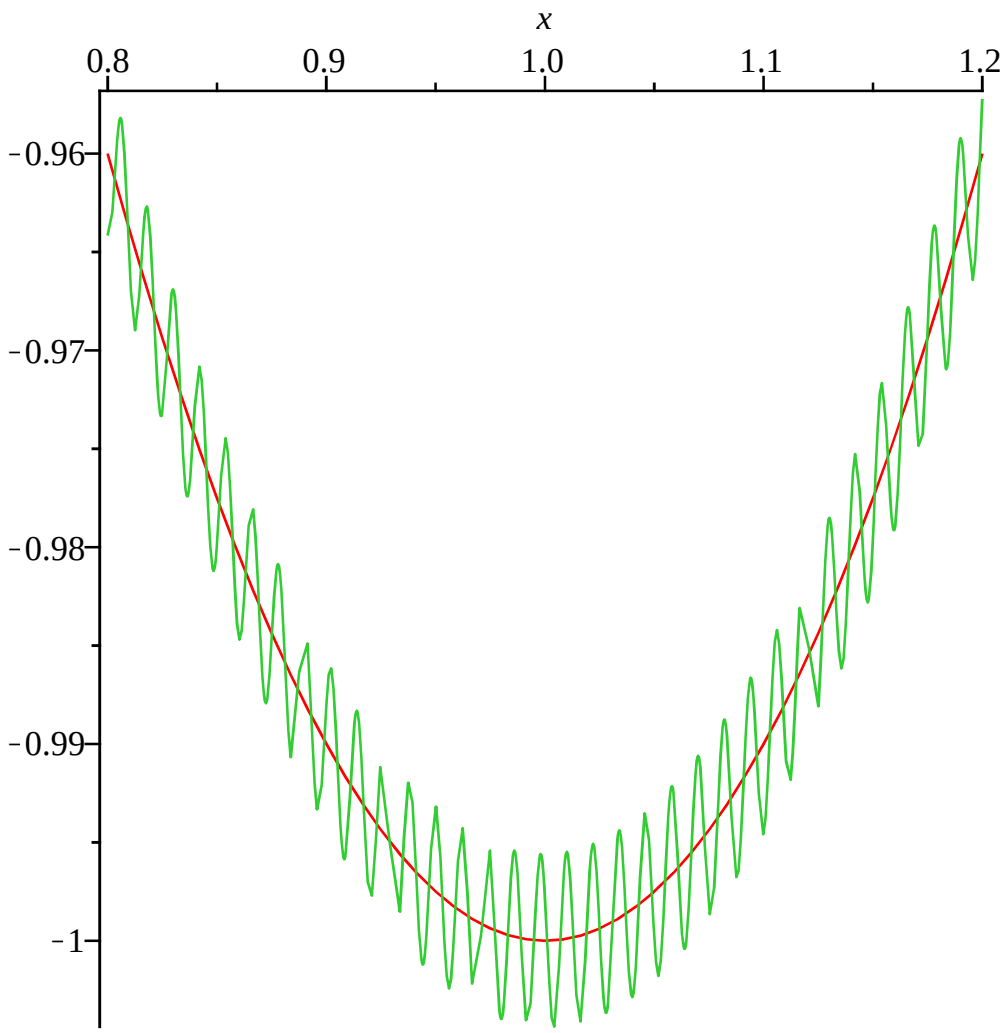
$$STF := 3 + \sum_{n=1}^{\infty} \left(\frac{36\,(-1)^n \cos\left(\frac{1}{3}\,n\,\pi\,x\right)}{n^2\,\pi^2} + \frac{12\,(-1)^n \sin\left(\frac{1}{3}\,n\,\pi\,x\right)}{n\,\pi} \right) \quad \textbf{(7)}$$

> $STF_{500} := C + sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1 \,..\, 500\right) :$

> $plot\left(STF_{500}, x = -2.9 \,..\, 2.9\right)$

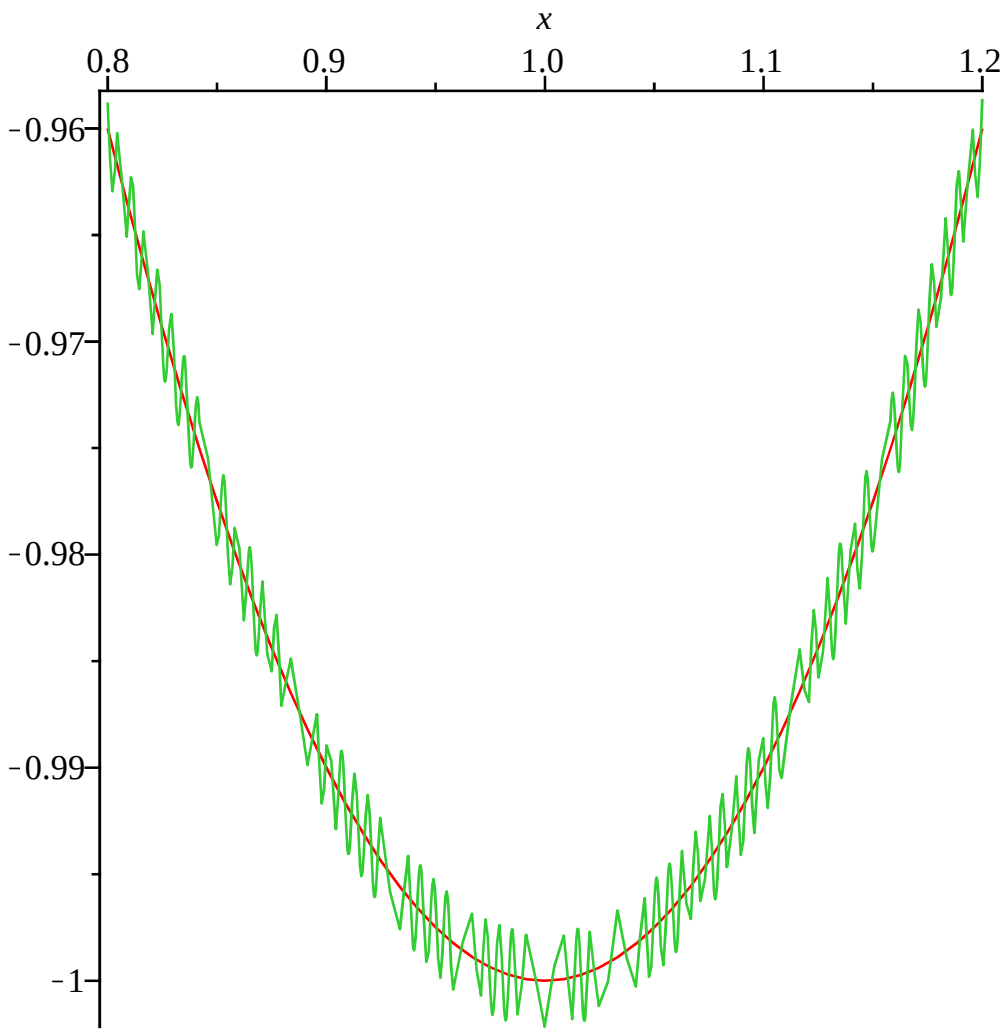


> $plot([f, STF_{500}], x = 0.8..1.2)$

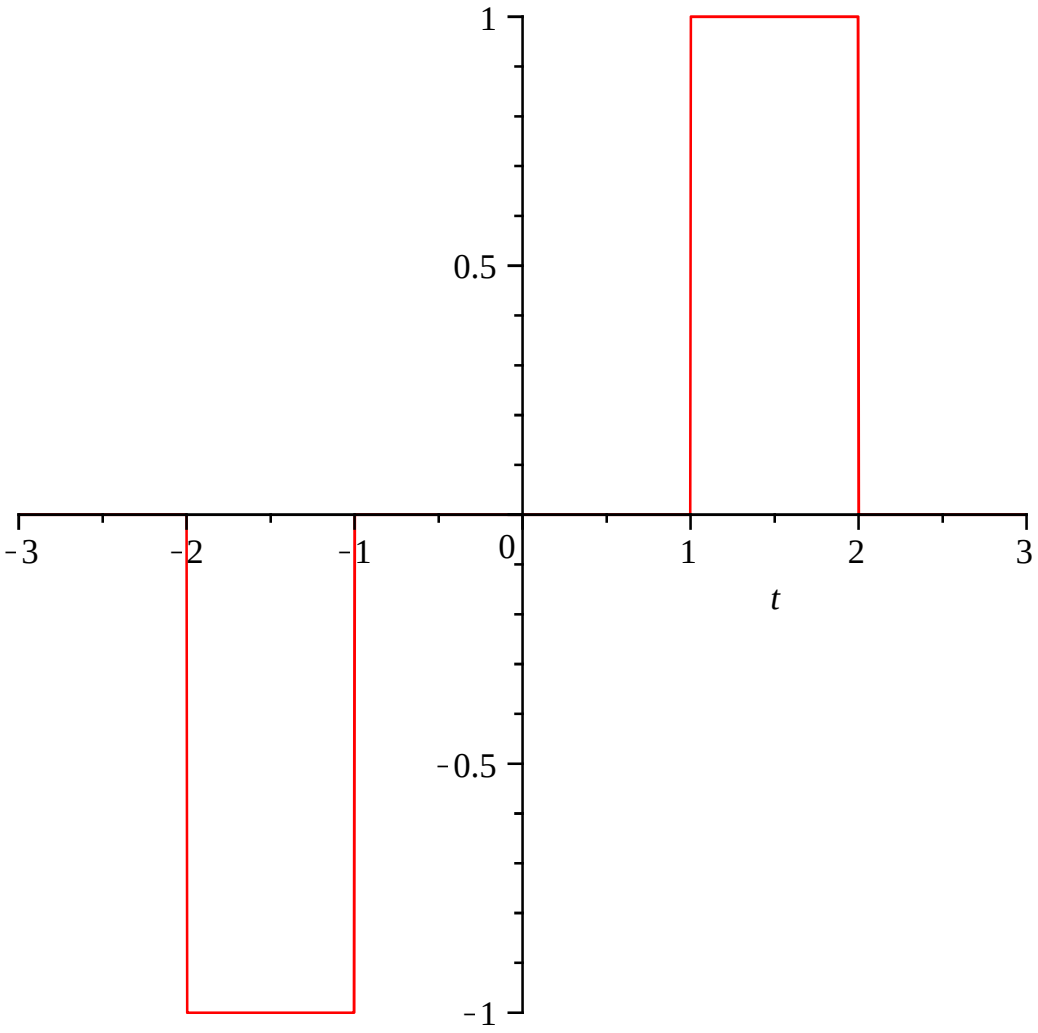


> $STF_{1000} := C + sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..1000\right)$:

> $plot([f, STF_{1000}], x = 0.8..1.2)$




```
> restart
> f := -Heaviside(t + 2) + Heaviside(t + 1) + Heaviside(t - 1) - Heaviside(t - 2) : plot(f, t =
      -3 ..3)
```



> $L := 3$

$$L := 3 \quad \textbf{(8)}$$

> $a_0 := \frac{1}{L} \cdot int(f, t = -L..L); C := \frac{a_0}{2}$

$$a_0 := 0$$

$$C := 0 \quad \textbf{(9)}$$

> $a_n := \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

$$a_n := 0 \quad \textbf{(10)}$$

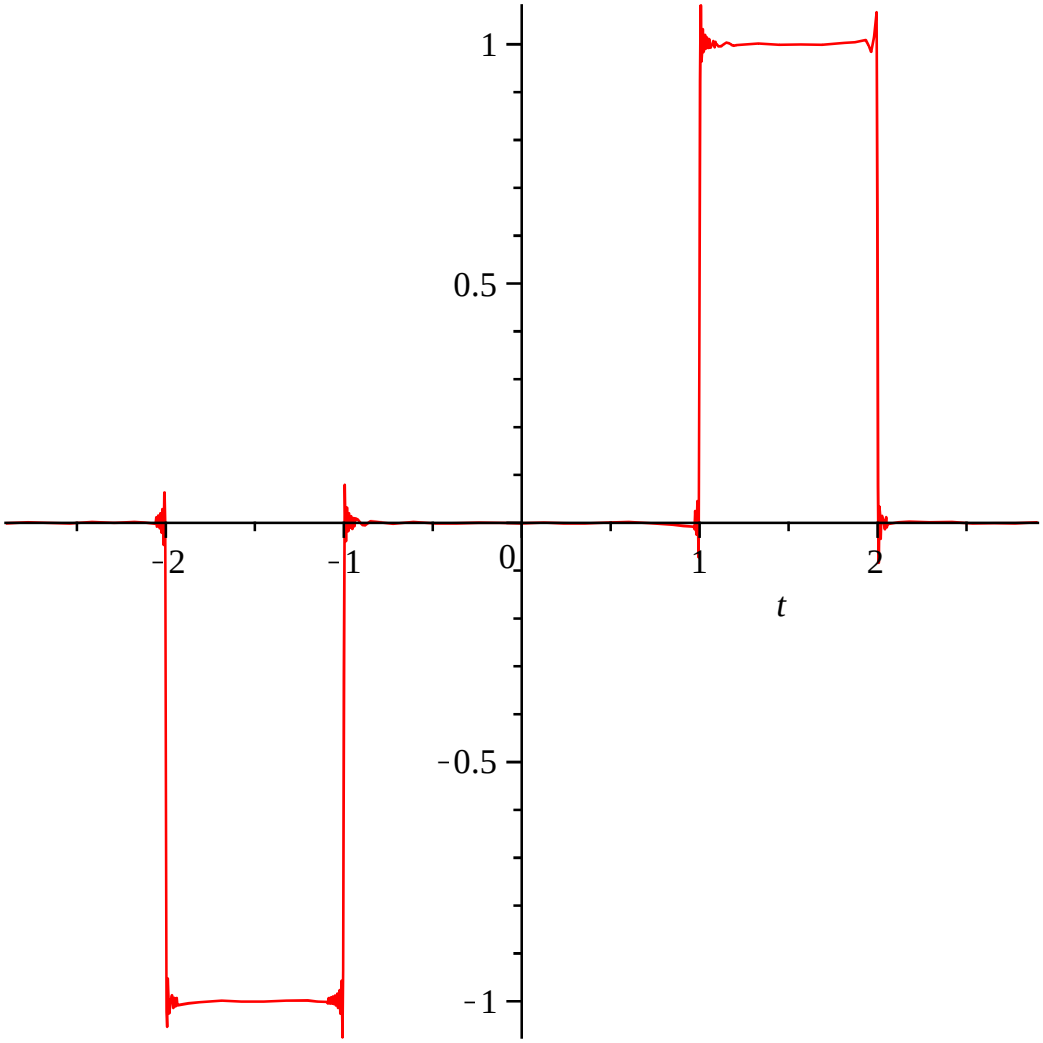
> $b_n := \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

$$b_n := -\frac{2\cos\left(\frac{2}{3} n \pi\right)}{n \pi} + \frac{2\cos\left(\frac{1}{3} n \pi\right)}{n \pi} \quad \textbf{(11)}$$

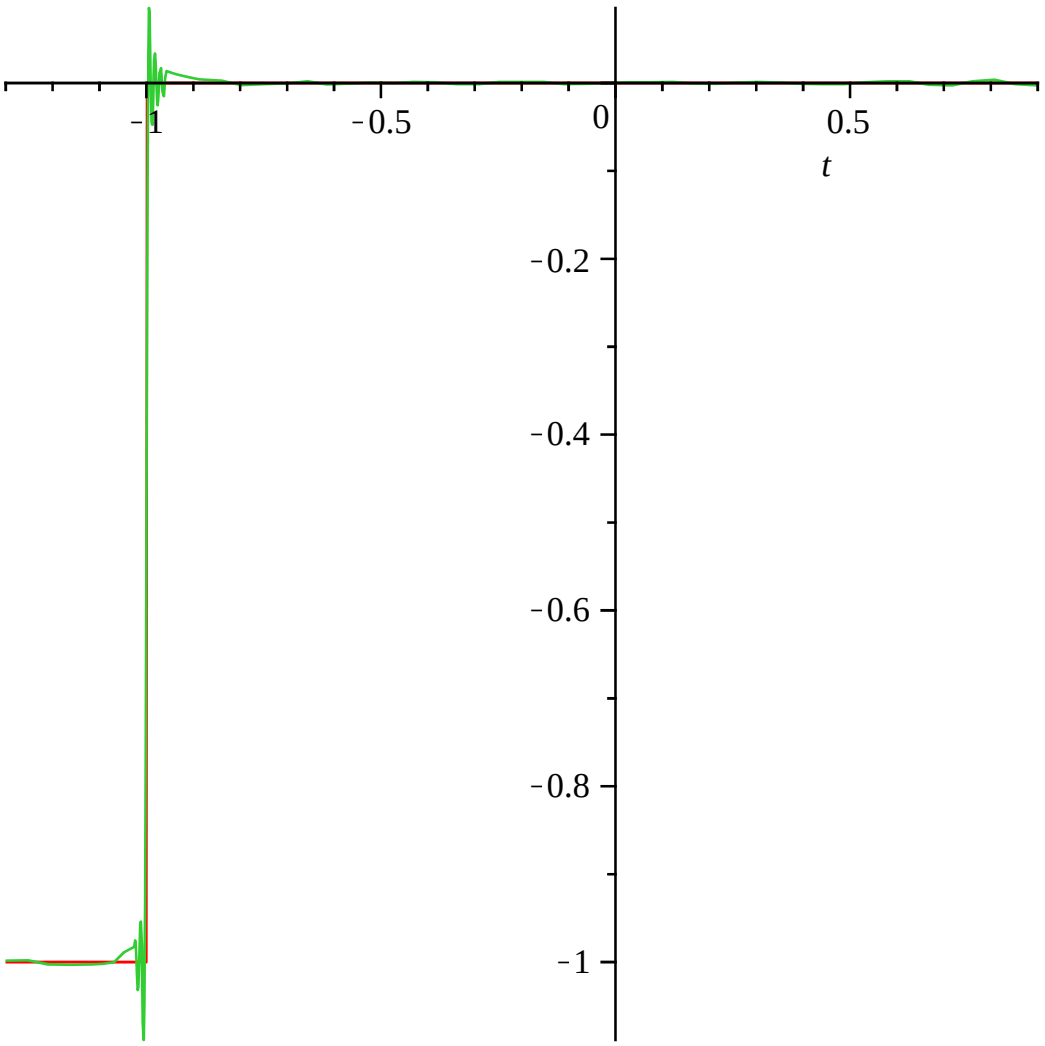
> $STF := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1..infinity\right)$

$$STF := \sum_{n=1}^{\infty} \left(-\frac{2\cos\left(\frac{2}{3}n\pi\right)}{n\pi} + \frac{2\cos\left(\frac{1}{3}n\pi\right)}{n\pi} \right) \sin\left(\frac{1}{3}n\pi t\right) \quad \textbf{(12)}$$

```
> STF500 := C + sum(a_n·cos((n·Pi/L)·t) + b_n·sin((n·Pi/L)·t), n = 1..500):
> plot(STF500, t = -2.9..2.9)
```




```
> plot([f, STF500], t = -1.3..0.9)
```



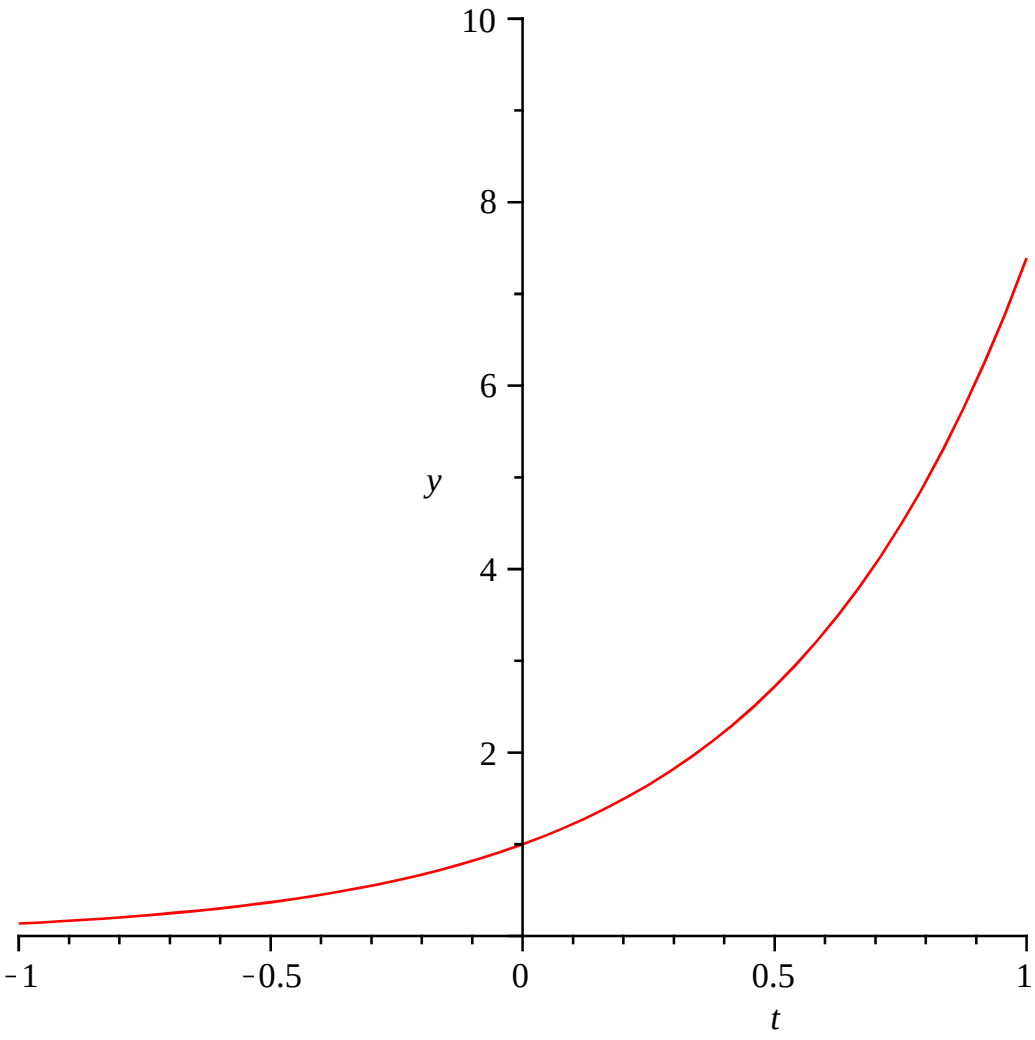

```
> restart
> f := exp(2 t)
```

$$f := e^{2t} \quad \textbf{(13)}$$

```
> L := 1
```

$$L := 1 \quad \textbf{(14)}$$

```
> plot( f, t = -L..L, y = 0..10 )
```



> $a_0 := \frac{1}{L} \cdot int(f, t = -L..L); evalf(\%, 3)$

$$a_0 := -\frac{1}{2}\, e^{-2} + \frac{1}{2}\, e^{2}$$

$$3.63 \qquad \textbf{(15)}$$

> $C := \frac{a_0}{2}; evalf(\%, 3)$

$$C := -\frac{1}{4}\, e^{-2} + \frac{1}{4}\, e^{2}$$

$$1.82 \qquad \textbf{(16)}$$

> $a_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\cdot\cdot n}, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

$$a_n := \frac{-2\, e^{-2}\, (-1)^n + 2\, e^{2}\, (-1)^n}{4 + n^2\, \pi^2} \qquad \textbf{(17)}$$

> $b_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\cdot\cdot n}, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)\right)$

(18)

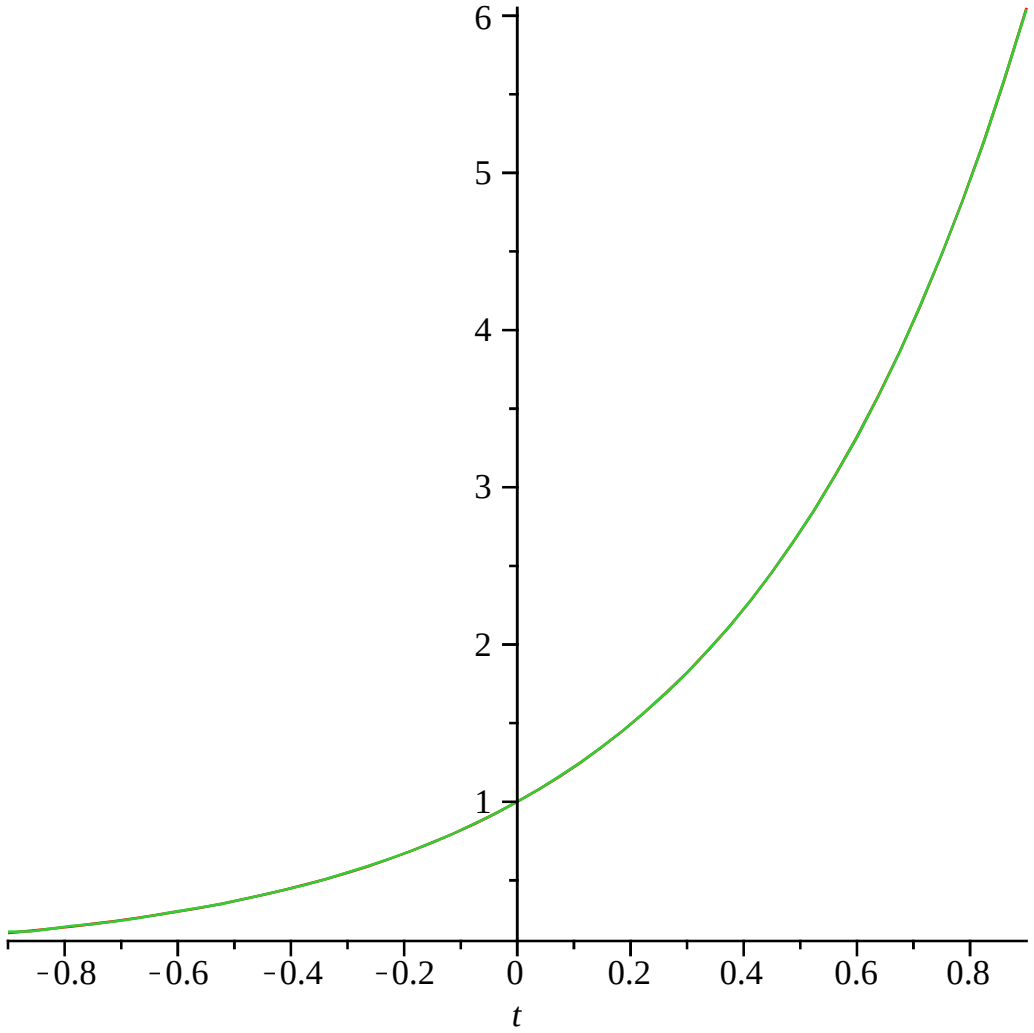
$$b_n := -\frac{-e^{-2} n \pi (-1)^n + e^2 n \pi (-1)^n}{4 + n^2 \pi^2} \quad \textbf{(18)}$$

> $STF := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1 \,..\, infinity\right)$

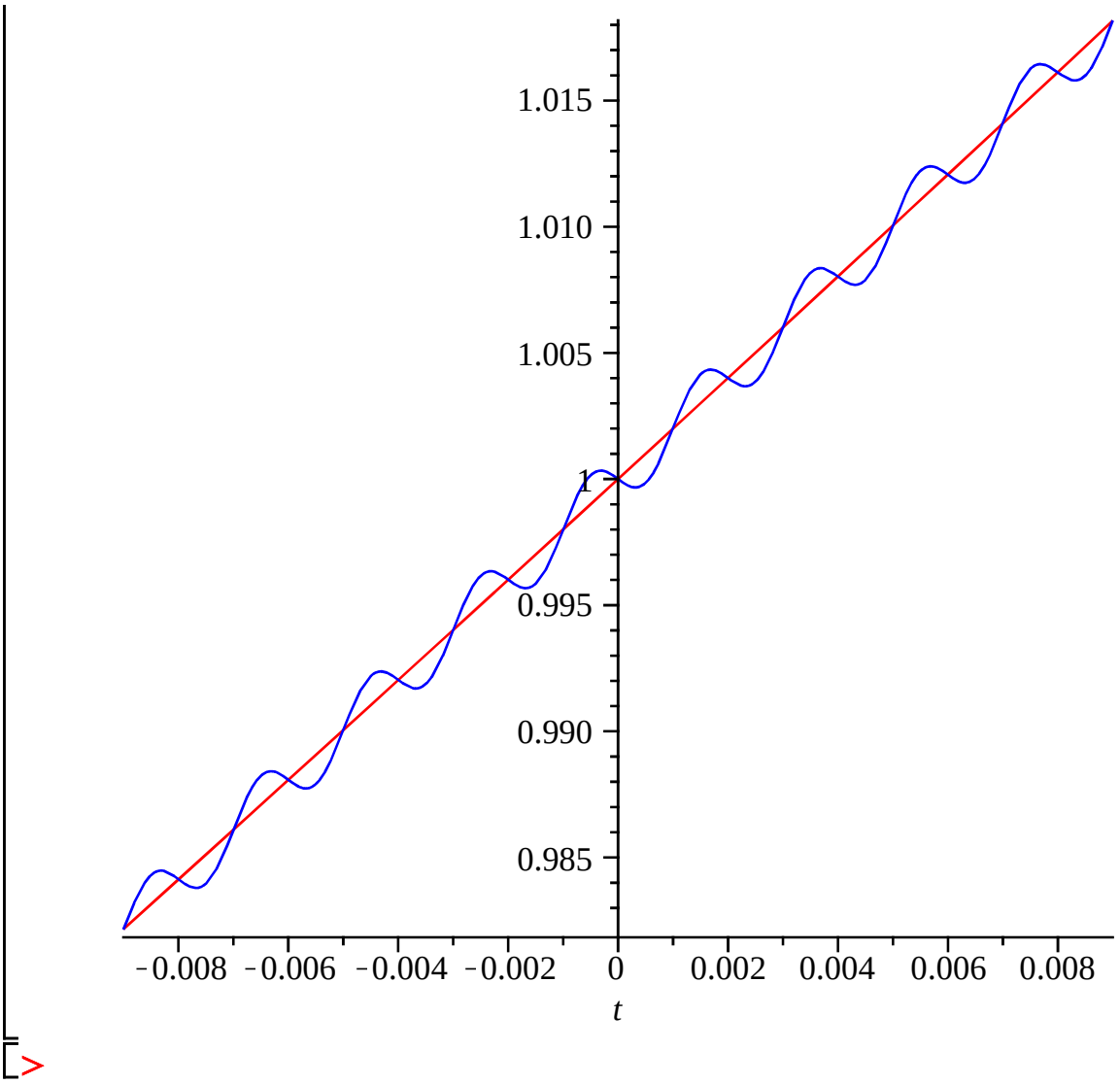
$$STF := -\frac{1}{4} e^{-2} + \frac{1}{4} e^2 + \sum_{n=1}^{\infty} \left(\frac{\left(-2 e^{-2} (-1)^n + 2 e^2 (-1)^n\right) \cos(n \pi t)}{4 + n^2 \pi^2} - \frac{\left(-e^{-2} n \pi (-1)^n + e^2 n \pi (-1)^n\right) \sin(n \pi t)}{4 + n^2 \pi^2} \right) \quad \textbf{(19)}$$

> $STF_{1000} := C + sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n = 1 \,..\, 1000\right) :$

> $plot([f, STF_{1000}], t = -0.9 \,..\, 0.9)$



> $plot([f, STF_{1000}], t = -0.009 \,..\, 0.009, color = [red, blue])$


1.015
1.010
1.005
1
0.995
0.990
0.985
-0.008
-0.006
-0.004
-0.002
0
0.002
0.004
0.006
0.008
t