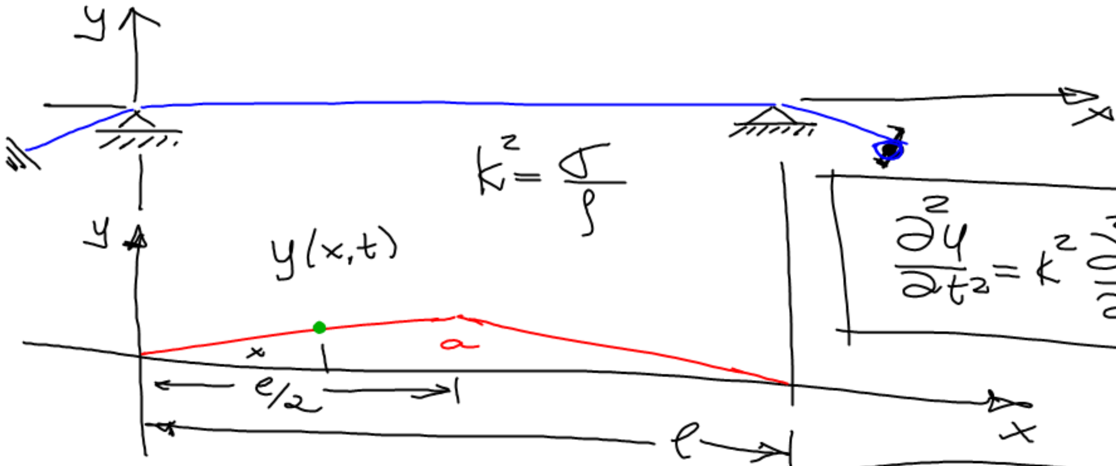
Aplicación final. EDenDP.

Problema de la Cuerda



$$k^2 = \frac{\sigma}{\rho}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = k^2 \frac{\partial^2 y}{\partial x^2}}$$

$$\boxed{\begin{array}{l} \ell = 1\,[m] \\ a = 0.005\,[m] \\ k^2 = 1 \end{array}}$$

Condiciones de frontera

$$\forall t \in \mathbb{R}^+ \quad \left.\begin{array}{l} y(0,t) = 0 \\ y(\ell,t) = 0 \end{array}\right\}$$

Condiciones Iniciales $t = 0$

$$y(x,0) = f(x) = \begin{cases} \frac{a}{\ell/2} x & : 0 \le x \le \ell/2 \\ 2a - \frac{a}{\ell/2} x & : \ell/2 < x \le \ell \end{cases}$$

$$v = \left.\frac{\partial y}{\partial t}\right|_{t=0} = 0$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \qquad y(x,t) = F(x)\cdot G(t)$$

$$F(x)\cdot G''(t) = F''(x)\cdot G(t)$$

$$\frac{G''(t)}{G(t)} = \frac{F''(x)}{F(x)}$$

$$\frac{F''(x)}{F(x)} = \alpha \qquad \frac{G''(t)}{G(t)} = \alpha.$$

$$F''(x) = \alpha F(x) \qquad G''(t) = \alpha G(t)$$

para $\alpha = 0$

$$F''(x) = 0 \quad F'(x) = k_1 \quad \boxed{F(x) = k_1 x + k_2}$$

$$y(0,t) = 0 \Rightarrow F(0)G(t) = 0 \quad G(t) \neq 0. \quad F(0) = 0$$

$$y(l,t) = 0 \Rightarrow F(l)G(t) = 0 \qquad F(l) = 0$$

$$F(0) \Rightarrow 0 = k_1(0) + k_2 \rightarrow k_2 = 0 \qquad F(x) = 0 \ \forall x \in \mathbb{R}$$

$$F(l) \Rightarrow 0 = k_1(l) \longrightarrow k_1 = 0$$

$F''(x) = \alpha F(x)$ para $\alpha > 0$ $\alpha = \beta^2$

$F''(x) = \beta^2 F(x) \longrightarrow F''(x) - \beta^2 F(x) = 0$

$\frac{d^2F}{dx^2} - \beta^2 F = 0$ EDO(2)L cc H.

$m^2 - \beta^2 = 0$ $(m-\beta)(m+\beta) = 0$

$\left.\begin{array}{l} m_1 = \beta \\ m_2 = -\beta. \end{array}\right\} m_1 \neq m_2$ CASO I.

$$\boxed{F(x) = k_1 e^{\beta x} + k_2 e^{-\beta x}}$$

$F(0) = 0$

$F(\ell) = 0$

$F(0) \Rightarrow 0 = k_1 e^{\beta(0)} + k_2 e^{-\beta(0)}$

$0 = k_1 + k_2$ $\boxed{k_2 = -k_1}$

$F(\ell) \Rightarrow 0 = k_1 e^{\beta(\ell)} - k_1 e^{-\beta(\ell)}$

$k_1 e^{-\beta(\ell)} = k_1 e^{\beta(\ell)}$

$\frac{k_1}{e^{\beta(\ell)}} = k_1 e^{\beta(\ell)}$

$k_1 = k_1 \left[e^{\beta(\ell)}\right]^2$

$1 = e^{2\beta(\ell)} \Rightarrow 2\beta\ell = 0$ $\boxed{\beta = 0}$

$F''(x) = \alpha F(x)$ para $\alpha < 0$ $\quad \alpha = -\beta^2$

$F''(x) = -\beta^2 F(x) \longrightarrow F''(x) + \beta^2 F(x) = 0$

$\frac{d^2F}{dx^2} + \beta^2 F = 0$ $\quad$ EDO(2)LccH.

$m^2 + \beta^2 = 0$ $\quad \left.\begin{array}{l} m_1 = \beta i \\ m_2 = -\beta i \end{array}\right\} \in \mathbb{C}$omp

CASO III

$$\boxed{F(x) = k_1 \cos(\beta x) + k_2 \operatorname{sen}(\beta x)}$$

$F(0) = 0$

$F(\ell) = 0$

$F(0) \Rightarrow 0 = k_1(1) + k_2(0)$ $\quad \boxed{k_1 = 0}$

$F(\ell) \Rightarrow 0 = k_2 \operatorname{sen}(\beta \ell)$ $\quad \operatorname{sen}(\beta\ell) = 0$

$\boxed{\beta = \frac{n\pi}{\ell}}$ $\quad k_2 \neq 0.$ $\quad \beta \ell = n\pi$ $\quad n \in \mathbb{Z}^+$

$$\boxed{F(x) = k_2 \operatorname{sen}\left(\frac{n\pi}{\ell} x\right)}$$

$G''(t) + \frac{n^2\pi^2}{\ell^2} G(t) = 0$

$\frac{d^2G}{dt^2} + \frac{n^2\pi^2}{\ell^2} G = 0$ $\quad$ EDO(2)LccH.

$m^2 + \frac{n^2\pi^2}{\ell^2} = 0$ $\quad m_1 = \frac{n\pi}{\ell} i$

$m_2 = -\frac{n\pi}{\ell} i$

$G(t) = C_1 \cos\left(\frac{n\pi}{\ell} t\right) + C_2 \operatorname{sen}\left(\frac{n\pi}{\ell} t\right).$

$$y(x,t) = k_2 \operatorname{sen}\left(\frac{n\pi}{\ell} x\right)\left(C_1 \cos\left(\frac{n\pi}{\ell} t\right) + C_2 \operatorname{sen}\left(\frac{n\pi}{\ell} t\right)\right).$$