1 Determine una solución completa de la ecuación diferencial en derivadas parciales

$$2\frac{\partial^3 z}{\partial x^2 \partial y} - 2\frac{\partial^2 z}{\partial x \partial y} = z$$

para una constante de separación igual a $-1$

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$H_0$: $z(x,y) = F(x)\cdot G(y)$

$$\frac{\partial^3 z}{\partial x^2 \partial y} = F''(x)\cdot G'(y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = F'(x)\cdot G'(y)$$

$$2F''(x)\cdot G'(y) - 2F'(x)\cdot G'(y) = F(x)\cdot G(y)$$

$$2\left(F''(x) - F'(x)\right)\cdot G'(y) = F(x)\cdot G(y)$$

$$\frac{F''(x) - F'(x)}{F(x)} = \frac{G(y)}{2G'(y)}$$

5

Obtener la solución completa de la ecuación diferencial parcial

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial u}{\partial y} = 0$$

Considerar una constante de separación $\alpha = 1$

$H_0$: $u(x,y) = F(x) \cdot G(y)$

$$\frac{\partial^2 u}{\partial x^2} = F''G \qquad \frac{\partial u}{\partial y} = FG'$$

$$\frac{\partial u}{\partial x \partial y} = F'G'$$

$$F''G + 4F'G' + 4FG' = 0$$

$$F''G = -4(F'G' + FG')$$

$$F''G = -4(F' + F)G'$$

$$\frac{F''}{F' + F} = -4\frac{G'}{G}$$

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$$H_1 = u(x,y) = F(x) + G(y)$$

7 Utilice la transformada de Laplace para resolver el sistema

$$x'' + y'' = e^{2t}$$
$$2x' + y'' = -e^{2t} + \delta(t-1)$$

sujeto a las condiciones iniciales $x(0) = 0$, $y(0) = 0$, $x'(0) = 0$, $y'(0) = 0$

$$\mathcal{L}\{x'' + y''\} = \frac{1}{s-2}$$
$$\mathcal{L}\{2x' + y''\} = -\frac{1}{s-2} + e^{-s}$$

$$\mathcal{L}\{x''\} + \mathcal{L}\{y''\} = \frac{1}{s-2}$$
$$2\mathcal{L}\{x'\} + \mathcal{L}\{y''\} = -\frac{1}{s-2} + e^{-s}$$

$$s^2\mathcal{L}\{x\} - s\cdot(0) - (0) + s^2\mathcal{L}\{y\} - s(0) - (0) = \frac{1}{s-2}$$
$$2s\mathcal{L}\{x\} - (0) + s^2\mathcal{L}\{y\} - s(0) - (0) = -\frac{1}{s-2} + e^{-s}$$

$$s^2\mathcal{L}\{x\} + s^2\mathcal{L}\{y\} = \frac{1}{s-2}$$
$$2s\mathcal{L}\{x\} + s^2\mathcal{L}\{y\} = -\frac{1}{s-2} + e^{-s}$$

$$s^2\mathcal{L}\{x\} - 2s\mathcal{L}\{x\} = \frac{2}{s-2} - e^{-s}$$

$$\mathcal{L}\{x\} = \frac{2 - (s-2)e^{-s}}{(s-2)(s^2-2s)}$$

$$\mathcal{L}\{y'\} = s\mathcal{L}\{y\} - y(0)$$

$$\mathcal{L}\{y''\} = s^2\mathcal{L}\{y\} - s\,y(0) - y'(0)$$

(A) $$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 2e^{5x} \qquad \begin{aligned} y(0) &= 4 \\ y'(0) &= -3. \end{aligned}$$

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(1) resolver y graficar
(2) convertirla a un sistema y resolver $e^{At}$
(3) resolver con TL.

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(B) $$x^2 \frac{\partial z}{\partial y^2} + 6 \frac{\partial z}{\partial x} = 0. \qquad \alpha > 0$$

1- no lineal de x
5- en derivadas parciales.