MÉTODO DE VARIABLES SEPARABLES

EDONL (1)

MVS → EDOL (1) CVH.

$$a_0(x)\frac{dy}{dx}+a_1(x)y=0 \Rightarrow \frac{dy}{dx}+p(x)y=0$$

FÓRMULA → $y=C_{10}e^{-\int p(x)dx}$

$$\frac{dy}{dx}-x^2y=0 \qquad p(x)=-x^2$$

$$-\int -x^2dx=+\int x^2dx \Rightarrow \frac{x^3}{3}$$

$$\boxed{y=Ce^{\frac{x^3}{3}}} \quad SG. \quad \frac{dy}{dx}=Ce^{\frac{x^3}{3}}\cdot x^2$$

$$\left[Cx^2e^{\frac{x^3}{3}}-x^2\left(Ce^{\frac{x^3}{3}}\right)\right]=0 \Rightarrow \underline{0=0}$$

FORMA GENERAL

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0 \quad \text{EDO(1)NL}$$

$$a_0(x)\frac{dy}{dx} + a_1(x)y = 0 \quad \text{EDO(1)} \mathcal{L}$$

$$\begin{cases} M(x,y) = a_1(x)\,y \\ N(x,y) = a_0(x) \end{cases}$$

Para que una EDO(1)NL sea de variables separables se requiere que:

$$M(x,y) = P(x)Q(y)$$
$$N(x,y) = R(x)S(y)$$

$$P(x)\,Q(y) + \left(R(x)S(y)\right)\cdot\frac{dy}{dx} = 0$$

$$\left[R(x)\,S(y)\right]\frac{dy}{dx} = -P(x)\cdot Q(y)$$

$$\boxed{\frac{S(y)}{Q(y)}\,dy = -\frac{P(x)}{R(x)}\,dx}$$

EJEMPLO

$$\underbrace{3e^{x}\tan(y)}_{M(x,y)} + \underbrace{(2-e^{x})\sec^2 y}_{N(x,y)} \cdot \frac{dy}{dx} = 0$$

$P(x) = 3e^{x}$ $\qquad$ $R(x) = (2-e^{x})$

$Q(y) = \tan(y)$ $\qquad$ $S(y) = \sec^2 y$

$$(2-e^{x})\sec^2 y \frac{dy}{dx} = -3e^{x}\tan(y)$$

$$\frac{\sec^2(y)}{\tan(y)} dy = \frac{-3e^{x}}{(2-e^{x})} dx$$

$$\int \frac{\sec^2 y}{\tan y} dy = 3\int \frac{-e^{x}}{2-e^{x}} dx$$

$u = \tan(y)$ $\qquad$ $V = 2-e^{x}$

$du = \sec^2 y\, dy$ $\qquad$ $dV = -e^{x} dx$

$$\int \frac{du}{u} = 3\int \frac{dV}{V}$$

$$\ln u + C_1 = 3\ln V + C_2$$

$$\ln u - \ln V^3 = C_2 - C_1$$

$$\ln \frac{u}{V^3} = C_2 - C_1$$

$$\frac{u}{V^3} = e^{C_2 - C_1}$$

$$\frac{u}{V^3} = C_{10}$$

$$u = C_{10} V^3$$

$$\tan(y) = C_{10}(2-e^{x})^3$$

SG $\boxed{y = \arg\tan\left(C_{10} \cdot (2-e^{x})^3\right)}$

$$y = \arg\tan\left(C_{10}(8-12e^{-x}+6e^{-2x}-e^{-3x})\right)$$

$$Y = \arg\tan\left(8C_{10}-12C_{10}e^{-x}+6C_{10}e^{-2x}-C_{10}e^{-3x}\right)$$

$$Solucion2 := y = -\arctan\left(-8\,C + 12\,C\,e^{x} - 6\,C\left(e^{x}\right)^{2} + C\left(e^{x}\right)^{3}\right)$$

$$Y = \operatorname{ang\,tan}\left(8C_{10} - 12C_{10}e^{-x} + 6C_{10}e^{-2x} - C_{10}e^{-3x}\right)$$

EJEMPLO 2

Resolver EDO(1)NL

$$\boxed{y' \operatorname{sen}(x) = y \ln y}$$

$-y\ln y + \operatorname{sen}(x) y' = 0$

$P(x) = 1 \quad R(x) = \operatorname{sen}(x)$

$Q(y) = -y\ln y \quad S(y) = 1$

$$\underbrace{\operatorname{sen}(x)}_{R(x)}\frac{dy}{dx} = \underbrace{+y\ln y}_{-Q(y)}$$

$$\frac{dy}{y\ln y} = \frac{dx}{\operatorname{sen}(x)}$$

$$\frac{\;\frac{a}{b}\;}{\frac{c}{d}}$$

$$\int \frac{dy}{y\ln y} = \int \frac{dx}{\operatorname{sen}(x)}$$

$$\frac{ad}{bc}$$

$$\int \frac{\frac{dy}{y}}{\ln y} = \int \csc(x)\,dx$$

$u = \ln y$

$du = \frac{1}{y}dy$

$$\int \frac{du}{u} = \ln\left(\tan\left(\frac{x}{2}\right)\right)$$

$$\ln(\ln y) = \ln\left(\tan\left(\frac{x}{2}\right)\right) + \ln C$$

$$\ln y = c\tan\left(\frac{x}{2}\right)$$

SG $\boxed{y = e^{c\tan\left(\frac{x}{2}\right)}}$

EJEMPLO 3.

$$(y^2+xy^2)y' + (x^2 - yx^2) = 0$$

$$\underbrace{(1+x)y^2}_{N(x,y)}\frac{dy}{dx} + \underbrace{x^2(1-y)}_{M(x,y)} = 0$$

$P(x) = (1+x)$

$Q(y) = y^2$

$R(x) = x^2$

$S(y) = (1-y)$

$$\frac{y^2}{y-1}\,dy = \frac{x^2\,dx}{x+1}$$

$$\int \frac{y^2\,dy}{y-1} = \int \frac{x^2\,dx}{x+1}$$

$$\begin{array}{r|l} & y+1 \\ \hline y-1 & y^2 \\ & -y^2+y \\ \hline & 0 \quad y \\ & -y+1 \\ \hline & 0 \quad 1 \end{array}$$

$$\begin{array}{r|l} & x-1 \\ \hline x+1 & x^2 \\ & -x^2-x \\ \hline & 0 \quad -x \\ & +x+1 \\ \hline & 0 \quad +1 \end{array}$$

$$\int \left(y+1+\frac{1}{y-1}\right)dy = \int \left(x-1+\frac{1}{x+1}\right)dx$$

$$\boxed{\frac{y^2}{2}+y+L(y-1) = \frac{x^2}{2}-x+L(x+1)+C} \quad SG$$

$$SolucionGral := \frac{1}{2}\,y^2 + y + \ln(y-1) = \frac{1}{2}\,x^2 - x + \ln(x+1) + C$$

$$\boxed{\frac{y^2}{2} + y + \ell(y-1) = \frac{x^2}{2} - x + \ell(x+1) + C}$$