MÉTODO SEPARACIÓN DE VARIABLES EDO NL (1)

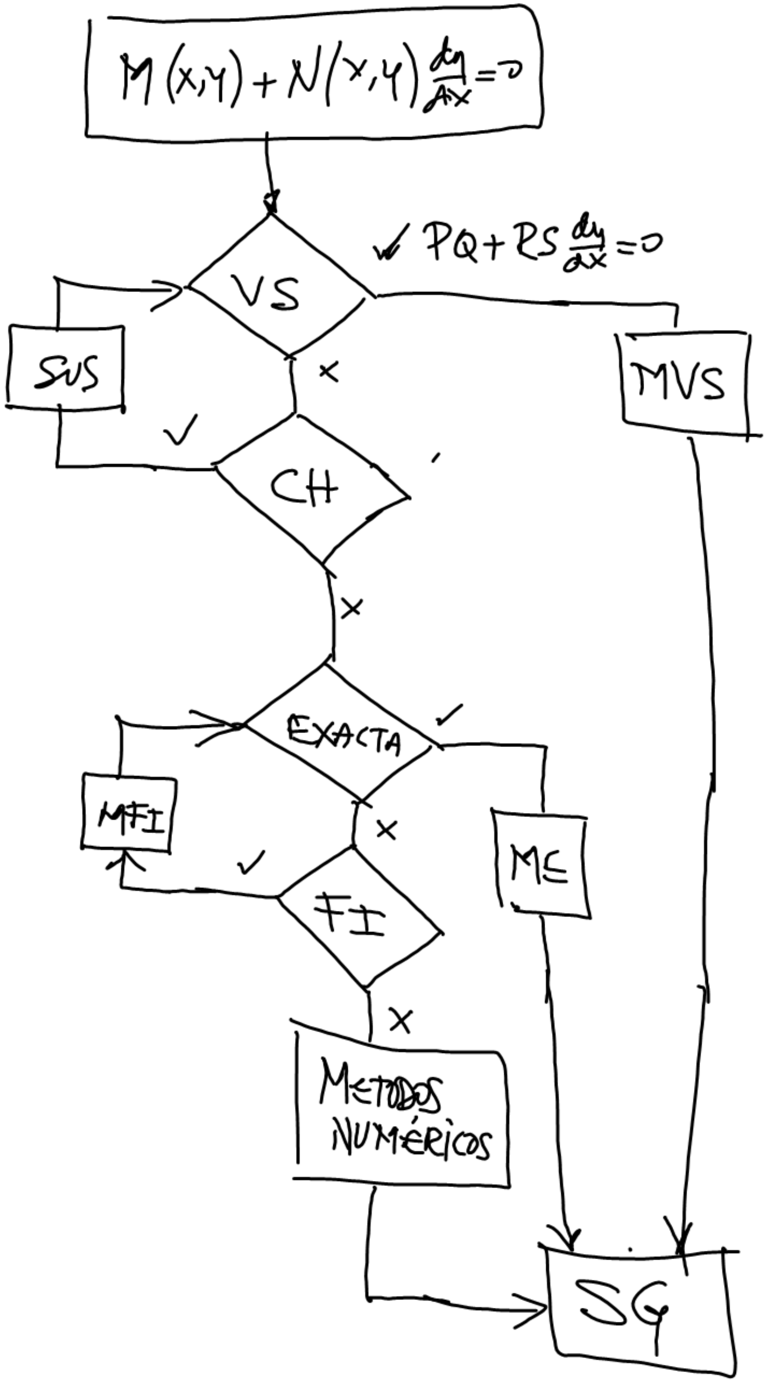
$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$P(x)\cdot Q(y) + R(x)\cdot S(y)\frac{dy}{dx} = 0$$

$$\frac{P(x)}{R(x)}dx + \frac{S(y)}{Q(y)}dy = 0$$

$$\int \frac{P(x)}{R(x)}dx + \int \frac{S(y)}{Q(y)}dy = C$$

$$F(x,y) = C$$ SOLUCIÓN GENERAL EDO NL (1).



$M(x,y) + N(x,y)\frac{dy}{dx} = 0$
VS
✓ $PQ + RS\frac{dy}{dx} = 0$
MVS
SUS
✓
×
CH
×
EXACTA
✓
MFI
×
ME
✓
FI
×
METODOS NUMÉRICOS
SG

MÉTODO DE COEFICIENTES HOMOGENEOS

TEORÍA: UNA FUNCIÓN IMPLÍCITA $F(x,y)$ ES DE COEFICIENTES HOMOGENEOS SI CUMPLE LA SIGUIENTE PROPIEDAD:

$$F(\lambda x, \lambda y) = \lambda^k F(x,y) \quad k \in \mathbb{N}.$$

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0$$

$$M(\lambda x, \lambda y) = \lambda^m M(x,y)$$
$$N(\lambda x, \lambda y) = \lambda^n N(x,y) \quad m = n$$

entonces EDO NL (1) CH.

EJEMPLO

$$x\frac{dy}{dx} = \sqrt{x^2-y^2} + y$$

$$\left(\sqrt{x^2-y^2} + y\right) - x\frac{dy}{dx} = 0$$

$$\begin{cases} M(x,y) = \sqrt{x^2-y^2} + y \\ N(x,y) = -x \end{cases}$$

$$\begin{aligned} M(\lambda x, \lambda y) &= \sqrt{(\lambda x)^2 - (\lambda y)^2} + (\lambda y) \\ &= \sqrt{\lambda^2(x^2-y^2)} + \lambda y \\ &= \sqrt{\lambda^2}\sqrt{x^2-y^2} + \lambda y \\ &= \lambda\sqrt{x^2-y^2} + \lambda y \\ &= \lambda\left(\sqrt{x^2-y^2} + y\right) \quad m = 1 \end{aligned}$$

$$\begin{aligned} N(\lambda x, \lambda y) &= -\lambda x \\ &= \lambda(-x) \quad n = 1 \quad m = n. \end{aligned}$$

MÉTODO COEF. HOM.

$$\begin{cases} y(x) = u(x)\cdot x \\ \dfrac{dy}{dx} = u(x) + \dfrac{du(x)}{dx}\cdot x \end{cases} \qquad \boxed{u(x) = \frac{y(x)}{x}}$$

EJEMPLO (sigue)

$$x\frac{dy}{dx} = \sqrt{x^2-y^2} + y$$

$$y = u\cdot x \qquad \frac{dy}{dx} = u + \frac{du}{dx}\cdot x$$

$$x\left(u + \frac{du}{dx}\cdot x\right) = \sqrt{x^2-(u\cdot x)^2} + ux$$

$$\cancel{xu} + x^2\frac{du}{dx} = \sqrt{x^2(1-u^2)} + \cancel{ux}$$

$$x^2\frac{du}{dx} = \sqrt{x^2}\sqrt{1-u^2}$$

$$x^2\frac{du}{dx} = x\sqrt{1-u^2}$$ EDO NL(1) VS.

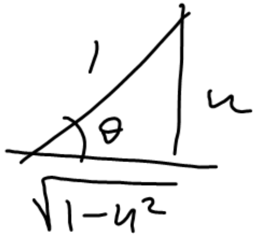
$$\frac{du}{\sqrt{1-u^2}} = \frac{x\,dx}{x^2}$$

$$\boxed{\frac{du}{\sqrt{1-u^2}} = \frac{dx}{x}}$$

$$\int\frac{du}{\sqrt{1-u^2}} = \int\frac{dx}{x}$$

$$= Lx + LC$$

$$\int \frac{du}{\sqrt{1-u^2}} = \int \frac{\cos\theta \, d\theta}{\cos\theta}$$



$$= \int d\theta$$

$u = \operatorname{sen}\theta$

$$= \theta$$

$\sqrt{1-u^2} = \cos\theta$

$$= \operatorname{ang\,sen}(u)$$

$du = \cos\theta \, d\theta$

$$\operatorname{ang\,sen}(u) = L(cx)$$

$$u = \operatorname{sen}(L(cx))$$

$u(x)$

$SGFinal := y(x) = \sin(\ln(x) + C)\, x$

$$\boxed{y(x) = x \operatorname{sen}(L(cx))} \quad SG$$

$$\boxed{x \frac{dy}{dx} = \sqrt{x^2 - y^2} + y} \quad \text{EDO NL}(1)$$

ECUACIÓN DIFERENCIAL EXACTA

$$F(x,y) = C \quad SG$$

función implícita.

$$\frac{dF}{dx} = 0 \quad \text{EDO NL (1)}$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dx} = 0 \quad \text{EDO NL (1)}$$

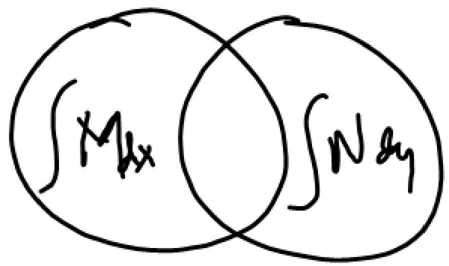
$$\left.\begin{array}{l} M(x,y) \Rightarrow \dfrac{\partial F}{\partial x} \\ N(x,y) \Rightarrow \dfrac{\partial F}{\partial y} \end{array}\right\} * \text{ojo}$$

$$\frac{\partial^2 F}{\partial y \partial x} = \frac{\partial^2 F}{\partial x \partial y}$$

$M(x,y) + N(x,y)\frac{dy}{dx} = 0$ EDO NL (1)

Será ED EXACTA si:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$



$$\left[\int M(x,y)\,dx\right] \cup \left[\int N(x,y)\,dy\right] = C.$$

EJEMPLO

$$x^4y + 8x^3y^2 - 16x^2y^3 + 9xy^4 = C \quad SG$$

$$\boxed{\left(4x^3y + 24x^2y^2 - 32xy^3 + 9y^4\right) + \left(x^4 + 16x^3y - 48x^2y^2 + 36xy^3\right)\frac{dy}{dx} = 0}$$

EDO NL (1).

$$M = 4x^3y + 24x^2y^2 - 32xy^3 + 9y^4$$

$$N = x^4 + 16x^3y - 48x^2y^2 + 36xy^3$$

$$\frac{\partial M}{\partial y} = 4x^3 + 48x^2y - 96xy^2 + 36y^3$$

$$\frac{\partial N}{\partial x} = 4x^3 + 48x^2y - 96xy^2 + 36y^3$$

SON IGUALES

$$\int M\,dx = 4y\int x^3dx + 24y^2\int x^2dx - 32y^3\int x\,dx + 9y^4\int dx$$

$$= 4y\left(\frac{x^4}{4}\right) + 24y^2\left(\frac{x^3}{3}\right) - 32y^3\left(\frac{x^2}{2}\right) + 9y^4x$$

$$= x^4y + 8x^3y^2 - 16x^2y^3 + 9xy^4$$

$$\int N\,dy = x^4\int dy + 16x^3\int y\,dy - 48x^2\int y^2dy + 36x\int y^3dy$$

$$= x^4y + 16x^3\left(\frac{y^2}{2}\right) - 48x^2\left(\frac{y^3}{3}\right) + 36x\left(\frac{y^4}{4}\right)$$

$$= x^4y + 8x^3y^2 - 16x^2y^3 + 9xy^4$$

$$\boxed{x^4y + 8x^3y^2 - 16x^2y^3 + 9xy^4 = C.} \quad SG.$$

$$M(x,y)+N(x,y)\frac{dy}{dx}=0 \quad \text{EDO NL (1)}$$

NO EXACTA. $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$

FACTOR INTEGRANTE.

$$\underbrace{\mu(x,y)M(x,y)} + \underbrace{\mu(x,y)N(x,y)}\frac{dy}{dx}=0$$

mu

$$\frac{\partial}{\partial y}\mu M = \frac{\partial}{\partial x}\mu N$$

$$\frac{\partial \mu}{\partial y}\cdot M + \mu\frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x}\cdot N + \mu\frac{\partial N}{\partial x}$$

$$\frac{\partial \mu}{\partial y}M - \frac{\partial \mu}{\partial x}N = \mu\frac{\partial N}{\partial x} - \mu\frac{\partial M}{\partial y}$$

$$\frac{\partial \mu}{\partial y}M - \frac{\partial \mu}{\partial x}N = \mu\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right).$$

E en DP

supongo que $\mu(x)$

$$-\frac{d\mu}{dx}N = \mu(x)\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$$

$$\frac{d\mu}{dx}N = \mu(x)\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$$

$$\frac{d\mu(x)}{\mu(x)} = \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\right)dx$$

$$\frac{d\mu(y)}{\mu(y)} = \left(\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\right)dy$$

EJEMPLO

$$(x^3+xy^2)+(x^2y+y^3)\frac{dy}{dx}=0$$

$$\frac{\partial M}{\partial y}=+2xy \qquad \frac{\partial N}{\partial x}=2xy$$

EXACTA

$$\int M\,dx=\int x^3dx+y^2\int x\,dx$$

$$=\frac{x^4}{4}+\frac{x^2y^2}{2}$$

$$\int N\,dy=x^2\int y\,dx+\int y^3dy$$

$$=\frac{x^2y^2}{2}+\frac{y^4}{4}$$

$$\frac{x^4}{4}+\frac{x^2y^2}{2}+\frac{y^4}{4}=C$$

EJEMPLO

$$(2xy^2-3y^3)+(7-3xy^2)\frac{dy}{dx}=0$$

$$\left.\begin{aligned}\frac{\partial M}{\partial y}&=4xy-9y^2\\ \frac{\partial N}{\partial x}&=-3y^2\end{aligned}\right\}$$ NO SON IGUALES $\therefore$ NO ES EXACTA.

$$\frac{d\mu(x)}{\mu(x)}=\left(\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}\right)dx \qquad \frac{d\mu(x)}{\mu(x)}=\left(\frac{4xy-9y^2+3y^2}{7-3xy^2}\right)dx$$

$$=\left(\frac{4xy-6y^2}{7-3xy^2}\right)$$

$$\frac{d\mu(y)}{\mu(y)}=\left(\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}\right)dy \qquad \frac{d\mu(y)}{\mu(y)}=\left(\frac{-3y^2-4xy+9y^2}{2xy^2-3y^3}\right)dy$$

$$=\left(\frac{-4xy+6y^2}{2xy^2-3y^3}\right)dy$$

$$=\left(\frac{-2(2xy-3y^2)}{y(2xy-3y^2)}\right)dy$$

$$\frac{du}{u}=-\frac{2}{y}\,dy$$

$$\int\frac{du}{u}=-2\int\frac{dy}{y}$$

$$Lu=-2Ly$$

$$Lu=Ly^{-2}$$

$$\mu=y^{-2} \qquad \boxed{\mu=\frac{1}{y^2}}$$

$$(2xy^2 - 3y^3) + (7 - 3xy^2)\frac{dy}{dx} = 0$$

$$\mu(y) = \frac{1}{y^2}$$

$$\left(\frac{2xy^2 - 3y^3}{y^2}\right) + \left(\frac{7 - 3xy^2}{y^2}\right)\frac{dy}{dx} = 0$$

$$(2x - 3y) + \left(\frac{7}{y^2} - 3x\right)\frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = -3 \qquad \frac{\partial N}{\partial x} = -3$$