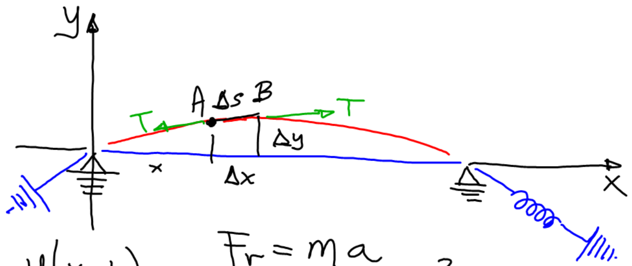
EJERCICIO FINAL.



$y(x,t)$

$F_r = ma$

$a = \frac{\partial^2 y(x,t)}{\partial t^2}$

$m = \rho \Delta s$

$F_r = T_{V_B} - T_{V_A}$

$\operatorname{sen}(\alpha) \doteq \tan(\alpha) = \frac{\Delta y}{\Delta x}$

$\alpha < 3^\circ$

$T_{V_A} = T \frac{\Delta y}{\Delta x}$

$\Delta x \to 0$

$T_{V_A} = T \frac{\partial y(x,t)}{\partial x}$

$T_{V_B} = T \frac{\partial y}{\partial x} + \frac{\partial}{\partial x} T \frac{\partial y}{\partial x} \cdot \Delta x$

$F_r = T \frac{\partial^2 y}{\partial x^2} \Delta x$

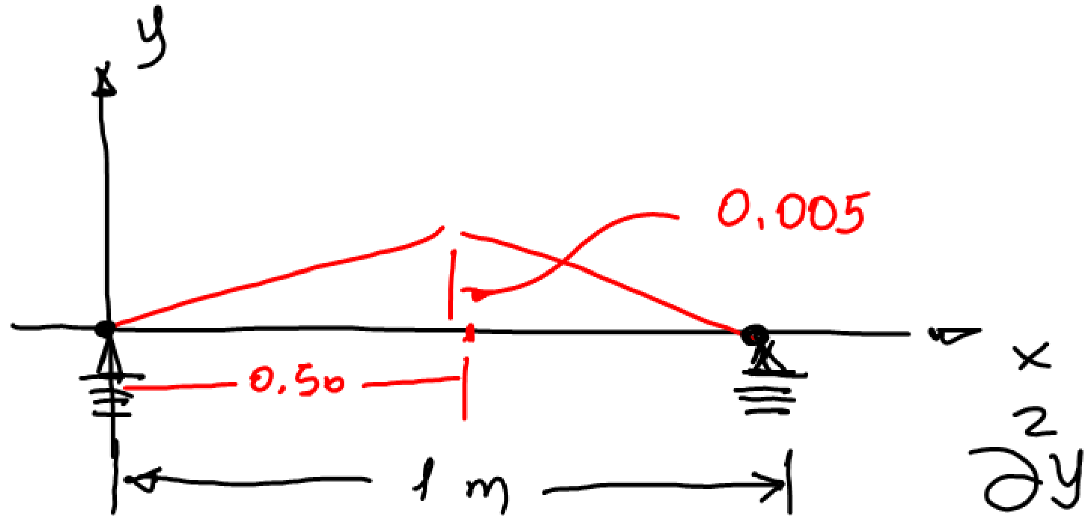
$F_r = ma$

$T \frac{\partial^2 y}{\partial x^2} \Delta x = \rho \Delta s \cdot \frac{\partial^2 y}{\partial t^2}$

$\Delta x \to 0 \quad \Delta s \to 0$

$\rho \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2} \qquad c^2 = \frac{T}{\rho}$

$$\boxed{\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0}$$



$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0$$

$$\underbrace{\begin{matrix} x = 0 \rightarrow y(0) = 0 \\ x = 1 \rightarrow y(1) = 0 \end{matrix}}_{\text{Condiciones en la frontera.}} \quad \forall t$$

$t = 0$

$$y(0) = \begin{cases} \dfrac{0{,}005}{0.5} x \; ; & 0 \leq x \leq 0{,}5 \\[2ex] -\dfrac{0{,}005}{0{,}5} x + 0{,}01 \; ; & 0{,}5 < x \leq 1 \end{cases}$$

$$V = \left. \frac{\partial y}{\partial t} \right|_{t=0} = 0$$

Condiciones iniciales del problema.

$$\frac{G''(t)}{G(t)} = c^2 \frac{F''(x)}{F(x)} \Rightarrow \alpha$$

$$\frac{G''(t)}{G(t)} = \alpha \qquad c^2 \frac{F''(x)}{F(x)} = \alpha$$

para $\alpha = 0$

$$F''(x) = 0$$
$$F'(x) = C_1$$
$$F(x) = C_1 x + C_2$$

$\forall t$

$$y(0,t) = 0 \qquad y(1,t) = 0$$
$$F(0) \cdot G(t) = 0 \qquad F(1)\, G(t) = 0$$
$$F(0) = 0 \qquad F(1) = 0$$

$$F(0) \Rightarrow C_1(0) + C_2 = 0 \qquad C_2 = 0$$
$$F(1) \Rightarrow C_1(1) = 0 \qquad C_1 = 0$$
$$F(x)\Big|_{\alpha=0} = 0 \qquad y(x,t)\Big|_{\alpha=0} = 0$$

$$\frac{G''(t)}{G(t)} = c^2 \frac{F''(x)}{F(x)}$$

$$c^2 \frac{F''(x)}{F(x)} = \alpha \qquad \alpha > 0 \qquad \alpha = \beta^2$$

$$c^2 \frac{F''(x)}{F(x)} = \beta^2 \rightarrow F''(x) = \frac{\beta^2}{c^2} F(x)$$

$$\left(D^2 - \frac{\beta^2}{c^2}\right) F(x) = 0$$

$$F(x) = C_1 e^{\frac{\beta}{c}x} + C_2 e^{-\frac{\beta}{c}x} \qquad \begin{cases} F(0) = 0 \\ F(1) = 0 \end{cases}$$

$$C_1 e^{\frac{\beta}{c}(0)} + C_2 e^{-\frac{\beta}{c}(0)} = 0$$

$$C_1 + C_2 = 0 \qquad C_2 = -C_1$$

$$F(1) \Rightarrow C_1 e^{\frac{\beta}{c}(1)} - C_1 e^{-\frac{\beta}{c}(1)} = 0$$

$$C_1 e^{\frac{\beta}{c}} = \frac{C_1}{e^{\frac{\beta}{c}}} \Rightarrow e^{2\frac{\beta}{c}} = 1$$

por lo tanto

$y(x,t)_{\alpha>0}$ no sirve.

$$\frac{G''(t)}{G(t)} = c^2 \frac{F''(x)}{F(x)} \qquad \alpha < 0 \qquad \alpha = -\beta^2$$

$$c^2 \frac{F''(x)}{F(x)} = -\beta^2 \longrightarrow F''(x) = -\frac{\beta^2}{c^2} F(x)$$

$$\left(D^2 + \frac{\beta^2}{c^2}\right) F(x) = 0$$

$$F(x) = C_1 \cos\left(\frac{\beta}{c} x\right) + C_2 \operatorname{sen}\left(\frac{\beta}{c} x\right). \qquad F(0) = 0 \qquad F(1) = 0$$

$$F(0) \Rightarrow C_1 \cos(0) + C_2 \operatorname{sen}(0) = 0$$

$$C_1 (1) + C_2 (0) = 0 \qquad \boxed{C_1 = 0}$$

$$F(1) \Rightarrow C_2 \operatorname{sen}\left(\frac{\beta}{c}\right) = 0 \qquad \operatorname{sen}\left(\frac{\beta}{c}\right) = 0 \qquad \frac{\beta}{c} = n\pi$$

$y(x,t)_{\alpha<0}$ sí existe solución. $\qquad C_2 \neq 0$

$$\beta = c n \pi$$

$$\frac{G''(t)}{G(t)} = -n^2 c^2 \pi^2 \qquad \alpha = -c^2 n^2 \pi^2$$

$$\text{Solución Inicial} = \sum_{n=1}^{\infty} b_n \operatorname{sen}(n\pi x)$$

$$b_n = \frac{1}{L}\int_0^L f(x)\operatorname{sen}\left(\frac{n\pi}{L}x\right)dx$$