TEMA 2.- EDOL(n) cc NH.

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} \frac{dy}{dx} + a_n y = Q(x)$$

EDOL(1) cc H

para la compañera:

$$a_0 \frac{dy}{dx} + a_1 y = 0$$

$$\frac{dy}{dx} + \frac{a_1}{a_0} y = 0$$

$$\frac{dy}{dx} + m y = 0$$

VAR. SEP.

$$\frac{dy}{dx} = -my$$

$$\int \frac{dy}{y} = -m \int dx$$

$$Ly = -mx + C$$

$$y = e^{(-mx+C)}$$

$$y = e^C e^{-mx}$$

SG $\boxed{y = C_1 e^{-mx}}$ $y_p = e^{-mx}$

EDOL $\boxed{\frac{dy}{dx} + my = 0}$

$$\frac{dy}{dx} - 3y = 0 \longrightarrow \boxed{y = C_1 e^{3x}}$$

$$\frac{dy}{dx} + \sqrt{2}\, y = 0 \qquad y = C_1 e^{-\sqrt{2}x}$$

$$\frac{dy}{dx} = 0 \qquad y = C_1 e^{(0)x} \Rightarrow C_1 \cdot (1) \Rightarrow C_1$$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$y_g = c_1 y_1 + c_2 y_2 \qquad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

$$y_p = e^{mx}$$

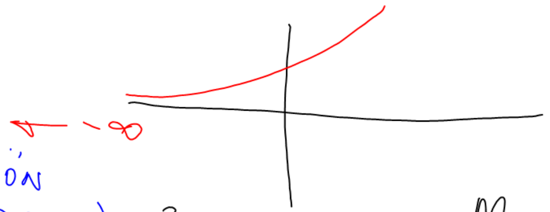
$$\frac{dy_p}{dx} = me^{mx}$$

$$\frac{d^2y_p}{dx^2} = m^2e^{mx}$$

$$\left[m^2e^{mx}\right] + a_1\left[me^{mx}\right] + a_2\left[e^{mx}\right] = 0$$

$$(m^2 + a_1 m + a_2)e^{mx} = 0$$

$$e^{mx} = 0 \quad \text{trivial EDOL(n)ccH.}$$



ECUACIÓN (ALGEBRAICA) CARACTERÍSTICA

$$m^2 + a_1 m + a_2 = 0 \qquad m_1 \quad m_2 \qquad m_1 \neq m_2 \qquad e^{m_1 x} \quad e^{m_2 x}$$

$$(m - m_1)\cdot(m - m_2) = 0$$

$$y_g = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

EDOL (2) cc H.

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0$$

CASO I.

$m_1 \neq m_2 \in \mathbb{R}$

$$m_1 = 2 \qquad m_2 = 3$$

$$y_1 = e^{2x} \qquad y_2 = e^{3x}$$

$$y_g = c_1 e^{2x} + c_2 e^{3x}$$

CASO III.

$$\left.\begin{array}{l} m_1 = a + bi \\ m_2 = a - bi \end{array}\right\} \begin{array}{l} a \in \mathbb{R} \\ b \in \mathbb{R}^+ \end{array}$$

$$m_1 \neq m_2$$

CASO II

$m_1 = m_2 \in \mathbb{R}$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \qquad \text{CASO III}$$

$$m_1 = a + bi \qquad i = \sqrt{-1}$$

$$m_2 = a - bi$$

$$y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x}$$

$$y \in \mathbb{R} \quad x \in \mathbb{R}$$

Teorema Euler

$$e^{\pi i} = -1$$

$$re^{\pi i} = r\cos(\pi) + i\, r \operatorname{sen}(\pi)$$

$$re^{\theta i} = r\cos(\theta) + i\, r \operatorname{sen}(\theta)$$

$$\boxed{\begin{aligned} e^{\theta i} &= \cos(\theta) + i \operatorname{sen}(\theta) \\ e^{-\theta i} &= \cos(\theta) - i \operatorname{sen}(\theta) \end{aligned}}$$

$$y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x}$$

$$y_g = c_1 e^{ax} e^{(bx)i} + c_2 e^{ax} e^{(-bx)i}$$

$$y_g = c_1 e^{ax}\left[\cos(bx) + i \operatorname{sen}(bx)\right] + c_2 e^{ax}\left[\cos(bx) - i \operatorname{sen}(bx)\right]$$

$$y_g = (c_1 + c_2) e^{ax}\cos(bx) + (c_1 i - c_2 i) e^{ax} \operatorname{sen}(bx)$$

$y_g \in \mathbb{R}$
$x \in \mathbb{R}$
$c_1, c_2 \in \mathbb{R}$

$$\boxed{y_g = c_{10} e^{ax}\cos(bx) + c_{20} e^{ax} \operatorname{sen}(bx)}$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

$$m^2 + m + 1 = 0$$

$$m_{1,2} = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}.$$

$$m_{1,2} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$m_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{3}\, i}{2}$$

$$\left.\begin{matrix} m_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i \\ m_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i \end{matrix}\right. \qquad m_1 \neq m_2$$

$$a = -\frac{1}{2} \qquad b = \frac{\sqrt{3}}{2}$$

$$\boxed{y_g = c_1 e^{-\frac{1}{2}x} \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 e^{-\frac{1}{2}x} \operatorname{sen}\left(\frac{\sqrt{3}}{2}x\right).}$$

$$y_g = C_1 \cos(3x) + C_2 \operatorname{sen}(3x)$$

$$m_1 = 0 + 3i$$
$$m_2 = 0 - 3i$$
$$(m - 3i)(m + 3i) = 0$$
$$m^2 + 9 = 0$$
$$\frac{d^2y}{dx^2} + 9y = 0$$

$$y_g = C_1 e^{-x} + C_2 e^{x} \quad \text{EDOL}(2)\text{ccH.}$$

$$m_1 = -1 \qquad m_1 \neq m_2 \quad \text{CASO I.}$$
$$m_2 = 1$$

Ec. $(m+1)(m-1) = 0$

$$m^2 - 1 = 0$$
$$\frac{d^2y}{dx^2} - y = 0$$

$$y_g = C_1 e^{x} \cos(x) + C_2 e^{x} \operatorname{sen}(x)$$

$$m_1 = 1 + i$$
$$m_2 = 1 - i$$

Ec. $(m - 1 - i)(m - 1 + i) = 0$

$$((m-1)^2 + 1) = 0$$
$$m^2 - 2m + 1 + 1 = 0$$
$$m^2 - 2m + 2 = 0$$
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

CASO II $\quad m_1 = m_2$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0 \quad m_1 = m_2 = 1.$$

$$\boxed{y_1 = e^x} \qquad y_2 = ?$$

$$m^2 + a_1 m + a_2 = 0$$

$m_1 \neq m_2$

$$(m - m_1)(m - m_2) = 0$$

no la satisfacen

$$\frac{d}{dm}\left(m^2 + a_1 m + a_2 = 0\right)$$

$$2m + a_1 = 0$$

$$2m_1 + a_1 = 0$$

$m_1 = m_2$

$$(m - m_1)^2 = 0$$

Si satisface

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0$$

$$m_1 = m_2$$

$$y = e^{mx} \xrightarrow{m = m_1} y_1 = e^{m_1 x}$$

$$\frac{d}{dm}\left(\right.$$

$$y = xe^{mx} \xrightarrow{m = m_1} y_2 = xe^{m_1 x}$$

$$y = C_1 e^{-\sqrt{2}x} + C_2 x e^{-\sqrt{2}x}$$

$$(m+\sqrt{2})^2 = 0$$

$$m^2 + 2\sqrt{2}\,m + 2 = 0$$

$$\frac{d^2y}{dx^2} + 2\sqrt{2}\frac{dy}{dx} + 2y = 0$$

$$y_g = C_1 e^{x}\cos(x) + C_2 e^{x}\operatorname{sen}(x) + C_3 x e^{x}\cos(x) + C_4 x e^{x}\operatorname{sen}(x)$$

$$m_1 = 1+i \qquad m_2 = 1-i$$

$$m_3 = 1+i \qquad m_4 = 1-i$$

$$(m-1-i)^2 \cdot (m-1+i)^2 = 0$$

$$\left((m-1)^2+1\right)\cdot\left((m-1)^2-1\right) = 0$$

$$(m-1)^4 - 1 = 0$$

$$m^4 - 4m^3 + 6m^2 - 4m + 1 - 1 = 0$$

$$m^4 - 4m^3 + 6m^2 - 4m = 0$$

$$\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0 \qquad m_1 = m_2 = 1$$

$$y = e^{mx} \xrightarrow{m=1} y_1 = e^x$$

$$\frac{d}{dm}$$

$$y = xe^{mx} \xrightarrow{m=1} y_2 = xe^x$$

$$y_2' = xe^x + e^x$$

$$y'' = xe^x + 2e^x$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$(xe^x + 2e^x) - 2(xe^x + e^x) + (xe^x) = 0$$

$$(1 - 2 + 1)xe^x + (+2 - 2)e^x = 0$$

$$0 \quad xe^x \qquad 0 \cdot e^x = 0$$

$$0 = 0$$

$$\begin{vmatrix} xe^x & e^x \\ xe^x + e^x & e^x \end{vmatrix} \neq 0$$

$$xe^x e^x - e^x(xe^x + e^x) \neq 0$$

$$\cancel{xe^x e^x} - \cancel{xe^x e^x} - e^x e^x \neq 0$$

$$-e^x e^x \neq 0$$

CASO II

$$y_g = c_1 e^x + c_2 x e^x$$

$$y_g = c_1 e^{2x} + c_2 x e^{2x} + c_3 x^2 e^{2x}$$

$$(m-2)^3 = 0 \qquad m_1 = m_2 = m_3 = 2.$$

$$m^3 - 6m^2 + 12m - 8 = 0$$

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} - 8y = 0$$