FACULTAD DE INGENIERÍA
ECUACIONES DIFERENCIALES
SERIE 2020-1-1

2019 SEPTIEMBRE 04

> *restart* :

1) Si conocemos la solución general de una ecuación diferencial ordinaria no lineal desconocida

>

> $SolucionGeneral := y(x)^2\,(1-y(x)) = (x-\_C1)^2$

$$SolucionGeneral := y(x)^2\,(1-y(x)) = (x-\_C1)^2 \tag{1}$$

a) obtenga su ecuación diferencial correspondiente.
b) demuestre porqué es una solución singular la siguiente función

> $y_1 = 1$

$$y_1 = 1 \tag{2}$$

c) obtenga la solución particular que satisface la siguiente condición inicial

> $CondicionInicial := y(3) = 12$

$$CondicionInicial := y(3) = 12 \tag{3}$$

> $DerSolucionGeneral := isolate(diff(SolucionGeneral, x), \_C1)$

$$DerSolucionGeneral := \_C1 = -y(x)\,(1-y(x))\left(\frac{d}{dx}\,y(x)\right) + \frac{1}{2}\,y(x)^2\left(\frac{d}{dx}\,y(x)\right) + x \tag{4}$$

> $EcuaDif := subs(\_C1 = rhs(DerSolucionGeneral), SolucionGeneral)$

$$EcuaDif := y(x)^2\,(1-y(x)) = \left(y(x)\,(1-y(x))\left(\frac{d}{dx}\,y(x)\right) - \frac{1}{2}\,y(x)^2\left(\frac{d}{dx}\,y(x)\right)\right)^2 \tag{5}$$

> $EcuaDifDos := \mathrm{sqrt}(lhs(EcuaDif)) = y(x)\,(1-y(x))\left(\frac{d}{dx}\,y(x)\right) - \frac{1}{2}\,y(x)^2\left(\frac{d}{dx}\,y(x)\right)$

$$EcuaDifDos := \sqrt{y(x)^2\,(1-y(x))} = y(x)\,(1-y(x))\left(\frac{d}{dx}\,y(x)\right) - \frac{1}{2}\,y(x)^2\left(\frac{d}{dx}\,y(x)\right) \tag{6}$$

> $EcuaDifTres := isolate(EcuaDifDos, diff(y(x), x))$

$$EcuaDifTres := \frac{d}{dx}\,y(x) = -\frac{\sqrt{y(x)^2\,(1-y(x))}}{-y(x)\,(1-y(x)) + \frac{1}{2}\,y(x)^2} \tag{7}$$

> $Comprob := eval(subs(y(x) = 1, EcuaDifTres))$

$$Comprob := 0 = 0 \tag{8}$$

> $SolucionGeneral$

$$y(x)^2\,(1-y(x)) = (x-\_C1)^2 \tag{9}$$

> $SolucionDos := y^2\,(1-y) = (x-\_C1)^2$

$$SolucionDos := y^2\,(1-y) = (x-\_C1)^2 \tag{10}$$

> $Para := isolate(subs(x = 3, y = 12, SolucionDos), \_C1)$

$$Para := \_C1 = -\sqrt{-1584} + 3 \tag{11}$$

> $SolucionParticular := subs(\_C1 = rhs(Para), SolucionGeneral)$

$$SolucionParticular := y(x)^2\,(1-y(x)) = \left(x+\sqrt{-1584}-3\right)^2 \tag{12}$$

>

> restart :

2) Obtener la solución general de la siguiente ecuación (sin usar dsolve) por ambos métodos posibles:

>

> $EcuacionDiferencial := 4\,x^2 + x\,y(x) - 3\,y(x)^2 + \left(-5\,x^2 + 2\,x\,y(x) + y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right) = 0$

$$EcuacionDiferencial := 4\,x^2 + x\,y(x) - 3\,y(x)^2 + \left(-5\,x^2 + 2\,x\,y(x) + y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{13}$$

> with(DEtools) :

> odeadvisor(EcuacionDiferencial)

$$[[\_homogeneous,\ class\ A],\ \_rational,\ \_dAlembert] \tag{14}$$

>

> EcuaDif := simplify(isolate(eval(subs(y(x) = u(x) ·x, EcuacionDiferencial)), diff(u(x), x)))

$$EcuaDif := \frac{d}{dx}\,u(x) = -\frac{u(x)^3 - u(x)^2 - 4\,u(x) + 4}{x\left(u(x)^2 + 2\,u(x) - 5\right)} \tag{15}$$

> $Solucion := int\left(\frac{1}{\frac{u^3 - u^2 - 4\,u + 4}{\left(u^2 + 2\,u - 5\right)}}, u\right) + int\left(\frac{1}{x}, x\right) = C[1]$

$$Solucion := \frac{2}{3}\ln(u-1) - \frac{5}{12}\ln(u+2) + \frac{3}{4}\ln(u-2) + \ln(x) = C_1 \tag{16}$$

> SolucionDos := simplify(isolate(Solucion, x))

$$SolucionDos := x = \frac{e^{C_1}\,(u+2)^{5/12}}{(u-1)^{2/3}\,(u-2)^{3/4}} \tag{17}$$

> $SolucionTres := subs\left(u = \frac{y}{x}, SolucionDos\right)$

$$SolucionTres := x = \frac{e^{C_1}\left(\frac{y}{x}+2\right)^{5/12}}{\left(\frac{y}{x}-1\right)^{2/3}\left(\frac{y}{x}-2\right)^{3/4}} \tag{18}$$

> $SolucionCuatro := x = \frac{C_1\left(\frac{y}{x}+2\right)^{5/12}}{\left(\frac{y}{x}-1\right)^{2/3}\left(\frac{y}{x}-2\right)^{3/4}}$

$$SolucionCuatro := x = \frac{C_1\left(\frac{y}{x}+2\right)^{5/12}}{\left(\frac{y}{x}-1\right)^{2/3}\left(\frac{y}{x}-2\right)^{3/4}} \tag{19}$$

> SolucionCinco := expand(isolate(SolucionCuatro, C[1]))

$$SolucionCinco := C_1 = \frac{x\left(\frac{y}{x}-1\right)^{2/3}\left(\frac{y}{x}-2\right)^{3/4}}{\left(\frac{y}{x}+2\right)^{5/12}} \tag{20}$$

> $SolucionSeis := C[1] = \frac{\left(\frac{x}{x^{\left(\frac{2}{3}\right)} \cdot x^{\left(\frac{3}{4}\right)}}\right)}{\left(\frac{1}{x^{\left(\frac{5}{12}\right)}}\right)} \cdot \frac{\left((y-x)^{\left(\frac{2}{3}\right)} \cdot (y-2\cdot x)^{\left(\frac{3}{4}\right)}\right)}{(y+2\cdot x)^{\left(\frac{5}{12}\right)}}$

$$SolucionSeis := C_1 = \frac{(y-x)^{2/3}(y-2x)^{3/4}}{(y+2x)^{5/12}} \tag{21}$$

>

> $intfactor(EcuacionDiferencial)$

$$\frac{1}{(y(x)-2x)\,(y(x)+2x)\,(y(x)-x)} \tag{22}$$

> $IntFact := \frac{1}{(y-2x)\,(y+2x)\,(y-x)}$

$$IntFact := \frac{1}{(y-x)\,(y-2x)\,(y+2x)} \tag{23}$$

> $M := 4x^2 + xy - 3y^2$

$$M := 4x^2 + xy - 3y^2 \tag{24}$$

> $N := -5x^2 + 2xy + y^2$

$$N := -5x^2 + 2xy + y^2 \tag{25}$$

> $CompUno := simplify(diff(M, y) - diff(N, x)) \neq 0$

$$CompUno := 11x - 8y \neq 0 \tag{26}$$

> $MM := simplify(M \cdot IntFact)$

$$MM := \frac{4x^2 + xy - 3y^2}{(-y+x)\,(4x^2-y^2)} \tag{27}$$

> $NN := simplify(N \cdot IntFact)$

$$NN := -\frac{5x^2 - 2xy - y^2}{(-y+x)\,(4x^2-y^2)} \tag{28}$$

> $CompDos := simplify(diff(MM, y) - diff(NN, x)) = 0$

$$CompDos := 0 = 0 \tag{29}$$

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{3}{4}\ln(-y+2x) - \frac{5}{12}\ln(y+2x) + \frac{2}{3}\ln(-y+x) \tag{30}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = C[1]$

$$SolGral := -\frac{5}{12}\ln(y+2x) + \frac{2}{3}\ln(y-x) + \frac{3}{4}\ln(y-2x) = C_1 \tag{31}$$

> $SolGralDos := simplify\left(\exp\left(-\frac{5}{12}\ln(y+2x)\right)\cdot\exp\left(\frac{3}{4}\ln(y-2x)\right)\cdot\exp\left(\frac{2}{3}\ln(y-x)\right) = C[1]\right)$

$$SolGralDos := \frac{(y-x)^{2/3}(y-2x)^{3/4}}{(y+2x)^{5/12}} = C_1 \quad \textbf{(32)}$$

> $SolucionSeis$

$$C_1 = \frac{(y-x)^{2/3}(y-2x)^{3/4}}{(y+2x)^{5/12}} \quad \textbf{(33)}$$

> $ComprobacionTres := expand(rhs(SolucionSeis) - lhs(SolGralDos)) = 0$

$$ComprobacionTres := 0 = 0 \quad \textbf{(34)}$$

> $SolucionSiete := \frac{(y(x)-2x)^{3/4}(y(x)-x)^{2/3}}{(y(x)+2x)^{5/12}} = C[1]$

$$SolucionSiete := \frac{(y(x)-2x)^{3/4}(y(x)-x)^{2/3}}{(y(x)+2x)^{5/12}} = C_1 \quad \textbf{(35)}$$

> $DerSolGral := isolate(diff(SolucionSiete, x), diff(y(x), x))$

$$DerSolGral := \frac{d}{dx}y(x) = \frac{3y(x)^2 - x\,y(x) - 4x^2}{-5x^2 + 2x\,y(x) + y(x)^2} \quad \textbf{(36)}$$

> $DerEcua := isolate(EcuacionDiferencial, diff(y(x), x))$

$$DerEcua := \frac{d}{dx}y(x) = \frac{3y(x)^2 - x\,y(x) - 4x^2}{-5x^2 + 2x\,y(x) + y(x)^2} \quad \textbf{(37)}$$

> $CompCuatro := lhs(DerSolGral) - lhs(DerEcua) = 0$

$$CompCuatro := 0 = 0 \quad \textbf{(38)}$$

>

> $restart:$

3) Dada la siguiente ecuación diferencial con condiciones iniciales:
a) Obtener su solución particular (**sin usar dsolve**)

>

b) Graficar dicha solución particular en un intervalo -4 < x < 4

> $restart:$

4) Obtenga la solución particular de la siguiente ecuación diferencial con la condición inicial dada - utilizando exclusivamente el método del factor integrante (**no utilizar dsolve**)

> $EcuacionDiferencial := 2x^2 + y(x) + (x^2 y(x) - x)\left(\frac{d}{dx}y(x)\right) = 0;\ CondicionInicial := y(1) = -2$

$$EcuacionDiferencial := 2x^2 + y(x) + (x^2 y(x) - x)\left(\frac{d}{dx}y(x)\right) = 0$$

$$CondicionInicial := y(1) = -2 \quad \textbf{(39)}$$

> $with(DEtools):$

> $IntFact := intfactor(EcuacionDiferencial)$

(40)

$$IntFact := \frac{1}{x^2} \quad \textbf{(40)}$$

> $M := 2x^2 + y$

$$M := 2x^2 + y \quad \textbf{(41)}$$

> $N := x^2 y - x$

$$N := x^2 y - x \quad \textbf{(42)}$$

> $CompUno := diff(M, y) - diff(N, x) = 0$

$$CompUno := -2xy + 2 = 0 \quad \textbf{(43)}$$

> $MM := expand(M \cdot IntFact); NN := expand(N \cdot IntFact)$

$$MM := 2 + \frac{y}{x^2}$$

$$NN := y - \frac{1}{x} \quad \textbf{(44)}$$

> $CompUno := diff(MM, y) - diff(NN, x) = 0$

$$CompUno := 0 = 0 \quad \textbf{(45)}$$

> $IntMMx := int(MM, x)$

$$IntMMx := 2x - \frac{y}{x} \quad \textbf{(46)}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = C[1]$

$$SolGral := 2x - \frac{y}{x} + \frac{1}{2}y^2 = C_1 \quad \textbf{(47)}$$

> $Para := subs(x = 1, y = -2, SolGral)$

$$Para := 6 = C_1 \quad \textbf{(48)}$$

> $SolPart := subs(C[1] = lhs(Para), SolGral)$

$$SolPart := 2x - \frac{y}{x} + \frac{1}{2}y^2 = 6 \quad \textbf{(49)}$$

>

> $restart:$

5) Dada la siguiente ecuación diferencial:

> $EcuacionDiferencial := 2x\,(x^2 + y(x)^2)\left(\frac{d}{dx}y(x)\right) = y(x)\,(y(x)^2 + 2x^2)$

$$EcuacionDiferencial := 2x\,(x^2 + y(x)^2)\left(\frac{d}{dx}y(x)\right) = y(x)\,(y(x)^2 + 2x^2) \quad \textbf{(50)}$$

> $with(DEtools):$

> $odeadvisor(EcuacionDiferencial)$

$$[[\_homogeneous, class\ A], \_rational, \_dAlembert] \quad \textbf{(51)}$$

> $intfactor(EcuacionDiferencial)$

$$\frac{1}{x\,y(x)^3} \quad \textbf{(52)}$$

> $FactInt := \frac{1}{x\,y^3}$

$$FactInt := \frac{1}{y^3\,x} \tag{53}$$

> $M := -y\,(y^2+2\,x^2)$

$$M := -y\,(2\,x^2+y^2) \tag{54}$$

> $N := 2\,x\,(x^2+y^2)$

$$N := 2\,x\,(x^2+y^2) \tag{55}$$

> $CompUno := diff(M, y) - diff(N, x) = 0$

$$CompUno := -8\,x^2 - 5\,y^2 = 0 \tag{56}$$

> $MM := expand(FactInt \cdot M)$

$$MM := -\frac{2\,x}{y^2} - \frac{1}{x} \tag{57}$$

> $NN := expand(FactInt \cdot N)$

$$NN := \frac{2\,x^2}{y^3} + \frac{2}{y} \tag{58}$$

> $CompDos := diff(MM, y) - diff(NN, x) = 0$

$$CompDos := 0 = 0 \tag{59}$$

> $IntMMx := int(MM, x)$

$$IntMMx := -\frac{x^2}{y^2} - \ln(x) \tag{60}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = C[1]$

$$SolGral := -\frac{x^2}{y^2} - \ln(x) + 2\,\ln(y) = C_1 \tag{61}$$

>

obtenga su solución general (**no se puede utilizar dsolve**)

> $restart:$

6) Obtener la solución general de la ecuación diferencial ordinaria siguiente **(sin utilizar dsolve)**

> $Ecua := x\ln(x)\left(\frac{d}{dx}\,y(x)\right) - (1+\ln(x))\,y(x) + \frac{1}{2}\sqrt{x}\,(2+\ln(x)) = 0$

$$Ecua := x\ln(x)\left(\frac{d}{dx}\,y(x)\right) - (1+\ln(x))\,y(x) + \frac{1}{2}\sqrt{x}\,(2+\ln(x)) = 0 \tag{62}$$

> $EcuaDos := expand\left(\frac{Ecua}{x\ln(x)}\right)$

$$EcuaDos := \frac{d}{dx}\,y(x) - \frac{y(x)}{x\ln(x)} - \frac{y(x)}{x} + \frac{1}{\sqrt{x}\,\ln(x)} + \frac{1}{2\sqrt{x}} = 0 \tag{63}$$

> $pp := -\frac{1}{x} - \frac{1}{x\ln(x)}$

(64)

$$pp := -\frac{1}{x} - \frac{1}{x\ln(x)} \quad (64)$$

> $qq := -\left(\frac{1}{2\sqrt{x}} + \frac{1}{\sqrt{x}\ln(x)}\right)$

$$qq := -\frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{x}\ln(x)} \quad (65)$$

> $IntPPx := int(pp, x)$

$$IntPPx := -\ln(x) - \ln(\ln(x)) \quad (66)$$

> $EEppMinX := expand(\exp(-IntPPx))$

$$EEppMinX := x\ln(x) \quad (67)$$

> $EEppMasX := expand(\exp(IntPPx))$

$$EEppMasX := \frac{1}{x\ln(x)} \quad (68)$$

> $SolGral := y(x) = C[1]\cdot EEppMinX + EEppMinX\cdot int(EEppMasX\cdot qq, x)$

$$SolGral := y(x) = C_1\, x\ln(x) + \sqrt{x} \quad (69)$$

> $SG := dsolve(Ecua)$

$$SG := y(x) = \sqrt{x} + x\ln(x)\ \_C1 \quad (70)$$

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FIN DE LA SERIE