> restart

> $Ecua := diff(y(t), t\$3) + diff(y(t), t\$2) + diff(y(t), t) + y(t) = 4\cdot\exp(-t)$

$$Ecua := \frac{d^3}{dt^3}\, y(t) + \frac{d^2}{dt^2}\, y(t) + \frac{d}{dt}\, y(t) + y(t) = 4\, e^{-t} \quad \textbf{(1)}$$

> $Cond := y(0) = 1, D(y)(0) = -2, D(D(y))(0) = 3$

$$Cond := y(0) = 1, D(y)(0) = -2, D^{(2)}(y)(0) = 3 \quad \textbf{(2)}$$

> $SolGral := dsolve(Ecua)$

$$SolGral := y(t) = \frac{2\,t}{e^t} + \_C1 \cos(t) + \_C2 \sin(t) + \_C3\, e^{-t} \quad \textbf{(3)}$$

> $DerSolGral := diff(SolGral, t)$

$$DerSolGral := \frac{d}{dt}\, y(t) = \frac{2}{e^t} - \frac{2\,t}{e^t} - \_C1 \sin(t) + \_C2 \cos(t) - \_C3\, e^{-t} \quad \textbf{(4)}$$

> $DerDerSolGral := diff(SolGral, t\$2)$

$$DerDerSolGral := \frac{d^2}{dt^2}\, y(t) = -\frac{4}{e^t} + \frac{2\,t}{e^t} - \_C1 \cos(t) - \_C2 \sin(t) + \_C3\, e^{-t} \quad \textbf{(5)}$$

> $SolPart := dsolve(\{Ecua, Cond\})$

$$SolPart := y(t) = \frac{2\,t}{e^t} - 3\cos(t) + 4\, e^{-t} \quad \textbf{(6)}$$

> $DerSolPart := diff(SolPart, t)$

$$DerSolPart := \frac{d}{dt}\, y(t) = \frac{2}{e^t} - \frac{2\,t}{e^t} + 3\sin(t) - 4\, e^{-t} \quad \textbf{(7)}$$

> $DerDerSolPart := diff(SolPart, t\$2)$

$$DerDerSolPart := \frac{d^2}{dt^2}\, y(t) = -\frac{4}{e^t} + \frac{2\,t}{e^t} + 3\cos(t) + 4\, e^{-t} \quad \textbf{(8)}$$

> $Sist := diff(y[1](t), t) = y[2](t), diff(y[2](t), t) = y[3](t), diff(y[3](t), t) = -y[1](t) - y[2](t) - y[3](t) + 4\cdot\exp(-t) : Sist[1]; Sist[2]; Sist[3]$

$$\frac{d}{dt}\, y_1(t) = y_2(t)$$

$$\frac{d}{dt}\, y_2(t) = y_3(t)$$

$$\frac{d}{dt}\, y_3(t) = -y_1(t) - y_2(t) - y_3(t) + 4\, e^{-t} \quad \textbf{(9)}$$

> $SolGralDos := dsolve(\{Sist\}) : SolGralDos[1]; SolGralDos[2]; SolGralDos[3]$

$$y_1(t) = 2\, e^{-t} + 2\, t\, e^{-t} - \_C3\, e^{-t} + \_C1 \sin(t) - \_C2 \cos(t)$$

$$y_2(t) = -2\, t\, e^{-t} + \_C1 \cos(t) + \_C2 \sin(t) + \_C3\, e^{-t}$$

$$y_3(t) = -2\, e^{-t} + 2\, t\, e^{-t} - \_C1 \sin(t) + \_C2 \cos(t) - \_C3\, e^{-t} \quad \textbf{(10)}$$

> $SolGral; DerSolGral; DerDerSolGral;$

$$y(t) = \frac{2\,t}{e^t} + \_C1 \cos(t) + \_C2 \sin(t) + \_C3\, e^{-t}$$

$$\frac{d}{dt}\,y(t)=\frac{2}{e^t}-\frac{2\,t}{e^t}-\_C1\sin(t)+\_C2\cos(t)-\_C3\,e^{-t}$$

$$\frac{d^2}{dt^2}\,y(t)=-\frac{4}{e^t}+\frac{2\,t}{e^t}-\_C1\cos(t)-\_C2\sin(t)+\_C3\,e^{-t} \quad \textbf{(11)}$$

> $BBuno := y[1](0)=1, y[2](0)=-2, y[3](0)=3$

$$BBuno := y_1(0)=1,\, y_2(0)=-2,\, y_3(0)=3 \quad \textbf{(12)}$$

> $SolPartDos := dsolve(\{Sist, BBuno\}) : SolPartDos[1]; SolPartDos[2]; SolPartDos[3]$

$$y_1(t)=4\,e^{-t}+2\,t\,e^{-t}-3\cos(t)$$

$$y_2(t)=-2\,t\,e^{-t}+3\sin(t)-2\,e^{-t}$$

$$y_3(t)=2\,t\,e^{-t}+3\cos(t) \quad \textbf{(13)}$$

> $SolPart$

$$y(t)=\frac{2\,t}{e^t}-3\cos(t)+4\,e^{-t} \quad \textbf{(14)}$$

> $expand(DerSolPart);$

$$\frac{d}{dt}\,y(t)=-\frac{2}{e^t}-\frac{2\,t}{e^t}+3\sin(t) \quad \textbf{(15)}$$

> $expand(DerDerSolPart)$

$$\frac{d^2}{dt^2}\,y(t)=\frac{2\,t}{e^t}+3\cos(t) \quad \textbf{(16)}$$

> $with(LinearAlgebra) :$

> $with(linalg) :$

> $AA := Matrix([[0, 1, 0], [0, 0, 1], [-1, -1, -1]])$

$$AA := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \quad \textbf{(17)}$$

> $BB := array([0, 0, rhs(Ecua)])$

$$BB := \begin{bmatrix} 0 & 0 & 4\,e^{-t} \end{bmatrix} \quad \textbf{(18)}$$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \frac{1}{2}\,e^{-t}+\frac{1}{2}\cos(t)+\frac{1}{2}\sin(t) & \sin(t) & \frac{1}{2}\sin(t)-\frac{1}{2}\cos(t)+\frac{1}{2}\,e^{-t} \\ -\frac{1}{2}\sin(t)+\frac{1}{2}\cos(t)-\frac{1}{2}\,e^{-t} & \cos(t) & \frac{1}{2}\cos(t)-\frac{1}{2}\,e^{-t}+\frac{1}{2}\sin(t) \\ -\frac{1}{2}\cos(t)+\frac{1}{2}\,e^{-t}-\frac{1}{2}\sin(t) & -\sin(t) & \frac{1}{2}\,e^{-t}-\frac{1}{2}\sin(t)+\frac{1}{2}\cos(t) \end{bmatrix} \quad \textbf{(19)}$$

> $MatExpTau := map(rcurry(eval, t='t-\mathrm{tau}'), MatExp) :$

> $BBtau := map(rcurry(eval, t='\mathrm{tau}'), BB)$

(20)

$$BBtau := \begin{bmatrix} 0 & 0 & 4\,e^{-\tau} \end{bmatrix} \quad \textbf{(20)}$$

> $ProdTau := evalm(MatExpTau\ \&*\ BBtau)$

$$ProdTau := \left[4\left(\frac{1}{2}\sin(t-\tau) - \frac{1}{2}\cos(t-\tau) + \frac{1}{2}e^{-t+\tau}\right)e^{-\tau}, 4\left(\frac{1}{2}\cos(t-\tau) - \frac{1}{2}e^{-t+\tau} + \frac{1}{2}\sin(t-\tau)\right)e^{-\tau}, 4\left(\frac{1}{2}e^{-t+\tau} - \frac{1}{2}\sin(t-\tau) + \frac{1}{2}\cos(t-\tau)\right)e^{-\tau}\right] \quad \textbf{(21)}$$

> $SolNoHom := simplify(map(int, ProdTau, \mathrm{tau} = 0..t))$

$$SolNoHom := \begin{bmatrix} -2\left(\cos(t)\,e^{t} - t - 1\right)e^{-t} & 2\left(\sin(t)\,e^{t} - t\right)e^{-t} & 2\left(\cos(t)\,e^{t} + t - 1\right)e^{-t} \end{bmatrix} \quad \textbf{(22)}$$

> $CompNoHom := map(rcurry(eval, t = '0'), SolNoHom)$

$$CompNoHom := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad \textbf{(23)}$$

> $Xcero := array([1, -2, 3])$

$$Xcero := \begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \quad \textbf{(24)}$$

> $SolHom := evalm(MatExp\ \&*\ Xcero)$

$$SolHom := \begin{bmatrix} 2\,e^{-t} - \cos(t) & \sin(t) - 2\,e^{-t} & \cos(t) + 2\,e^{-t} \end{bmatrix} \quad \textbf{(25)}$$

> $yy[1](t) = simplify(evalm(SolHom[1] + SolNoHom[1])); yy[2](t) = simplify(evalm(SolHom[2] + SolNoHom[2])); yy[3](t) = simplify(evalm(SolHom[3] + SolNoHom[3]))$

$$yy_1(t) = 4\,e^{-t} + 2\,t\,e^{-t} - 3\cos(t)$$

$$yy_2(t) = -2\,t\,e^{-t} + 3\sin(t) - 2\,e^{-t}$$

$$yy_3(t) = 2\,t\,e^{-t} + 3\cos(t) \quad \textbf{(26)}$$

> $SolPartDos[1]; SolPartDos[2]; SolPartDos[3]$

$$y_1(t) = 4\,e^{-t} + 2\,t\,e^{-t} - 3\cos(t)$$

$$y_2(t) = -2\,t\,e^{-t} + 3\sin(t) - 2\,e^{-t}$$

$$y_3(t) = 2\,t\,e^{-t} + 3\cos(t) \quad \textbf{(27)}$$

> $with(inttrans):$

> $II := Matrix([[1, 0, 0], [0, 1, 0], [0, 0, 1]])$

$$II := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \textbf{(28)}$$

> $MatExpLap := inverse(evalm(s \cdot II - AA))$

(29)

$$MatExpLap := \begin{bmatrix} \frac{s^2+s+1}{s^3+s^2+s+1} & \frac{1}{s^2+1} & \frac{1}{s^3+s^2+s+1} \\ -\frac{1}{s^3+s^2+s+1} & \frac{s}{s^2+1} & \frac{s}{s^3+s^2+s+1} \\ -\frac{s}{s^3+s^2+s+1} & -\frac{1}{s^2+1} & \frac{s^2}{s^3+s^2+s+1} \end{bmatrix} \quad \mathbf{(29)}$$

> $MatExpTres := map(invlaplace, MatExpLap, s, t)$

$$MatExpTres := \begin{bmatrix} \frac{1}{2}e^{-t}+\frac{1}{2}\cos(t)+\frac{1}{2}\sin(t) & \sin(t) & \frac{1}{2}\sin(t)-\frac{1}{2}\cos(t)+\frac{1}{2}e^{-t} \\ -\frac{1}{2}\sin(t)+\frac{1}{2}\cos(t)-\frac{1}{2}e^{-t} & \cos(t) & \frac{1}{2}\cos(t)-\frac{1}{2}e^{-t}+\frac{1}{2}\sin(t) \\ -\frac{1}{2}\cos(t)+\frac{1}{2}e^{-t}-\frac{1}{2}\sin(t) & -\sin(t) & \frac{1}{2}e^{-t}-\frac{1}{2}\sin(t)+\frac{1}{2}\cos(t) \end{bmatrix} \quad \mathbf{(30)}$$

> $evalm(MatExp)$

$$\begin{bmatrix} \frac{1}{2}e^{-t}+\frac{1}{2}\cos(t)+\frac{1}{2}\sin(t) & \sin(t) & \frac{1}{2}\sin(t)-\frac{1}{2}\cos(t)+\frac{1}{2}e^{-t} \\ -\frac{1}{2}\sin(t)+\frac{1}{2}\cos(t)-\frac{1}{2}e^{-t} & \cos(t) & \frac{1}{2}\cos(t)-\frac{1}{2}e^{-t}+\frac{1}{2}\sin(t) \\ -\frac{1}{2}\cos(t)+\frac{1}{2}e^{-t}-\frac{1}{2}\sin(t) & -\sin(t) & \frac{1}{2}e^{-t}-\frac{1}{2}\sin(t)+\frac{1}{2}\cos(t) \end{bmatrix} \quad \mathbf{(31)}$$

> $restart$

> $Sist := diff(x[1](t), t) = x[3](t), diff(x[2](t), t) = x[4](t), diff(x[3](t), t) = -10\cdot x[1](t) + 4 \cdot x[2](t), diff(x[4](t), t) = 4\cdot x[1](t) - 4\cdot x[2](t) : Sist[1]; Sist[2]; Sist[3]; Sist[4]$

$$\frac{d}{dt} x_1(t) = x_3(t)$$

$$\frac{d}{dt} x_2(t) = x_4(t)$$

$$\frac{d}{dt} x_3(t) = -10\, x_1(t) + 4\, x_2(t)$$

$$\frac{d}{dt} x_4(t) = 4\, x_1(t) - 4\, x_2(t) \quad \mathbf{(32)}$$

> $Cond := x[1](0) = \frac{4}{3}, x[2](0) = 2, x[3](0) = 0, x[4](0) = 0$

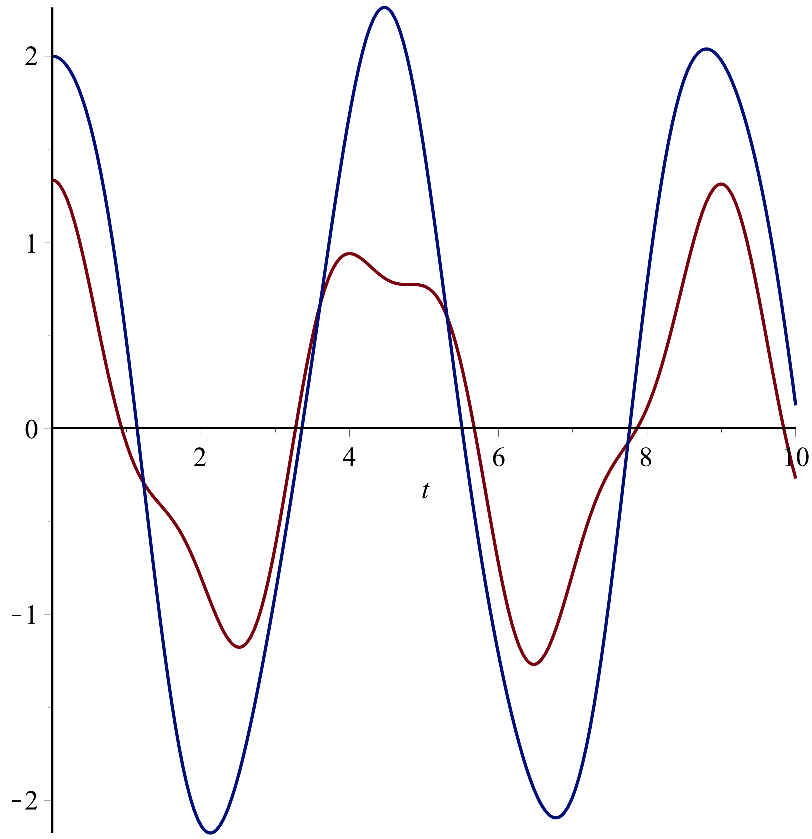
$$Cond := x_1(0) = \frac{4}{3}, x_2(0) = 2, x_3(0) = 0, x_4(0) = 0 \quad \mathbf{(33)}$$

> $SolPart := dsolve(\{Sist, Cond\}) : SolPart[1]; SolPart[2]$

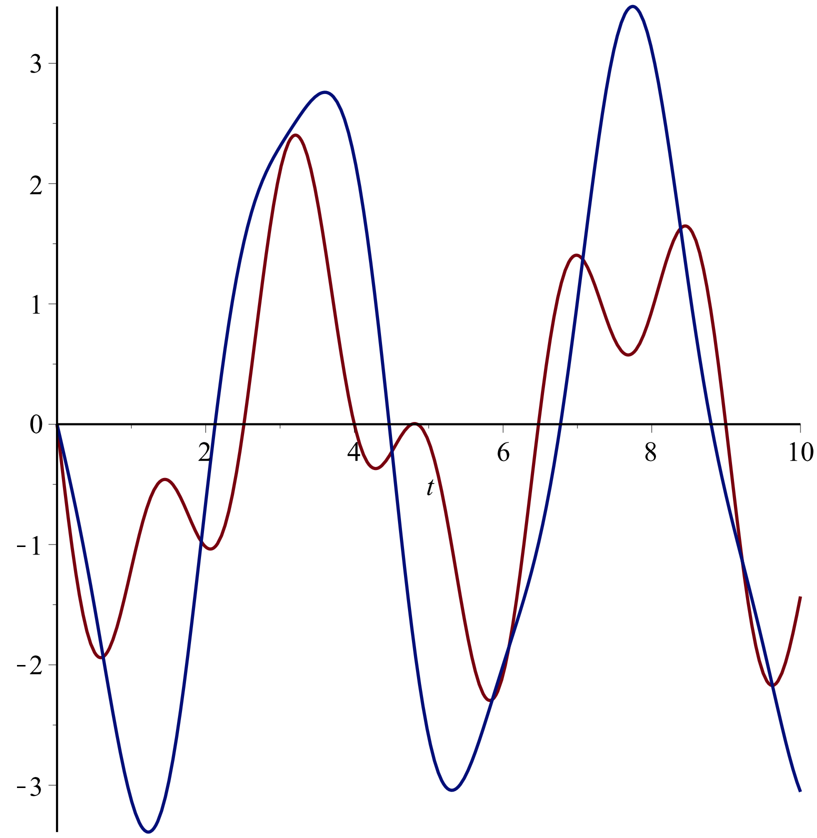
$$x_1(t) = \frac{16}{15}\cos\left(\sqrt{2}\, t\right) + \frac{4}{15}\cos\left(2\sqrt{3}\, t\right)$$

$$x_2(t) = \frac{32}{15}\cos\left(\sqrt{2}\, t\right) - \frac{2}{15}\cos\left(2\sqrt{3}\, t\right) \quad \mathbf{(34)}$$

> $plot([rhs(SolPart[1]), rhs(SolPart[2])], t=0..10)$



> $plot([rhs(diff(SolPart[1], t)), rhs(diff(SolPart[2], t))], t=0..10)$



> restart

> Sist := diff(x[1](t), t) = x[3](t), diff(x[2](t), t) = x[4](t), diff(x[3](t), t) = -10·x[1](t) + 6·x[2](t), diff(x[4](t), t) = 6·x[1](t) − 6·x[2](t) : Sist[1]; Sist[2]; Sist[3]; Sist[4]

$$\frac{d}{dt} x_1(t) = x_3(t)$$

$$\frac{d}{dt} x_2(t) = x_4(t)$$

$$\frac{d}{dt} x_3(t) = -10\, x_1(t) + 6\, x_2(t)$$

$$\frac{d}{dt} x_4(t) = 6\, x_1(t) - 6\, x_2(t) \tag{35}$$

> Cond := x[1](0) = 3, x[2](0) = 2, x[3](0) = 0, x[4](0) = 0

$$Cond := x_1(0) = 3, x_2(0) = 2, x_3(0) = 0, x_4(0) = 0 \tag{36}$$

> SolPart := dsolve({Sist, Cond}) : SolPart[1]; SolPart[2]

$$x_1(t) = \frac{3}{5}\left(\sqrt{10} + 2\right)\sqrt{10}\cos\left(\sqrt{8 + 2\sqrt{10}}\; t\right) + \frac{3}{5}\sqrt{10}\left(-2\right.$$

$$\left. + \sqrt{10}\right)\cos\left(\sqrt{8 - 2\sqrt{10}}\; t\right) - \frac{3}{80}\left(\sqrt{10} + 2\right)\left(8\right.$$

$$+2\sqrt{10}\,)\sqrt{10}\cos\left(\sqrt{8+2\sqrt{10}}\;t\right)-\frac{3}{80}\left(8-2\sqrt{10}\right)\sqrt{10}\,(-2$$

$$+\sqrt{10}\,)\cos\left(\sqrt{8-2\sqrt{10}}\;t\right)$$

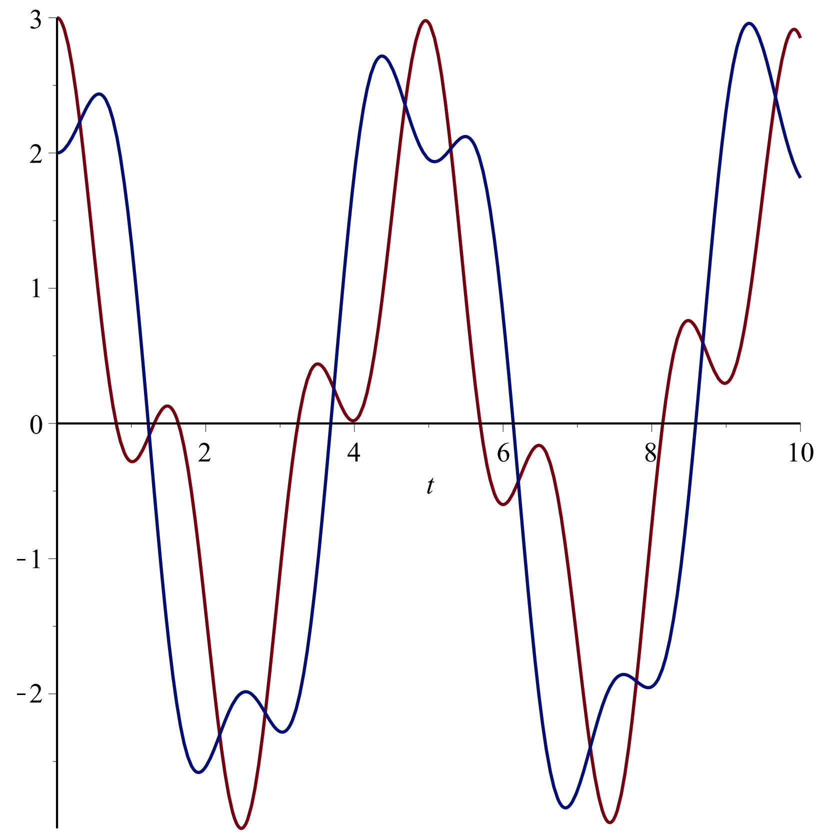
$$x_2(t)=\frac{17}{20}\left(\sqrt{10}+2\right)\sqrt{10}\cos\left(\sqrt{8+2\sqrt{10}}\;t\right)+\frac{17}{20}\sqrt{10}\,(-2 \tag{37}$$

$$+\sqrt{10}\,)\cos\left(\sqrt{8-2\sqrt{10}}\;t\right)-\frac{1}{16}\left(\sqrt{10}+2\right)(8$$

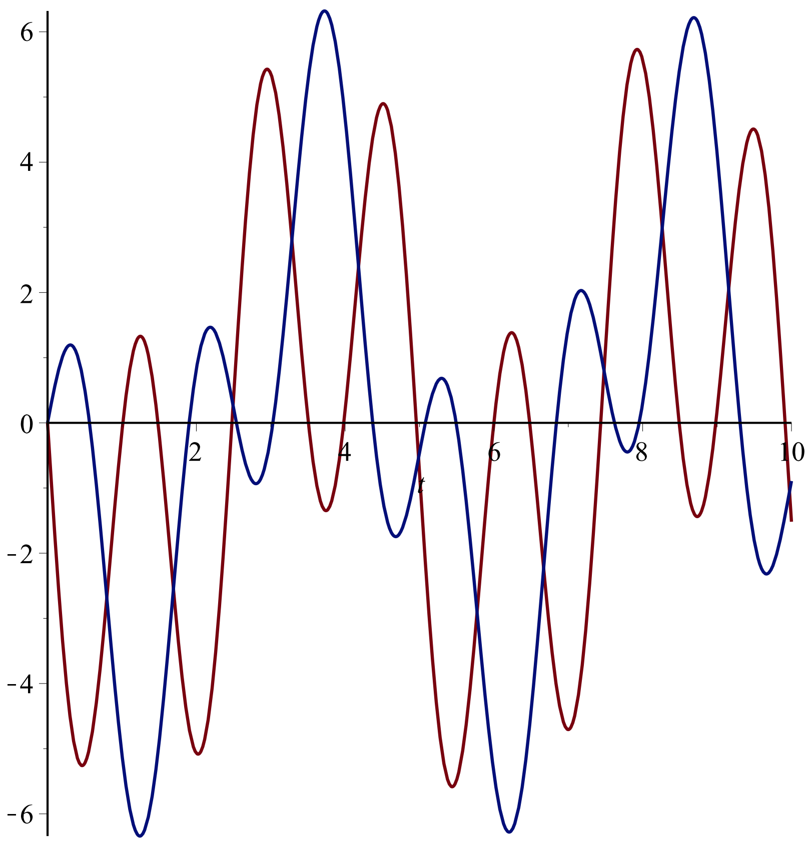
$$+2\sqrt{10}\,)\sqrt{10}\cos\left(\sqrt{8+2\sqrt{10}}\;t\right)-\frac{1}{16}\left(8-2\sqrt{10}\right)\sqrt{10}\,(-2$$

$$+\sqrt{10}\,)\cos\left(\sqrt{8-2\sqrt{10}}\;t\right)$$

```
> plot([rhs(SolPart[1]), rhs(SolPart[2])], t=0..10)
```




```
> plot([rhs(diff(SolPart[1], t)), rhs(diff(SolPart[2], t))], t=0..10)
```


6
4
2
0
-2
-4
-6
2
4
6
8
10
t