TRANSFORMADA DE LAPLACE.

- PROBLEMAS EDOL CON C.I.
- FUNCIONES SECCIONALMENTE CONTINUAS

$u(t-a)$ ESCALON

$r(t-a)$ RAMPA

$\delta(t-a)$. DIRAC.

$\mathcal{L}\{f(t)\} = F(s)$ ES ÚNICA.

$f, t, F \in \mathbb{R}$ $t > 0$

$s \in \mathbb{C}$.

$L^{-1}\{F(s)\} = f(t)$ NO ES ÚNICA

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

$$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s)$$

$a, b \in \mathbb{R}$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - \left(\sum_{i=0}^{n} s^{n-1} f^{(i)}(0)\right)$$

$$L^{-1}\{F'(s)\} = -t\,f(t)$$

$$L^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

$$\mathcal{L}\left\{ \int_0^t f(z)\,dz \right\} = \frac{F(s)}{s}$$

$$\mathcal{L}^{-1}\left\{ \int_s^\infty F(\sigma)\,d\sigma \right\} = \frac{f(t)}{t}$$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$e^{at} f(t)$	$F(s-a)$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\operatorname{sen}(bt)$	$\frac{b}{s^2+b^2}$

$$\mathcal{L}\{t^3e^{3t}\} = \frac{3!}{(s-3)^4} \qquad \mathcal{L}\{te^{2t}\} = \frac{1}{(s-2)^2}$$

$$\mathcal{L}\{t^3\} = \frac{3!}{s^4} \qquad \mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\frac{d^2y(t)}{dt^2} - 5\frac{dy}{dt} = 4e^{2t} + 2te^{2t}$$

$$y(0) = -2 \quad y'(0) = 4$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 5\frac{dy}{dt}\right\} = \mathcal{L}\left\{4e^{2t} + 2te^{2t}\right\}$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 5\mathcal{L}\left\{\frac{dy}{dt}\right\} = 4\mathcal{L}\{e^{2t}\} + 2\mathcal{L}\{te^{2t}\}$$

$$\left[s^2Y(s) - s(-2) - (4)\right] - 5\left[sY(s) - (-2)\right] = 4\left[\frac{1}{s-2}\right] + 2\left[\frac{1}{(s-2)^2}\right]$$

$$\left[s^2 - 5s\right]Y(s) + 2s - 14 = \frac{4}{s-2} + \frac{2}{(s-2)^2}$$

$$(s^2 - 5s)Y(s) = \frac{4}{s-2} + \frac{2}{(s-2)^2} - 2s + 14$$

$$= \frac{4(s-2) + 2 + (-2s+14)(s-2)^2}{(s-2)^2}$$

$$= \frac{4s - 8 + 2 + (-2s+14)(s^2-4s+4)}{(s-2)^2}$$

$$= \frac{4s - 6 - 2s^3 + 8s^2 - 8s + 14s^2 - 56s + 56}{(s-2)^2}$$

$$(s^2 - 5s)Y(s) = \frac{-2s^3 + 22s^2 - 60s + 50}{(s-2)^2}$$

$$Y(s) = \frac{-2s^3 + 22s^2 - 60s + 50}{s(s-5)(s-2)^2}$$

$$= \frac{A}{s} + \frac{B}{s-5} + \frac{D}{(s-2)^2} + \frac{E}{(s-2)}$$

$$-2s^3 + 22s^2 - 60s + 50 = A(s-5)(s-2)^2 + B(s)(s-2)^2 + D(s)(s-5) + E(s)(s-5)(s-2)$$

$$= A(s-5)(s^2-4s+4) + B(s)(s^2-4s+4) + D(s^2-5s) + E(s)(s^2-7s+10)$$

$$= A(s^3 - 4s^2 + 4s - 5s^2 + 20s - 20) +$$

$$+ B(s^3 - 4s^2 + 4s) + D(s^2 - 5s) + E(s^3 - 7s^2 + 10s)$$

$$= s^3(A+B+E) + s^2(-9A - 4B + D - 7E) +$$

$$+ s(24A + 4B - 5D + 10E) +$$

$$+ (-20A)$$

$$A + B + E = -2$$

$$-9A - 4B + D - 7E = 22$$

$$24A + 4B - 5D + 10E = -60$$

$$-20A = 50$$

$$Y(s) = \frac{A}{s} + \frac{B}{s-5} + \frac{D}{(s-2)^2} + \frac{E}{(s-2)}$$

$$y(t) = A\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + B\mathcal{L}^{-1}\left\{\frac{1}{s-5}\right\} + D\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\} + E\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

$$\boxed{y(t) = A + Be^{5t} + Dte^{2t} + Ee^{2t}}$$

$\rightarrow y(t_f) = 225$

$y'(t_f) = 0$

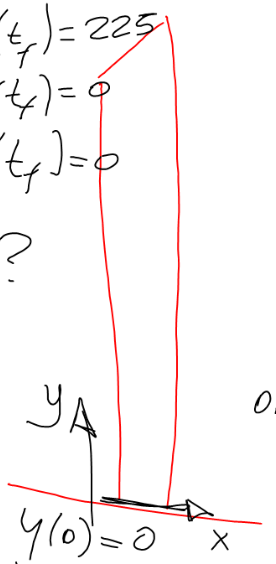
$y''(t_f) = 0$

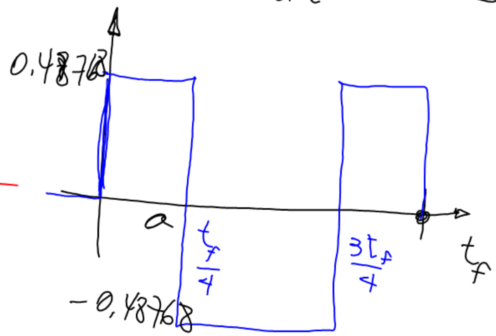
$t_f = ?$

TORRE MAYOR

$225 = 55$ pisos

$$\text{impulso} = \frac{d^3y}{dt^3} < 1.6\,\frac{ft}{s^3}$$





$y(0) = 0$

$y'(0) = 0$

$y''(0) = 0$

$$\frac{48768}{10000} \quad 1{,}6\,\frac{ft}{s^3} = 1{,}6 \times 0{,}3048 = 0{,}48768$$

$$\frac{d^3y}{dt^3} = 0{,}48\,u(t) - 2\times 0{,}48\,u(t-a) + 2\times 0{,}48\,u(t-3a) - 0.48\,u(t-4a)$$

$t_f = 4a$

$$\frac{dy}{dt} = 3y + 4x$$

$$\frac{dx}{dt} = -2y + 5x + e^{3t}$$

$$\frac{d}{dt}\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} y \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ e^{3t} \end{bmatrix}$$

$$\frac{d}{dt}\bar{x} = A\bar{x} + \bar{b}(t)$$

$$\bar{x} = e^{At}\bar{x}(0) + \int_0^t e^{A(t-z)} \bar{b}(z)\,dz.$$

$$e^{At} = B(0)I + B(1)A.$$

$$A = \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} \qquad \det(A - \lambda I) = 0$$

$$\begin{vmatrix} 3-\lambda & 4 \\ -2 & 5-\lambda \end{vmatrix} = 0 \qquad (3-\lambda)(5-\lambda) - (-2)(4) = 0$$

$$\lambda^2 - 8\lambda + 23 = 0$$

$$\lambda = \frac{8 \pm \sqrt{64 - 4(23)}}{2}$$

$$\lambda_1 = \frac{8 + \sqrt{28}\,i}{2} = 4 + \sqrt{7}\,i$$

$$\lambda_2 = \frac{8 - \sqrt{28}\,i}{2} = 4 - \sqrt{7}\,i$$

$$e^{\lambda t} = B_0(t) + \lambda_1 B_1(1)$$

$$e^{(4+\sqrt{7}i)t} = B_0(t) + (4+\sqrt{7}i)B_1(t)$$

$$e^{(4-\sqrt{7}i)t} = B_0(t) + (4-\sqrt{7}i)B_1(t)$$

$$e^{4t}e^{\sqrt{7}it} = [B_0(t) + 4B_1(t)] + \sqrt{7}B_1(t)\,i$$

$$e^{4t}e^{-\sqrt{7}it} = [B_0(t) + 4B_1(t)] - \sqrt{7}B_1(t)\,i$$

$$e^{4t}\left(\cos(\sqrt{7}t) + i\,\mathrm{sen}(\sqrt{7}t)\right) = [B_0(t) + 4B_1(t)] - \sqrt{7}B_1(t)\,i$$

$$B_0(t) + 4B_1(t) = e^{4t}\cos(\sqrt{7}t).$$

$$\sqrt{7}B_1(t) = e^{4t}\,\mathrm{sen}(\sqrt{7}t).$$

$$B_1(t) = \frac{1}{\sqrt{7}}e^{4t}\,\mathrm{sen}(\sqrt{7}t)$$

$$B_0(t) = e^{4t}\cos(\sqrt{7}t) - \frac{4}{\sqrt{7}}e^{4t}\,\mathrm{sen}(\sqrt{7}t).$$

$$e^{At} = e^{4t}\cos(\sqrt{7}t)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(e^{4t}\cos(\sqrt{7}t) - \frac{4}{\sqrt{7}}e^{4t}\,\mathrm{sen}(\sqrt{7}t)\right)\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$$