

$$\underbrace{(x^2 + y^2 + 1)}_M - \underbrace{2xy}_N \frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = (0) + 2y + (0)$$

$$\frac{\partial N}{\partial x} = -2y \quad \text{NO EXACT.$$

$$f(x) \quad f \frac{\partial M}{\partial y} + \cancel{\frac{\partial f}{\partial y}} M = f \frac{\partial N}{\partial x} + \cancel{N \frac{\partial f}{\partial x}} \quad \frac{df}{dx}$$

$$N \frac{df}{dx} = f \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\begin{aligned} \rightarrow \frac{df}{f} &= \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \right) dx \\ &= \left(\frac{2y - (-2y)}{-2xy} \right) dx \\ &= \frac{4y}{-2xy} \Rightarrow -\frac{2}{x} \end{aligned}$$

$$\int \frac{df}{f} = \int -\frac{2}{x} dx \Rightarrow -2 \int \frac{dx}{x} \Rightarrow -2 \ln x$$

$$\ln f = -2 \ln x$$

$$\ln f = \ln \left(\frac{1}{x^2} \right)$$

$$f(x) = \frac{1}{x^2}$$

$$(x^2 + y^2 + 1) - 2xy \frac{dy}{dx} = 0$$

$$f(x)(x^2 + y^2 + 1) - f(x)2xy \frac{dy}{dx} = 0$$

$$\frac{1}{x^2}(x^2 + y^2 + 1) - \frac{2y}{x} \frac{dy}{dx} = 0$$

$$\underbrace{\left(1 + \frac{y^2}{x^2} + \frac{1}{x^2}\right)}_{MM} - \underbrace{\frac{2y}{x} \frac{dy}{dx}}_{NN} = 0$$

$$\frac{\partial MM}{\partial y} = \frac{2y}{x^2} \quad \frac{\partial NN}{\partial x} = +\frac{2y}{x^2}$$

$$\int MM dx = \int dx + y^2 \int \frac{dx}{x^2} + \int \frac{dx}{x^2}$$

$$= x + y^2 \left(\frac{-1}{x} \right) + \left(\frac{-1}{x} \right)$$

$$= x - \left(\frac{y^2 + 1}{x} \right)$$

$$S_6 = \left(x - \left(\frac{y^2 + 1}{x} \right) \right) + \int \left(\left(-\frac{2y}{x} - \left(-\frac{2y}{x} \right) \right) dy \right) = C_1$$

$$x - \frac{y^2}{x} - \frac{1}{x} = C_1$$

$$\boxed{x^2 - y^2 - 1 = C_1 x}$$

$$x + y \frac{dy}{dx} + x \left(x \frac{dy}{dx} - y \right) = 0$$

$$\underbrace{(x - xy)}_M + \underbrace{(y + x^2)}_N \frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = (0) - x \quad \frac{\partial N}{\partial x} = (0) + 2x$$

NO ES EXACTA

$f(x) \Rightarrow$

$$f \frac{\partial M}{\partial y} + (0) = f \frac{\partial N}{\partial x} + N \frac{df}{dx}$$

$f(x, y)$

$$N \frac{df}{dx} = f \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\frac{df}{f} = \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \right) dx$$

$$\frac{df}{f} = \left(\frac{-x - (2x)}{y + x^2} \right) dx$$

$$= \frac{-3x}{y + x^2} dx$$

$f(x)$ NO ES
FACTOR
INTEGRANTE.

$$(x - xy) + (y + x^2) \frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = -x \quad \frac{\partial N}{\partial x} = 2x$$

$g(y)$

$$g \frac{\partial M}{\partial y} + M \frac{dg}{dy} = g \frac{\partial N}{\partial x} + (0)$$

$$M \frac{dg}{dy} = g \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$$

$$\frac{dg}{g} = \left(\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} \right) dy$$

$$\frac{dg}{g} = \left(\frac{3x}{x - xy} \right) dy$$

$$\frac{dg}{g} = \left(\frac{3}{1-y} \right) dy$$

$$\int \frac{dg}{g} = -3 \int \frac{dy}{(y-1)}$$

$$\ln g = -3 \ln(y-1)$$

$$\ln g = \ln \left(\frac{1}{(y-1)^3} \right)$$

$$g = \frac{1}{(y-1)^3}$$

EDO $L(1)$ cv NA.

$$\frac{dy}{dx} + p(x)y = q(x)$$

Regla de Oro. \rightarrow LINEALES

$$y_{g/n-H} = y_{g/H_A} + y_{p/q(x)}$$

Homogénea asociada

$$\frac{dy}{dx} + p(x)y = 0$$

$$\frac{dy}{dx} = -p(x)y \quad \text{Variables Separables}$$

$$\int \frac{dy}{y} = -\int p(x) dx$$

$$\mathcal{L}y = -\int p(x) dx + C_1$$

$$\mathcal{L}y - C_1 = -\int p(x) dx$$

$$\mathcal{L}y - \mathcal{L}C = -\int p(x) dx$$

$$\mathcal{L}\left(\frac{y}{C}\right) = -\int p(x) dx$$

$$\frac{y}{C} = e^{-\int p(x) dx}$$

$$\underline{y = C_1 e^{-\int p(x) dx}}$$

Soln.

$$\frac{dy}{dx} + p(x)y = 0$$

$$M + N \frac{dy}{dx} = 0$$

$$p(x)y + \frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = p(x) \quad \frac{\partial N}{\partial x} = 0$$

$f(x)$

$$f \frac{\partial M}{\partial y} + (0) = f \frac{\partial N}{\partial x} + N \frac{df}{dx}$$

$$\frac{df}{dx} = f \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\frac{df}{f} = (p(x) - (0)) dx$$

$$\int \frac{df}{f} = \int p(x) dx$$

$$\ln f = \int p(x) dx$$

FACTOR
INTEGRANTS. $f(x) = e^{\int p(x) dx}$

$$e^{\int p(x) dx} \frac{dy}{dx} + \left[e^{\int p(x) dx} \right] p(x) y = 0$$

$$\frac{\partial MM}{\partial y} = e^{\int p(x) dx} p(x)$$

$$\frac{\partial NN}{\partial x} = e^{\int p(x) dx} p(x) \quad \left. \vphantom{\frac{\partial NN}{\partial x}} \right] \text{EXACTA.}$$

$$\frac{d}{dx} \left(e^{\int p(x) dx} y \right) = 0$$

$$e^{\int p(x) dx} y = C,$$

$$y = C e^{-\int p(x) dx}$$

$$\frac{dy}{dx} + p(x)y = q(x)$$

$$e^{\int p(x) dx} \left(\frac{dy}{dx} + p(x)y \right) = e^{\int p(x) dx} q(x)$$

$$\frac{d}{dx} \left(e^{\int p(x) dx} y \right) = e^{\int p(x) dx} q(x)$$

$$d \left(e^{\int p(x) dx} y \right) = e^{\int p(x) dx} q(x) dx$$

$$e^{\int p(x) dx} y = \int e^{\int p(x) dx} q(x) dx + C_1$$

$$y = C_1 e^{-\int p(x) dx} + e^{-\int p(x) dx} \int e^{\int p(x) dx} q(x) dx.$$

$$\frac{dy}{dx} - \frac{y}{x} = x^2$$

$$p(x) = -\frac{1}{x}$$

$$q(x) = x^2$$

$$\int p(x) dx = -\int \frac{dx}{x} = -\ln x$$

$$-\int p(x) dx = \ln x$$

$$e^{\int p(x) dx}$$

$$= e^{-\ln x} \Rightarrow e^{\ln \frac{1}{x}} \Rightarrow \frac{1}{x}$$

$$e^{-\int p(x) dx}$$

$$= e^{\ln x} \Rightarrow x$$

$$y = C_1 x + x \int \frac{1}{x} x^2 dx$$

$$y = C_1 x + x \left[\int x dx \right]$$

$$y = C_1 x + x \left(\frac{x^2}{2} \right)$$

$$\underline{y = C_1 x + \frac{x^3}{2} \rightarrow \frac{dy}{dx} - \frac{y}{x} = x^2}$$