TEMA 2.- LA ECUACIÓN DIFERENCIAL ORDINARIA LINEAL DE ORDEN "n" COEFICIENTES CONSTANTES

$$\begin{cases} \text{HOMOGÉNEA} \\ \text{NO HOMOGÉNEA.} \end{cases}$$

EDO(n) L cc NH.

1. $$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = Q(x)$$

$$y_{g/NH} = c_1 y_1 + c_2 y_2 + \cdots + c_n y_n + y_{p/Q}.$$

$$W = \begin{bmatrix} y_1 & y_2 & y_3 & \cdots & y_n \\ y_1' & y_2' & y_3' & \cdots & y_n' \\ \vdots & & & & \\ y_1^{(n-1)} & y_2^{(n-1)} & y_3^{(n-1)} & \cdots & y_n^{(n-1)} \end{bmatrix}$$

$$|W| \neq 0.$$

$$\frac{dy}{dx} + p(x)\,y = q(x)$$

$$y(x) = c_1 e^{-\int p(x)dx} + e^{-\int p(x)dx} \int e^{\int p(x)dx} \cdot q(x) \cdot dx$$

$$\frac{dy}{dx} + a_1 y = q(x)$$

$$\therefore \quad y(x) = c_1 e^{-a_1 x} + e^{-a_1 x} \int e^{a_1 x} q(x)\,dx$$

$$y_{g/nh}(x) = \Big[\underbrace{c_1 + \int e^{a_1 x} q(x)\,dx}\Big] e^{-a_1 x}$$

$$y(x) = A(x)\, e^{-a_1 x}$$

$$\frac{dy}{dx} + a_1 y = 0 \qquad y_h(x) = c_1 e^{-a_1 x}$$

$$\boxed{y_{g/nh} = y_{g/h_A} + y_{p/q}.}$$

S6 $y = c_1 e^{3x} + 4x^2$

$$y_{g/nh} = \boxed{y_{g/h} + y_{p/q}.}$$

$$\frac{dy}{dx} + a_1 y = q(x)$$

$$\frac{dy}{dx} = 8x$$

$$\frac{dy}{dx} - 3y = q(x)$$

$$(8x) - 3(4x^2) = q$$

$$-12x^2 + 8x = q.$$

$$\boxed{\frac{dy}{dx} - 3y = -12x^2 + 8x}$$

$$y = c_1 e^{3x} + 4x^2$$

$$+ \quad \frac{dy}{dx} = +3c_1 e^{3x} + 8x$$

$$-3y = -3c_1 e^{3x} - 3(4x^2)$$

$$\frac{dy}{dx} - 3y = (0)e^{3x} - 12x^2 + 8x$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 8y = 0 \quad \text{EDO(2)LCCH.}$$

Hipótesis $\Rightarrow y = e^{mx}$

$$y = e^{mx}$$
$$\frac{dy}{dx} = me^{mx}$$
$$\frac{d^2y}{dx^2} = m^2e^{mx}$$

$$\left(m^2e^{mx} - 4me^{mx} - 8e^{mx}\right) = 0$$

E(A) $\quad (m^2 - 4m - 8)e^{mx} = 0$

$e^{mx} = 0$

INÚTIL

$$m^2 - 4m - 8 = 0$$

CARACTERÍSTICA.

$$m_{1,2} = \frac{4 \pm \sqrt{16 - 4(-8)}}{2(1)}$$
$$m_{1,2} = \frac{4 \pm \sqrt{48}}{2}$$
$$m_{1,2} = \frac{4 \pm 4\sqrt{3}}{2}$$
$$m_{1,2} = 2 \pm 2\sqrt{3}$$
$$m_1 = 2 + 2\sqrt{3} \qquad m_2 = 2 - 2\sqrt{3}$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 8y = 0$$

$$\boxed{y_{g/H} = C_1e^{(2+2\sqrt{3})x} + C_2e^{(2-2\sqrt{3})x}}$$

CASO I

$m_1 \neq m_2$

CASO II $m_1 = m_2 \in \mathbb{R}$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0$$

$$(m-a)^2 = 0 \qquad m_1 = m_2$$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} \qquad m_1 \neq m_2$$

$$m_1 = a \longrightarrow e^{m_1 x} \checkmark$$

$$m^2 + a_1 m + a_2 = 0 \qquad m_1 \neq m_2$$

$$(m_1)^2 + a_1 m_1 + a_2 = 0 \rightarrow 0 \equiv 0$$

$$(m_2)^2 + a_1 m_2 + a_2 = 0 \rightarrow 0 \equiv 0$$

$$(m-m_1)(m-m_2) = 0$$

$$\frac{d}{dm}\left((m-m_1)(m-m_2)\right) = 0$$

$$(m-m_1) + (m-m_2) = 0$$

$$m^2 + a_1 m + a_2 = 0 \qquad m_1 = m_2$$

$$(m-m_1)^2 = 0$$

$$\frac{d}{dm}(m-m_1)^2 = 0$$

$$2(m-m_1) = 0 \quad 0 \equiv 0$$

$$2m + a_1 = 0 \quad 0 \equiv 0$$

$$\frac{d}{dm}\begin{cases} e^{mx} \longrightarrow m_1 \longrightarrow e^{m_1 x} \\ xe^{mx} \longrightarrow m_1 \longrightarrow xe^{m_1 x} \end{cases}$$

$$y_{g/H} = c_1 e^{m_1 x} + c_2 x e^{m_1 x}$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

$$m^2 + 4m + 4 = 0$$

$$(m+2)^2 = 0 \qquad m_1 = m_2 = -2$$

$$\boxed{y = c_1 e^{-2x} + c_2 x e^{-2x}}$$