TEMA 3b) SISTEMAS DE ECUACIONES DIFERENCIALES LINEALES.

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + b_1(t)$$

$$\frac{dx_2}{dt} = \underbrace{a_{21}x_1 + a_{22}x_2}_{\text{Hom}} + \underbrace{b_2(t)}_{\text{N.H.}}$$

$$\text{I} \quad \frac{dx_1}{dt} = 2x_1 + 3x_2$$

$$\text{II} \quad \frac{dx_2}{dt} = x_1 + 4x_2$$

S(2)EDO(1)L H CC.

De II despejo x_1

$$x_1 = \frac{dx_2}{dt} - 4x_2$$

$$\frac{d}{dt}: \quad \frac{dx_1}{dt} = \frac{d^2x_2}{dt^2} - 4\frac{dx_2}{dt}$$

$$\frac{d^2x_2}{dt^2} - 4\frac{dx_2}{dt} = 2\left(\frac{dx_2}{dt} - 4x_2\right) + 3x_2$$

$$\frac{d^2x_2}{dt^2} - 6\frac{dx_2}{dt} + 5x_2 = 0 \quad \text{EDO(2)LCCH.}$$

$$(D^2 - 6D + 5)x_2 = 0$$

$$(D-1)(D-5)\,x_2 = 0$$

$$x_2 = c_1e^{t} + c_2e^{5t}$$

$$\frac{d}{dt}: \quad \frac{dx_2}{dt} = c_1e^{t} + 5c_2e^{5t}$$

$$x_1 = \left(c_1e^{t} + 5c_2e^{5t}\right) - 4\left(c_1e^{t} + c_2e^{5t}\right)$$

$$\boxed{\begin{aligned} x_1 &= -3c_1e^{t} + c_2e^{5t} \\ x_2 &= c_1e^{t} + c_2e^{5t} \end{aligned}}$$

SOL. GRAL SISTEMA.

$$\frac{d^3y}{dt^3} - 6\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 2y = 0$$

$$y = y_1$$

$$\frac{dy}{dt} = \frac{dy_1}{dt} = y_2$$

$$\frac{d^2y}{dt^2} = \frac{dy_2}{dt} = y_3$$

$$\frac{d^3y}{dt^3} = \frac{dy_3}{dt} = 2y_1 - 4y_2 + 6y_3$$

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + b_1(t)$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + b_2(t)$$

$$\bar{b} = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} \quad \bar{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad \frac{d}{dt}\bar{x} = \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix}$$

$$\frac{d}{dt}\bar{x} = A\bar{x}$$

$$\bar{x} = \left[e^{At}\right]\bar{x}(0)$$

$$\frac{d}{dt}\left[e^{At}\right] = A \times e^{At}$$

$$\left[e^{At}\right]_{t=0} = I.$$

$$\left[e^{At}\right]^{-1} = \left[e^{A(-t)}\right]$$

$$\left[e^{At}\right] \times \left[e^{A(-t)}\right] = I.$$