TEMA 4.- ECUACIÓN EN DERIVADAS PARCIALES

SG = ÚNICA

ED { ORDINARIAS → SOLUCIÓN $f(x)$

SG = NO SER ÚNICAS

EN DERIV. PARCIALES → SOLUCIÓN
$F(x,y)$
$F(x,y,z)$
$F(x,y,z,t)$

LINEALES

CUASILINEALES

NO LINEALES

$$\frac{\partial F}{\partial x} \quad \frac{\partial F}{\partial y} \quad \frac{\partial F}{\partial z}$$

$$\frac{\partial F}{\partial t}$$

$$\frac{\partial F}{\partial x} - \frac{\partial F}{\partial y} = 0$$

VAR. SEP. $F(x,y) \Rightarrow f(y+mx)$

$$\frac{\partial F}{\partial x} = m \cdot f'(y+mx)$$

$$\frac{\partial F}{\partial y} = (1)\, f'(y+mx)$$

$$[m \cdot f'] - [f'] = 0$$

$(m-1)\, f' = 0$ $\quad f' = 0 \quad F(x,y) = C_1$

$m - 1 = 0$ $\quad \boxed{f(y+mx) = C_1}$

$m = 1$ $\quad$ INÚTIL.

SG $\quad F(x,y) = f(y+x)$

SP $\quad F(x,y) = (y+x)^3$

$F(x,y) = \cos(y+x)$

$F(x,y) = e^{3(y+x)}$

$F(x,y) = (y+x)^2 + 5(y+x) + 6$

SERIE TRIGONOMÉTRICA FOURIER

CUERDA DE GUITARRA

$$\frac{\partial^2 F}{\partial y^2} - 5\frac{\partial^2 F}{\partial x \partial y} + 6\frac{\partial^2 F}{\partial x^2} = 0$$

$f(x+my)$ $\quad \frac{\partial f}{\partial x} = (1)f'$

$\frac{\partial^2 f}{\partial y^2} = m^2 f''$ $\quad \frac{\partial f}{\partial y} = mf'$

$\frac{\partial^2 f}{\partial x \partial y} = mf''$ $\quad m^2 f'' - 5mf'' + 6f'' = 0$

$\frac{\partial^2 f}{\partial x^2} = f''$ $\quad (m^2 - 5m + 6)f'' = 0$

INÚTIL $\left\{ \begin{array}{l} f'' = 0 \quad f'(x+my) = C_1 \\ f(x+my) = C_1(x+my) + C_2 \end{array} \right.$

$m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$ $\quad m_1 = 2$, $m_2 = 3$

SG $\quad F(x,y) = f_1(x+2y) + f_2(x+3y)$

SP $\Rightarrow F(x,y) = (x+2y)^3 + e^{5(x+3y)} \operatorname{sen}(x+3y)$

$$\frac{\partial^2 z}{\partial y^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial x^2} = 0$$

$f(x+my)$ $\quad \frac{\partial z}{\partial y} = f' \cdot m$

$\frac{\partial^2 z}{\partial y^2} = m^2 f''$ $\quad \frac{\partial z}{\partial x} = f' \cdot (1)$

$\frac{\partial^2 z}{\partial x \partial y} = m f''$

$\frac{\partial^2 z}{\partial x^2} = f''$

$$m^2 f'' + 2mf'' + f'' = 0$$

$$(m^2 + 2m + 1) = 0$$

$$(m+1)^2 = 0 \quad m_1 = -1$$

$$m_2 = -1$$

$$z(x,y) = f_1(x-y) + f_2(x-y) \cdot x$$

$$z(x,y) = f_1(x-y) + f_2(x-y) \cdot y$$