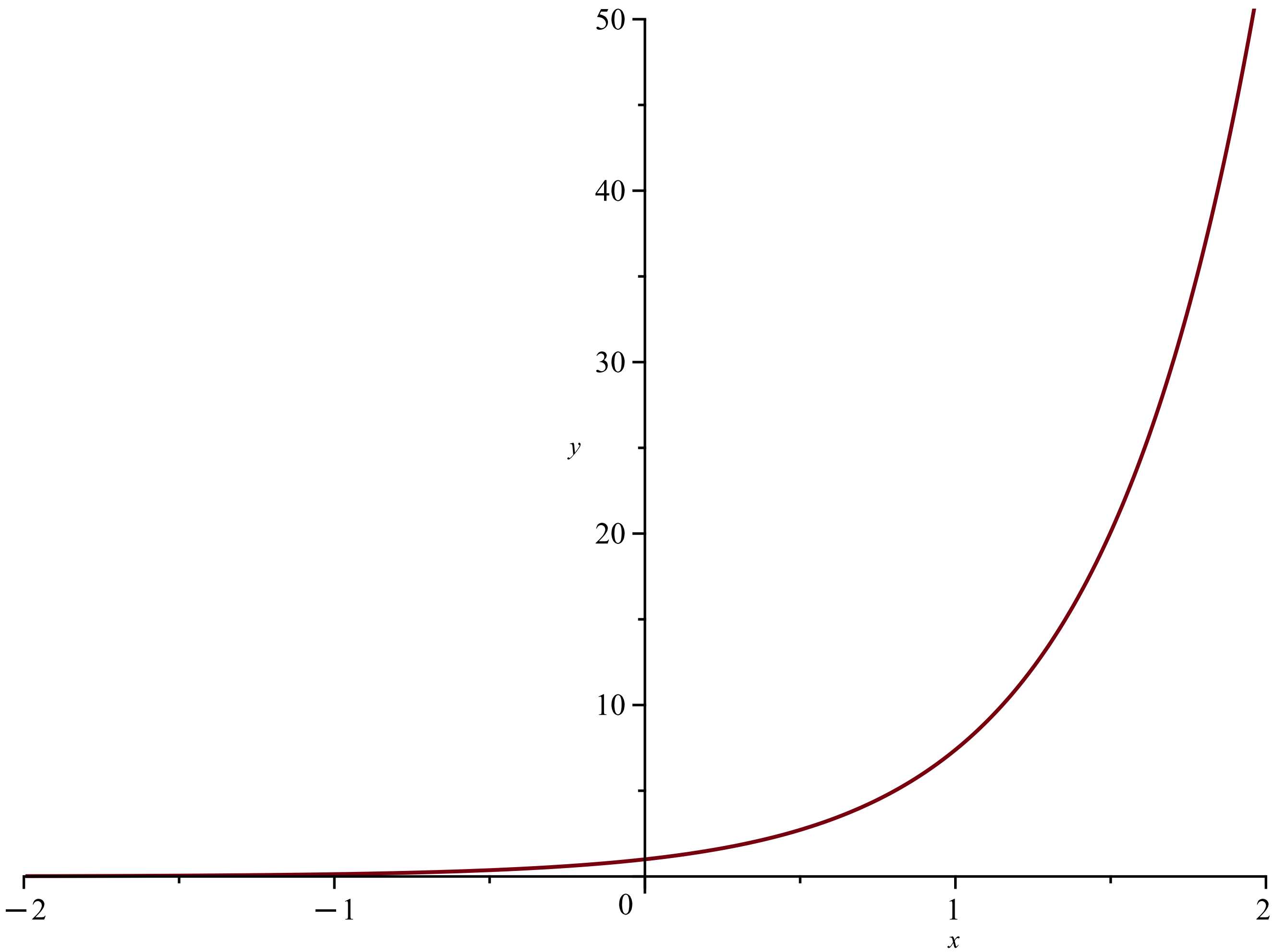
> $restart$

> $f := \exp(2 \cdot x)$

$$f := \mathrm{e}^{2x} \qquad (1)$$

> $plot(f, x = -2..2, y = -1..50)$



> $L := 2$

$$L := 2 \qquad (2)$$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L);\ evalf(\%)$

$$a_0 := -\frac{\mathrm{e}^{-4}}{4} + \frac{\mathrm{e}^{4}}{4}$$

$$13.64495860 \qquad (3)$$

> $a[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

$$a_n := \frac{4\,\mathrm{e}^{4}\,(-1)^n - 4\,\mathrm{e}^{-4}\,(-1)^n}{n^2\,\pi^2 + 16} \qquad (4)$$

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

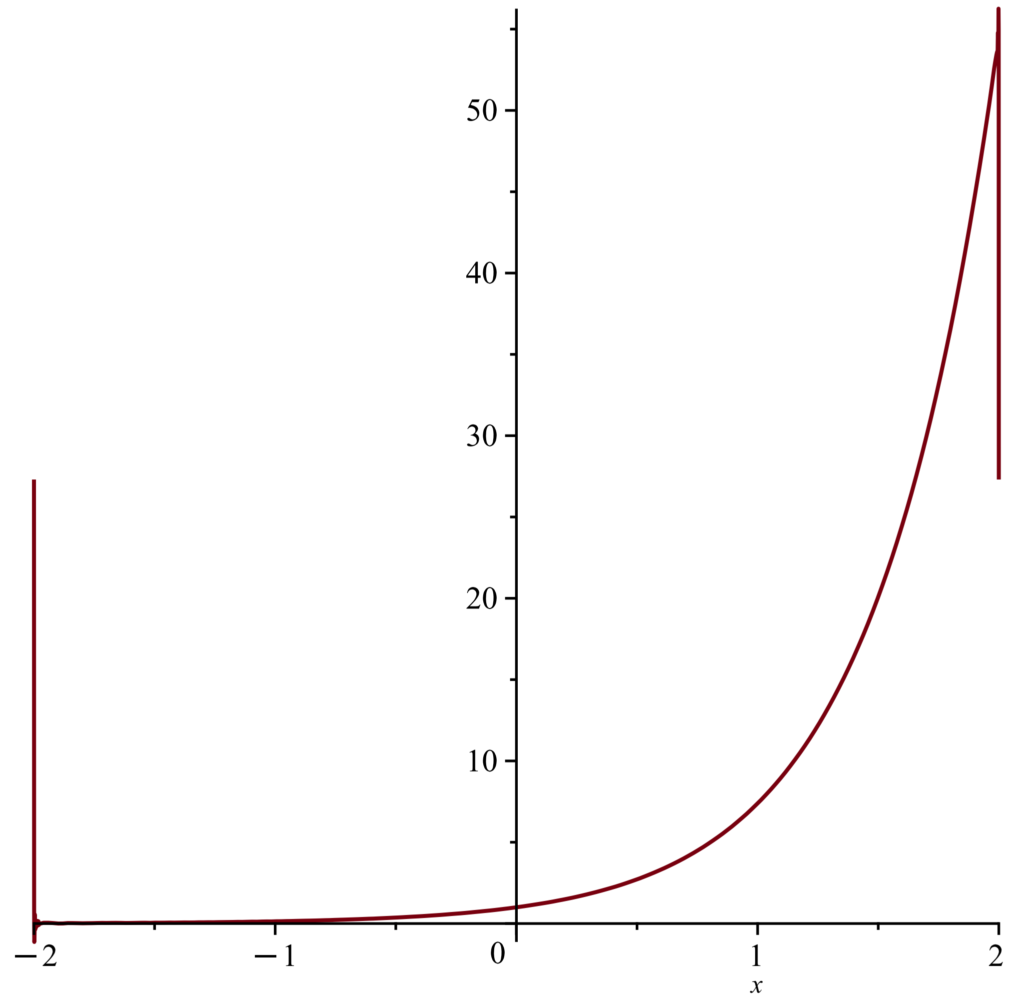
$$b_n := -\frac{e^4 (-1)^n \pi n - e^{-4} (-1)^n \pi n}{n^2 \pi^2 + 16} \tag{5}$$

> $STF\_f := \frac{a[0]}{2} + Sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1 .. \mathrm{infinity}\right)$

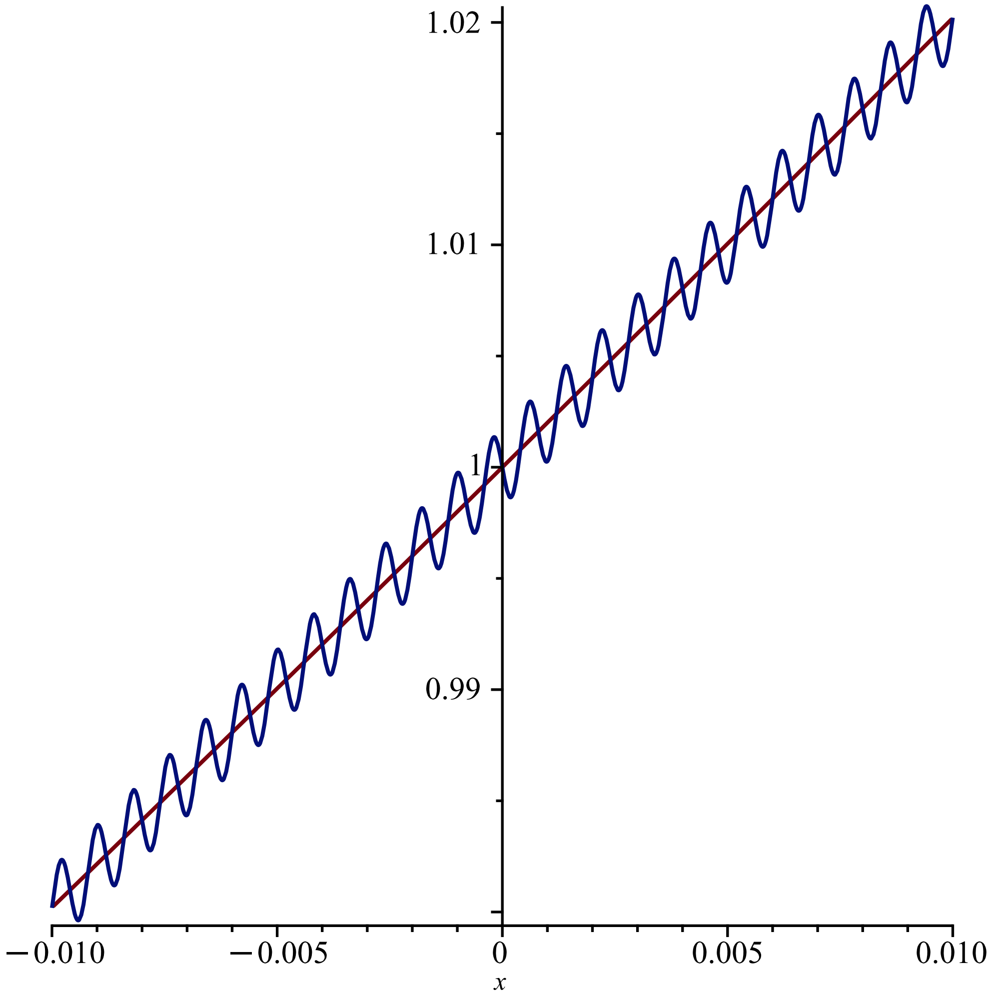
$$STF\_f := -\frac{e^{-4}}{8} + \frac{e^4}{8} + \sum_{n=1}^{\infty} \left(\frac{(4 e^4 (-1)^n - 4 e^{-4} (-1)^n) \cos\left(\frac{n \pi x}{2}\right)}{n^2 \pi^2 + 16} - \frac{(e^4 (-1)^n \pi n - e^{-4} (-1)^n \pi n) \sin\left(\frac{n \pi x}{2}\right)}{n^2 \pi^2 + 16} \right) \tag{6}$$

> $STF\_f5000 := \frac{a[0]}{2} + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1 .. 5000\right) :$

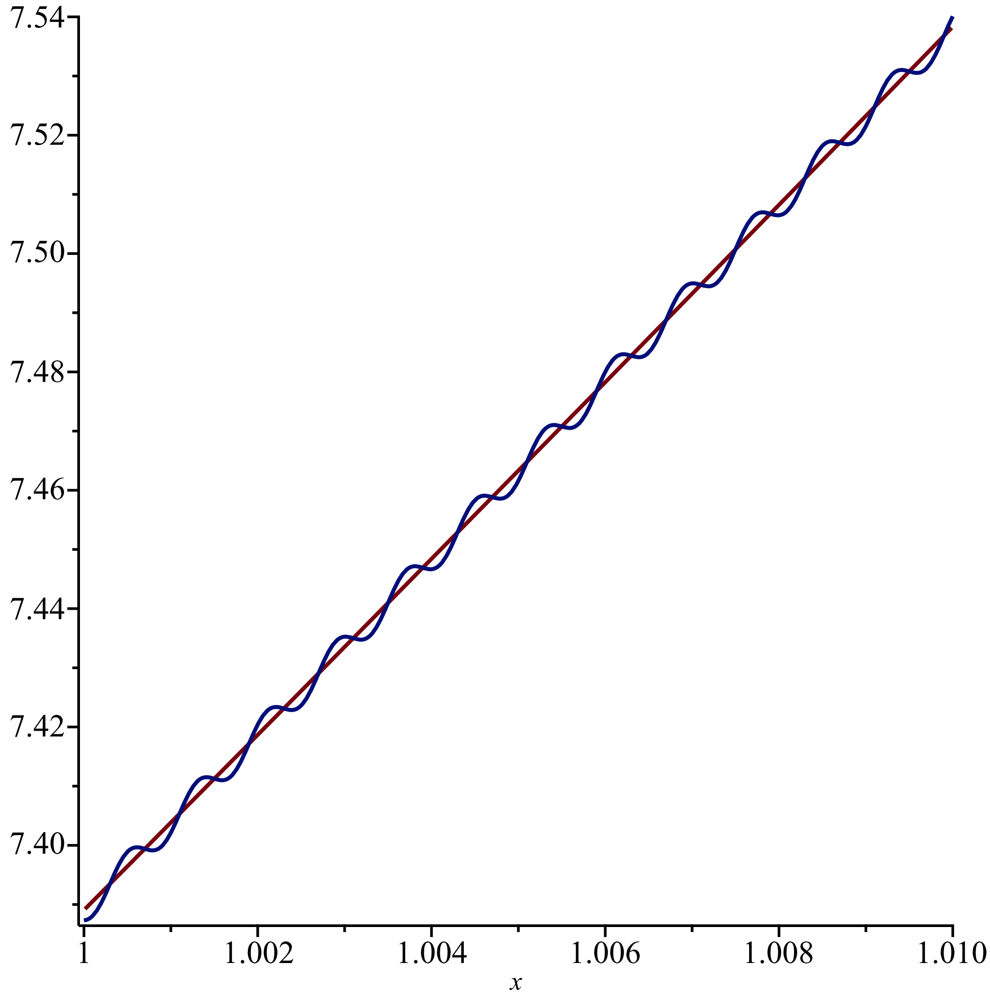
> $plot(STF\_f5000, x = -L .. L)$



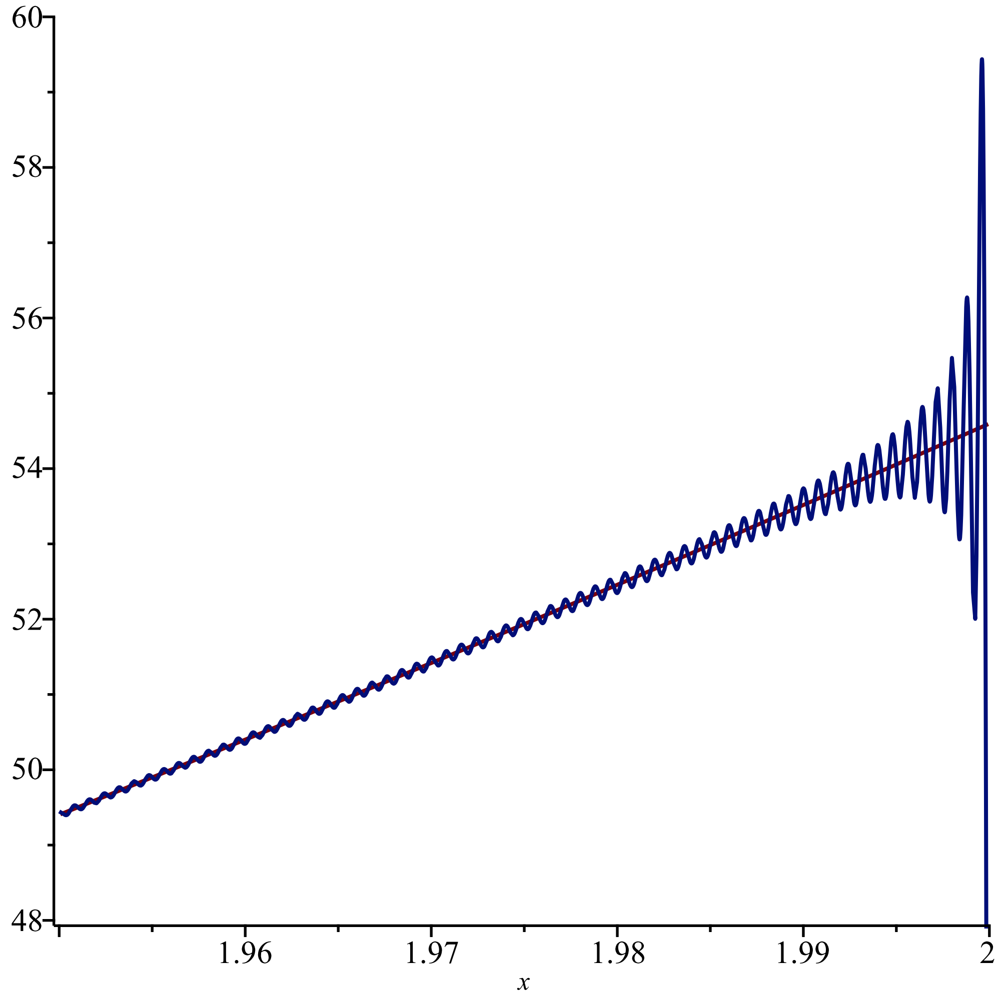
> $plot([f, STF\_f5000], x = -0.01 .. 0.01)$



> $plot([f, STF\_f5000], x = 1..1.01)$



> $plot([f, STF\_f5000], x = L - 0.05 .. L)$

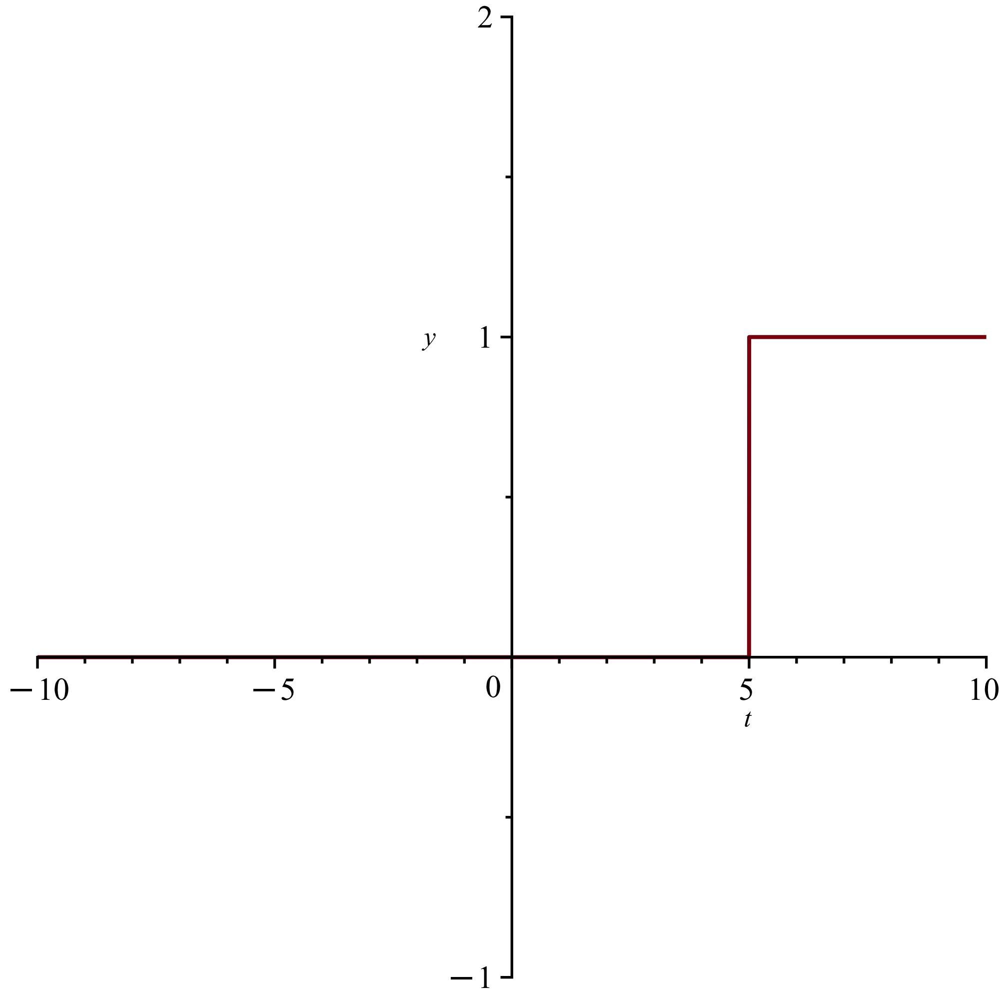


> *restart*

> $g := \text{Heaviside}(t-5)$

$$g := \text{Heaviside}(t-5) \tag{7}$$

> $plot(g, t = -10..10, y = -1..2)$



> $L := 10$

$$L := 10 \tag{8}$$

> $a[0] := \frac{1}{L} \cdot int(g, t = -L..L); evalf(\%)$

$$a_0 := \frac{1}{2}$$

$$0.5000000000 \tag{9}$$

> $a[n] := \frac{1}{L} \cdot int\left(g \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

$$a_n := \frac{-\sin\left(\frac{n\,\pi}{2}\right) + \sin(n\,\pi)}{n\,\pi} \tag{10}$$

> $b[n] := \frac{1}{L} \cdot int\left(g \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t = -L..L\right)$

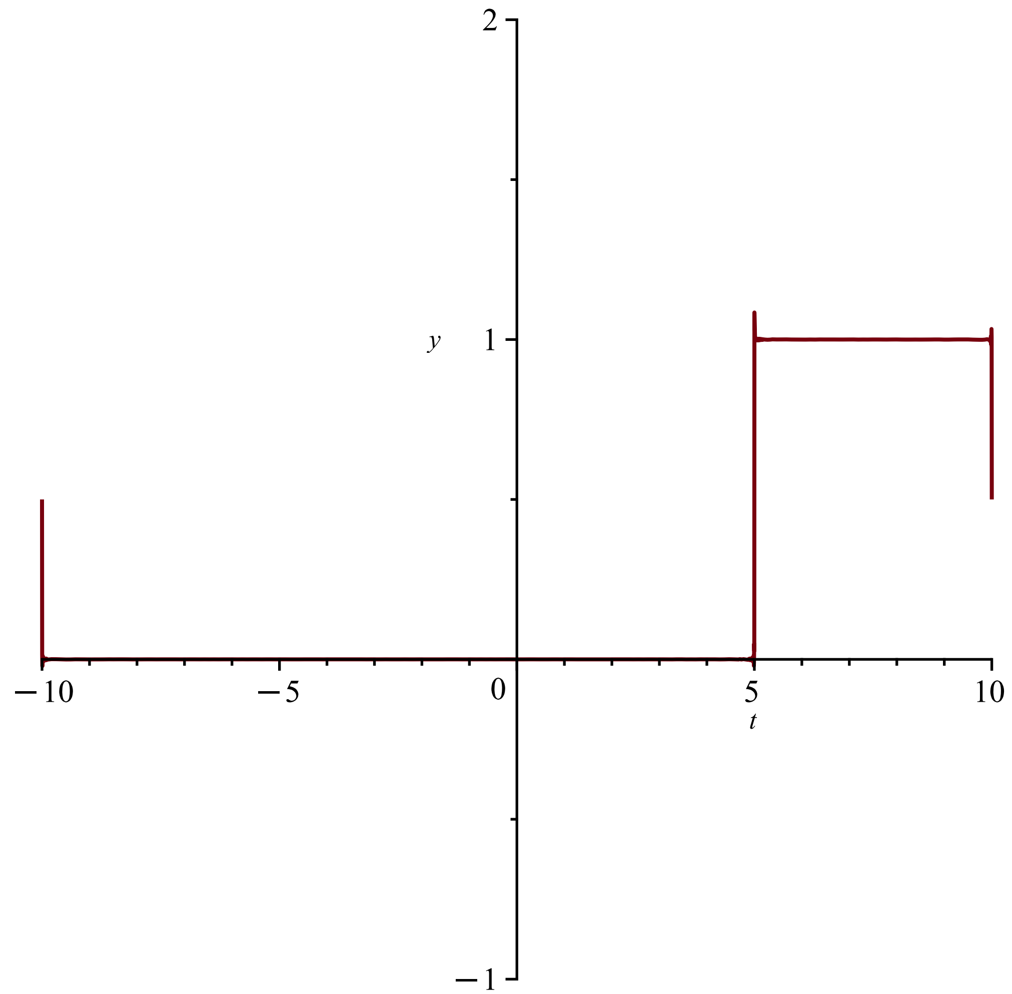
$$b_n := -\frac{-\cos\left(\frac{n\,\pi}{2}\right) + \cos(n\,\pi)}{n\,\pi} \tag{11}$$

> $STF\_g := \frac{a[0]}{2} + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot t\right) + b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot t\right), n = 1 ..\mathrm{infinity}\right)$

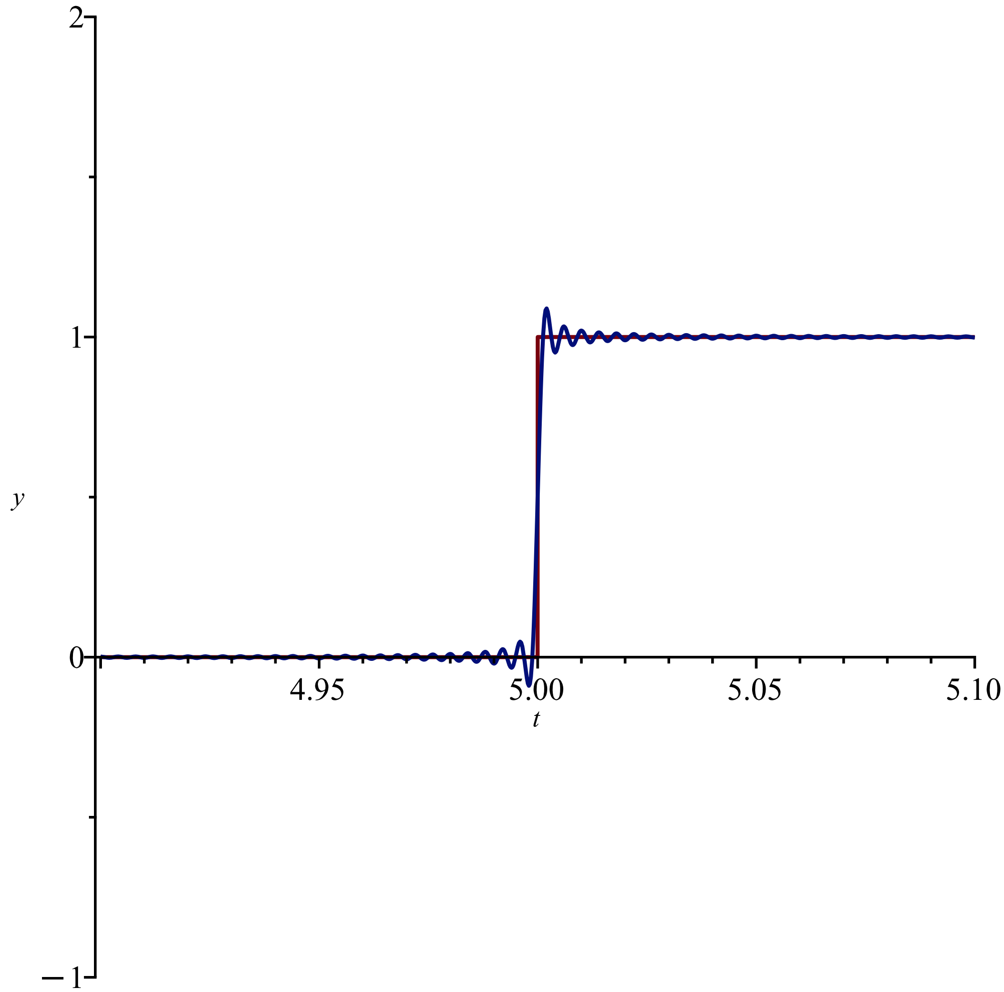
$$STF\_g := \frac{1}{4} + \sum_{n=1}^{\infty}\left(\frac{\left(-\sin\left(\frac{n\,p}{2}\right) + \sin(n\,p)\right)\cos\left(\frac{n\,p\,t}{10}\right)}{n\,p} - \frac{\left(-\cos\left(\frac{n\,p}{2}\right) + \cos(n\,p)\right)\sin\left(\frac{n\,p\,t}{10}\right)}{n\,p}\right) \tag{12}$$

> $STF\_g5000 := \frac{a[0]}{2} + Sum\left(a[n]\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot t\right) + b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot t\right), n = 1 ..5000\right)$:

> $plot(STF\_g5000, t = -L ..L, y = -1 ..2)$

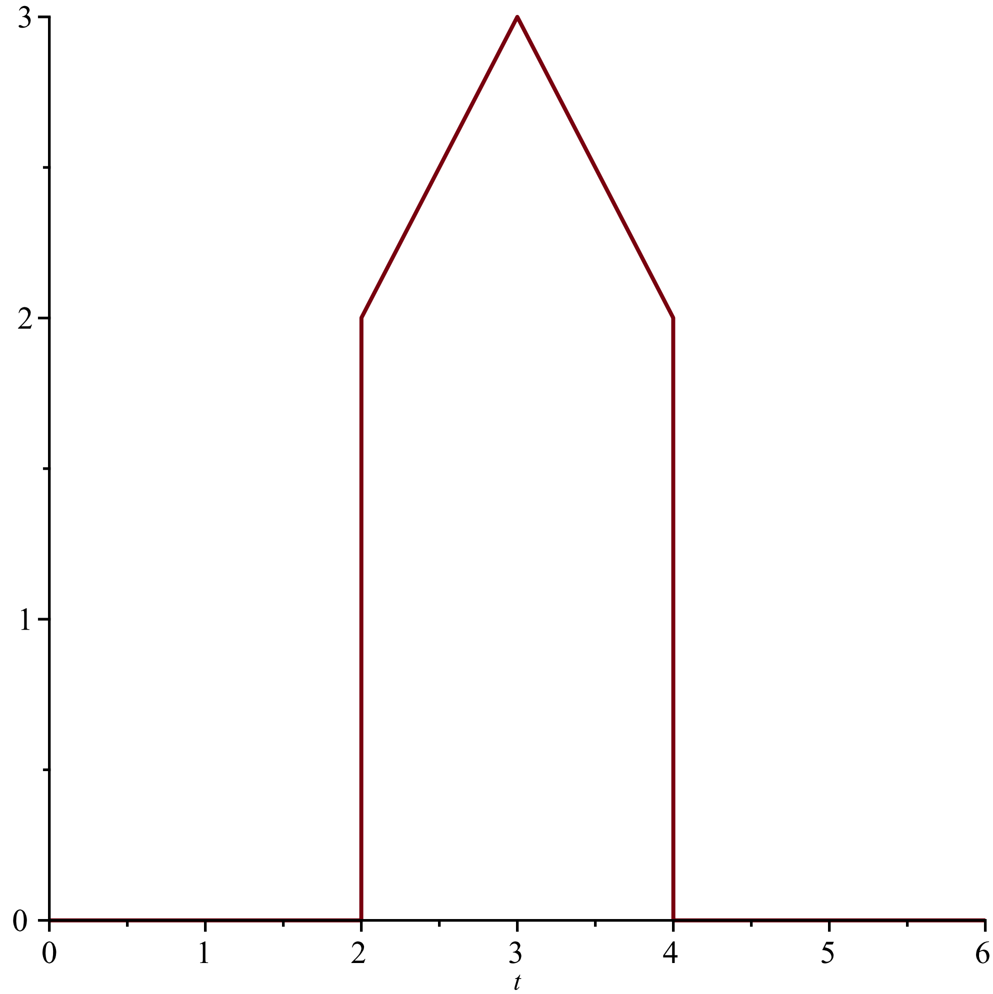


> $plot([g, STF\_g5000], t = 4.9 ..5.1, y = -1 ..2)$

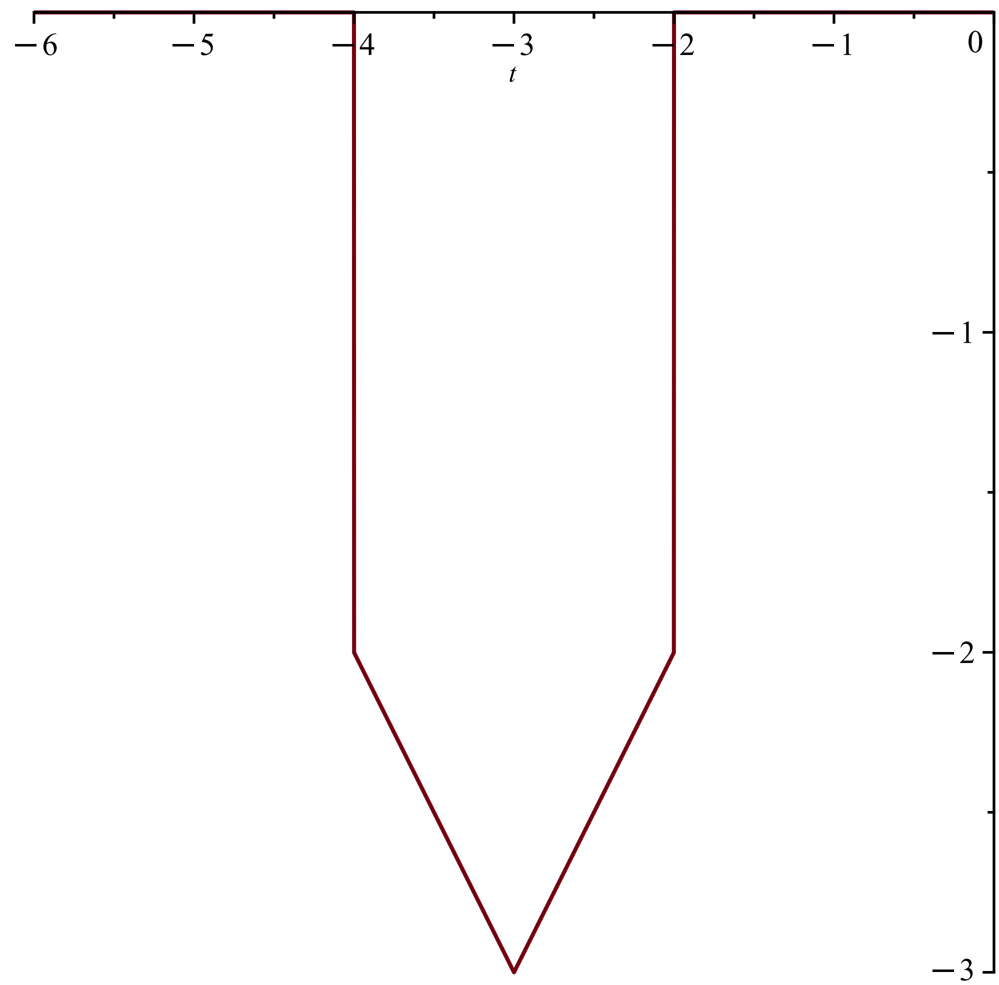


> *restart*

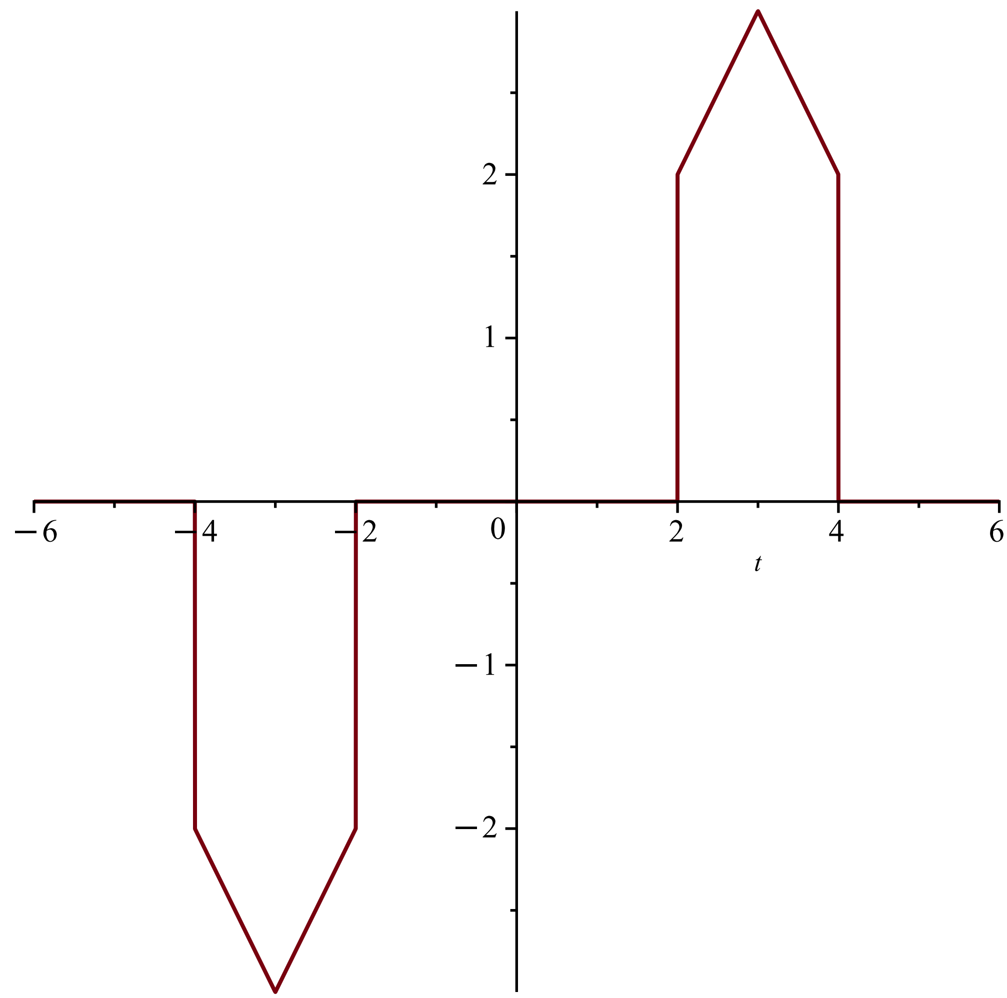
> $f := 2\cdot \text{Heaviside}(t-2) + (t-2)\cdot\text{Heaviside}(t-2) - 2\cdot(t-3)\cdot\text{Heaviside}(t-3) + (t-4)\cdot\text{Heaviside}(t-4) - 2\cdot\text{Heaviside}(t-4)$: $plot(f, t = 0..6)$



> $g := -2\cdot\mathrm{Heaviside}(t+4) - (t+4)\cdot\mathrm{Heaviside}(t+4) + 2\cdot(t+3)\cdot\mathrm{Heaviside}(t+3) - (t+2)\cdot\mathrm{Heaviside}(t+2) + 2\cdot\ \mathrm{Heaviside}(t+2) : plot(g, t = -6\,..0)$



> $plot([g+f], t=-6..6)$



> $L := 6$

$$L := 6 \tag{13}$$

> $a[0] := \frac{1}{L} \cdot int((f+g), t=-L..L)$

$$a_0 := 0 \tag{14}$$

> $a[n] := simplify\left(\frac{1}{L} \cdot int\left((f+g)\cdot\cos\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot t\right), t=-L..L\right)\right)$

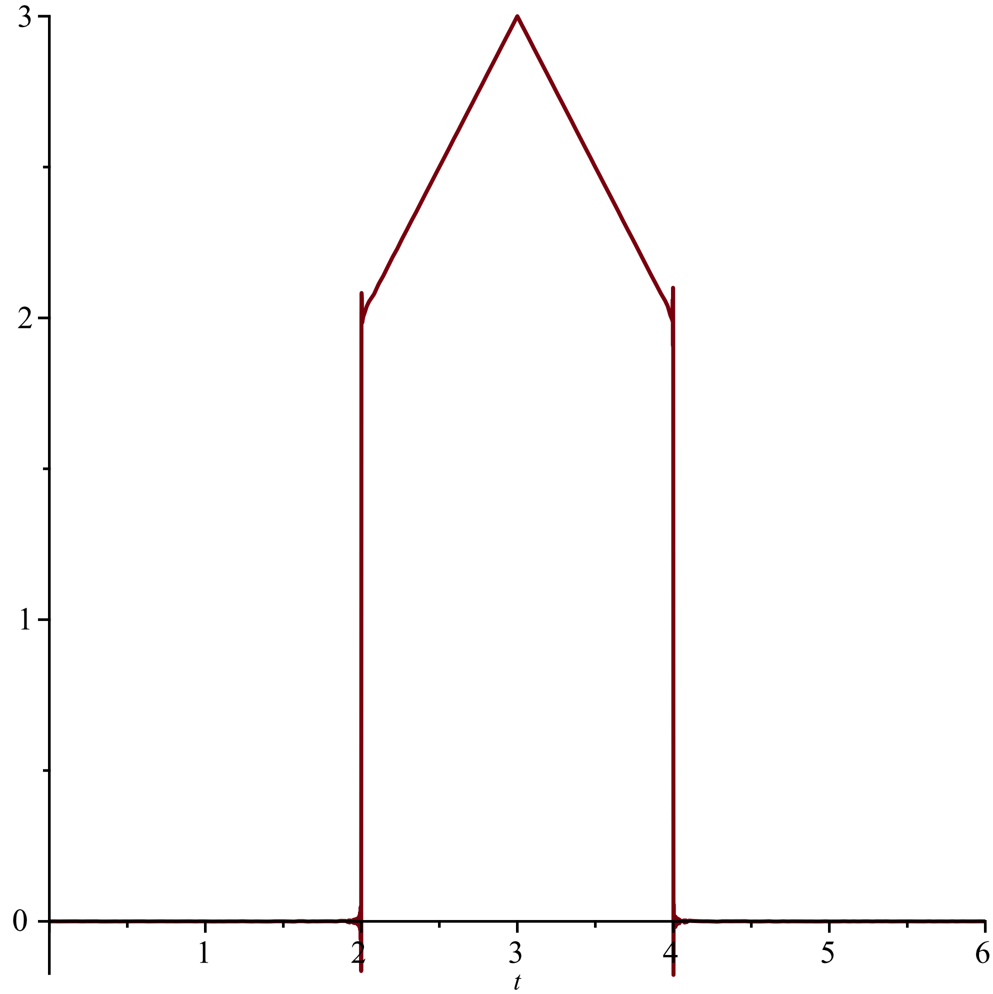
$$a_n := 0 \tag{15}$$

> $b[n] := simplify\left(\frac{1}{L} \cdot int\left((f+g)\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot t\right), t=-L..L\right)\right)$

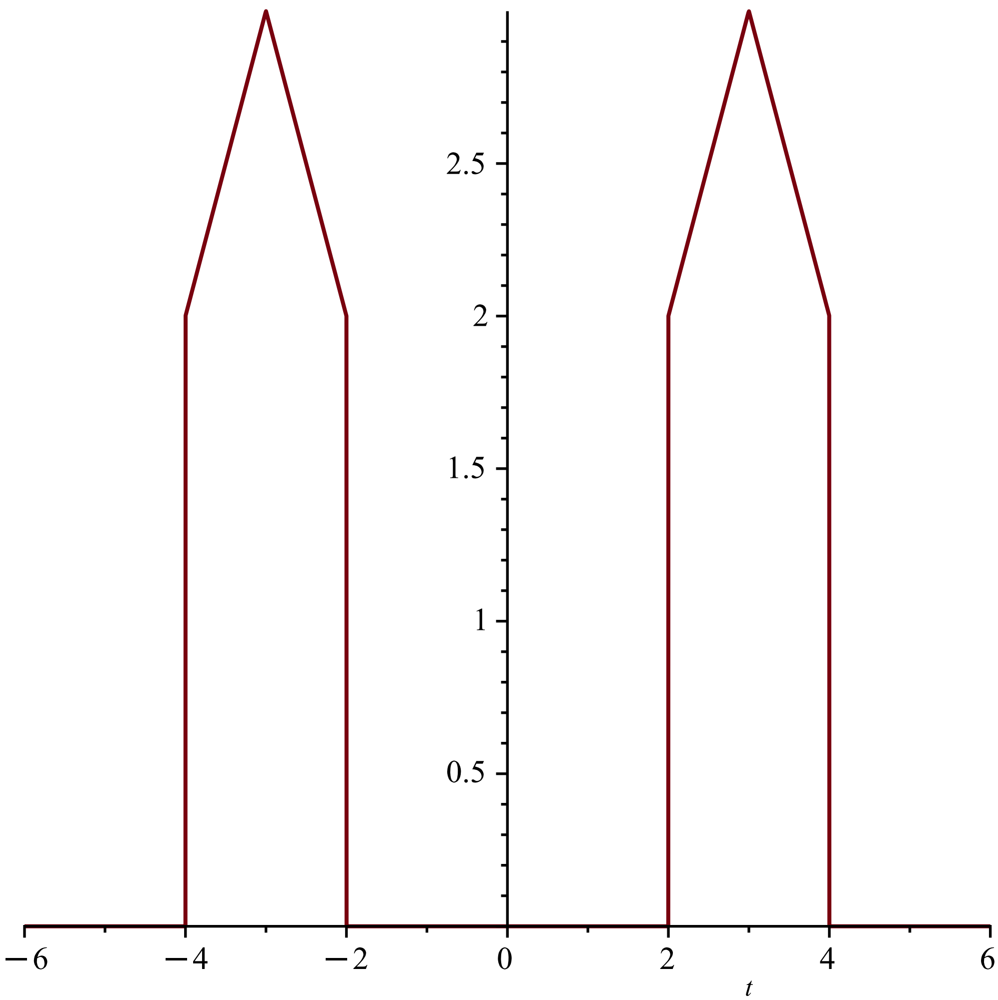
$$b_n := \frac{1}{n^2 \pi^2}\left(-4\, n\, \pi \cos\left(\frac{2\, n\, \pi}{3}\right) + 4\, n\, \pi \cos\left(\frac{n\, \pi}{3}\right) + 24 \sin\left(\frac{n\, \pi}{2}\right) - 12 \sin\left(\frac{2\, n\, \pi}{3}\right) - 12 \sin\left(\frac{n\, \pi}{3}\right)\right) \tag{16}$$

> $STF\_fg5000 := sum\left(b[n]\cdot\sin\left(\frac{n\cdot \mathrm{Pi}}{L}\cdot t\right), n=1..5000\right):$

> $plot(STF\_fg5000, t=0..L)$



> $h := 2\cdot \mathrm{Heaviside}(t+4) + (t+4)\cdot \mathrm{Heaviside}(t+4) - 2\cdot (t+3)\cdot \mathrm{Heaviside}(t+3) + (t+2)$
$\cdot \mathrm{Heaviside}(t+2) - 2\cdot\ \mathrm{Heaviside}(t+2) : plot((h+f), t=-6..6)$



> $aa[0] := \frac{1}{L} \cdot int((f+h), t=-L..L)$

$$aa_0 := \frac{5}{3} \tag{17}$$

> $aa[n] := simplify\left(\frac{1}{L} \cdot int\left((f+h) \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t=-L..L\right)\right)$

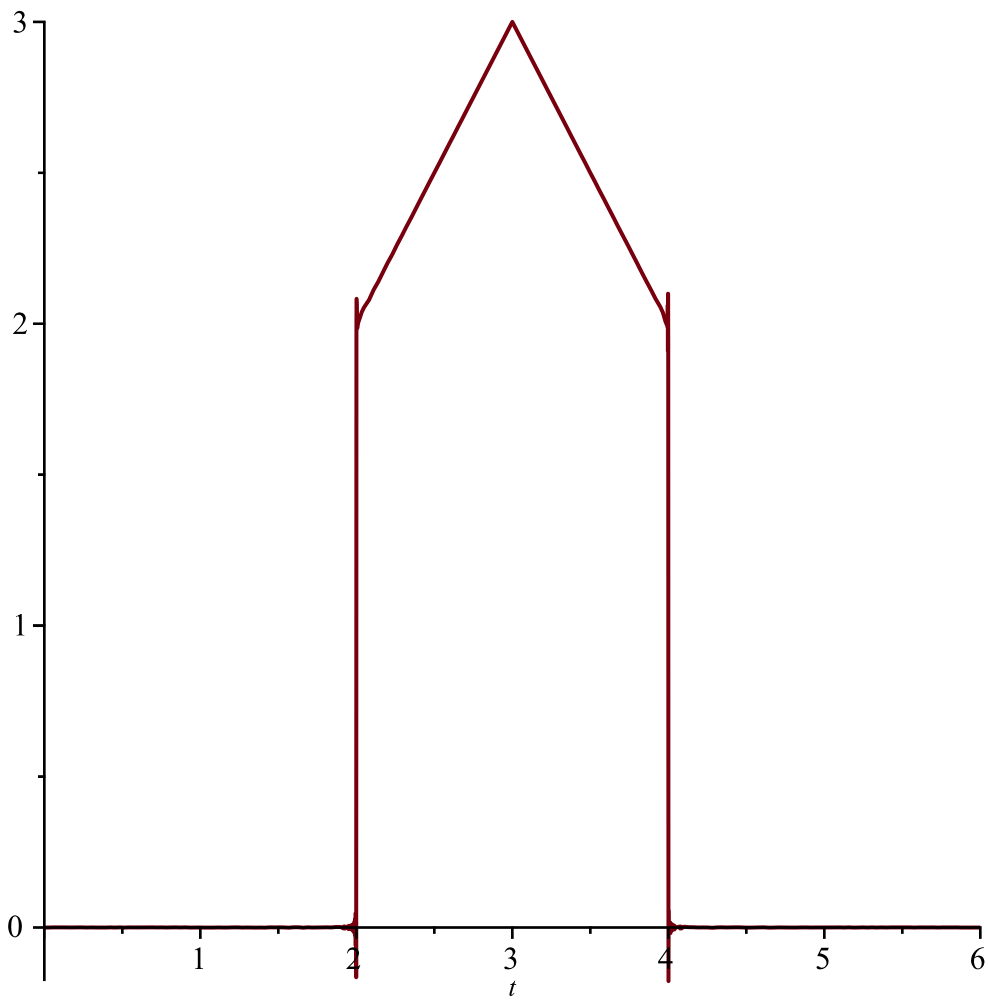
$$aa_n := \frac{32 \cos\left(\frac{n\pi}{6}\right) \left(\cos\left(\frac{n\pi}{6}\right)^2 - \frac{3}{4}\right) \left(\sin\left(\frac{n\pi}{6}\right) \pi n - 3 \cos\left(\frac{n\pi}{6}\right) + 3\right)}{n^2 \pi^2} \tag{18}$$

> $bb[n] := simplify\left(\frac{1}{L} \cdot int\left((f+h) \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), t=-L..L\right)\right)$

$$bb_n := 0 \tag{19}$$

> $STF\_fh5000 := \frac{aa[0]}{2} + sum\left(aa[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot t\right), n=1..5000\right) :$

> $plot(STF\_fh5000, t=0..L)$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