TEMA 3.- SISTEMAS Y TRANSFORMADA LAPLACE.

$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3e^{2x}$ $\quad y(0)=3,\ y'(0)=5$

$\mathcal{L}\{\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y\} = 3\mathcal{L}\{e^{2x}\}$

$\mathcal{L}\{\frac{d^2y}{dx^2}\} - 5\mathcal{L}\{\frac{dy}{dx}\} + 6\mathcal{L}\{y\} = \frac{3}{s-2}$

$\left(s^2\mathcal{L}\{y\} - s\cdot(3) - (5)\right) - 5\left(s\mathcal{L}\{y\} - (3)\right) + 6\mathcal{L}\{y\} = \frac{3}{s-2}$

$(s^2-5s+6)\mathcal{L}\{y\} - 3s + 10 = \frac{3}{s-2}$

$(s-2)(s-3)\mathcal{L}\{y\} = \frac{3}{s-2} + 3s - 10$

$= \frac{3 + (s-2)(3s-10)}{(s-2)}$

$\mathcal{L}\{y\} = \frac{3s^2-16s+23}{(s-2)^2(s-3)}$

$\frac{3s^2-16s+23}{(s-2)^2(s-3)} = \frac{A}{(s-2)^2} + \frac{B}{s-2} + \frac{D}{s-3}$

$3s^2-16s+23 = A(s-3) + B(s-2)(s-3) + D(s-2)^2$

$= A(s-3) + B(s^2-5s+6) + D(s^2-4s+4)$

$3s^2-16s+23 = (B+D)s^2 + (A-5B-4D)s + (-3A+6B+4D)$

$B+D=3$

$A-5B-4D=-16$

$-3A+6B+4D=23$ ⊕

$3A-15B-12D=-48$

$-9B-8D=-25$

$9B+8D=25$

$-8B-8D=-24$

$B=1$

$D=3-B$

$D=2$

$A=-16+5(1)+4(2)$

$A=-16+5+8$

$A=-3$

$\mathcal{L}\{y\} = \frac{-3}{(s-2)^2} + \frac{1}{(s-2)} + \frac{2}{(s-3)}$

$y = -3xe^{2x} + e^{2x} + 2e^{3x}$

$$G = \frac{e^{-4s} + s - 3}{s^2 - 6s - 7}$$

$$\mathcal{L}\{u(t-4)\} = \frac{e^{-4s}}{s}$$

$$\mathcal{L}\{e^{2t}u(t-4)\} = \frac{e^{-4s}}{s-2}$$

$$G = \frac{e^{-4s}}{(s-7)(s+1)} + \frac{s-3}{(s-7)(s+1)}$$

$$\frac{1}{(s-7)(s+1)} = \frac{A}{s-7} + \frac{B}{s+1}$$

$A + B = 0$

$A - 7B = 1$

$A = -B$

$-B - 7B = 1 \quad -8B = 1 \quad B = -\frac{1}{8} \quad A = \frac{1}{8}$

$$1 = A(s+1) + B(s-7)$$

$$1 = (A+B)s + (A-7B)$$

$$G_1 = \frac{-e^{-4s}}{8(s-7)} + \frac{e^{-4s}}{8(s+1)}$$

$$g_1 = \frac{-1}{8}e^{7t}u(t-4) + \frac{1}{8}e^{-t}u(t-4)$$

$$\frac{dy_1(x)}{dx} = 2y_1(x) + y_2(x) + e^{2x}$$

$$\frac{dy_2(x)}{dx} = y_1(x) - 2y_2(x) + 3x$$

$$y_2(x) = \frac{dy_1(x)}{dx} - 2y_1(x) - e^{2x}$$

$$\frac{dy_2}{dx} = \frac{d^2y_1(x)}{dx^2} - 2\frac{dy_1(x)}{dx} - 2e^{2x}$$

$$\left[\frac{d^2y_1(x)}{dx^2} - 2\frac{dy_1(x)}{dx} - 2e^{2x}\right] = y_1(x) - 2\left(\frac{dy_1(x)}{dx} - 2y_1 - e^{2x} + 3x\right.$$

$$\frac{d^2y_1(x)}{dx^2} - 5y_1 = 4e^{2x} + 3x$$

$$m^2 - 5 = 0 \rightarrow m_1 = +\sqrt{5} \quad -\sqrt{5} = m_2$$

$$y_1(x) = c_1e^{\sqrt{5}x} + c_2e^{-\sqrt{5}x}$$