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> restart
> Ecua := x·y'' + (1 - 2 x)·y' + (x - 1)·y = x·exp(x)
      Ecua := x ⎛  $\frac{d^2}{dx^2} y(x)$  ⎞ + (1 - 2 x) ⎛  $\frac{d}{dx} y(x)$  ⎞ + (x - 1) y(x) = x ex
(1)
=
> yy[1] := exp(x); yy[2] := exp(x)·log(x)
      yy1 := ex
      yy2 := ex ln(x)
(2)
=
> EcuaHom := lhs(Ecua) = 0
      EcuaHom := x ⎛  $\frac{d^2}{dx^2} y(x)$  ⎞ + (1 - 2 x) ⎛  $\frac{d}{dx} y(x)$  ⎞ + (x - 1) y(x) = 0
(3)
=
Respuesta
> SolGralHom := y(x) = _C1·yy[1] + _C2·yy[2]
      SolGralHom := y(x) = _C1 ex + _C2 ex ln(x)
(4)
=
> SolNoHom := y(x) = A·yy[1] + B·yy[2]
      SolNoHom := y(x) = A ex + B ex ln(x)
(5)
=
> ComprobarUno := simplify(eval(subs(y(x) = rhs(SolGralHom), EcuaHom)))
      ComprobarUno := 0 = 0
(6)
=
> EcuaHomDos := expand( $\frac{EcuaHom}{x}$ )
      EcuaHomDos :=  $\frac{d^2}{dx^2} y(x) - 2 \frac{d}{dx} y(x) + y(x) + \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{x} = 0$ 
(7)
=
> ComprobarDos := simplify(eval(subs(y(x) = rhs(SolGralHom), EcuaHomDos)))
      ComprobarDos := 0 = 0
(8)
=
> Ecua
      x ⎛  $\frac{d^2}{dx^2} y(x)$  ⎞ + (1 - 2 x) ⎛  $\frac{d}{dx} y(x)$  ⎞ + (x - 1) y(x) = x ex
(9)
=
> EcuaDos := expand( $\frac{lhs(Ecua)}{x}$ ) =  $\frac{rhs(Ecua)}{x}$ 
      EcuaDos :=  $\frac{d^2}{dx^2} y(x) - 2 \frac{d}{dx} y(x) + y(x) + \frac{\frac{d}{dx} y(x)}{x} - \frac{y(x)}{x} = e^x$ 
(10)
=
> Q := rhs(EcuaDos)
      Q := ex
(11)
=
Aplicar el método de parametros variables
> with(linalg) :
> WW := wronskian([yy[1], yy[2]], x)
(12)

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$$WW := \begin{bmatrix} e^x & e^x \ln(x) \\ e^x & e^x \ln(x) + \frac{e^x}{x} \end{bmatrix} \quad (12)$$

> $BB := \text{array}([0, Q])$

$$BB := \begin{bmatrix} 0 & e^x \end{bmatrix} \quad (13)$$

> $\text{ParamVar} := \text{linsolve}(WW, BB)$

$$\text{ParamVar} := \begin{bmatrix} -\ln(x) & x & x \end{bmatrix} \quad (14)$$

> $\text{Aprima} := \text{ParamVar}[1]; \text{Bprima} := \text{ParamVar}[2]$

$$\begin{aligned} \text{Aprima} &:= -\ln(x) & \text{Bprima} &:= x \end{aligned} \quad (15)$$

> $A := \text{int}(\text{Aprima}, x) + _C1$

$$A := -\frac{\ln(x) x^2}{2} + \frac{x^2}{4} + _C1 \quad (16)$$

> $B := \text{int}(\text{Bprima}, x) + _C2$

$$B := \frac{x^2}{2} + _C2 \quad (17)$$

> $\text{SolFinal} := \text{expand}(\text{SolNoHom})$

$$\text{SolFinal} := y(x) = \frac{e^x x^2}{4} + _C1 e^x + _C2 e^x \ln(x) \quad (18)$$

> $\text{SolUltima} := \text{dsolve}(\text{Ecua})$

$$\text{SolUltima} := y(x) = e^x c_2 + e^x \ln(x) c_1 + \frac{e^x x^2}{4} \quad (19)$$

> $\text{ComprobarTres} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolFinal}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0)))$

$$\text{ComprobarTres} := 0 = 0 \quad (20)$$

> restart

> $\text{EcuaCarac} := m^3 + m^2 + m + 1 = 0$

$$\text{EcuaCarac} := m^3 + m^2 + m + 1 = 0 \quad (21)$$

> $\text{Raiz} := \text{solve}(\text{EcuaCarac})$

$$\text{Raiz} := -1, I, -I \quad (22)$$

> $\text{yy}[1] := \exp(-x); \text{yy}[2] := \cos(x); \text{yy}[3] := \sin(x)$

$$\text{yy}_1 := e^{-x}$$

$$\text{yy}_2 := \cos(x)$$

$$\text{yy}_3 := \sin(x) \quad (23)$$

> $\text{with}(\text{linalg}) :$

> $WW := \text{wronskian}([\text{yy}[1], \text{yy}[2], \text{yy}[3]], x)$

(24)

$$WW := \begin{bmatrix} e^{-x} & \cos(x) & \sin(x) \\ -e^{-x} & -\sin(x) & \cos(x) \\ e^{-x} & -\cos(x) & -\sin(x) \end{bmatrix} \quad (24)$$

$$\begin{aligned} &> BB := \text{array}([0, 0, 2 \cdot \exp(-x) + 5 \cdot x + 6 \cdot \sin(2x)]) \\ &BB := \begin{bmatrix} 0 & 0 & 2e^{-x} + 5x + 6\sin(2x) \end{bmatrix} \end{aligned} \quad (25)$$

$$\begin{aligned} &> ParaVar := \text{expand}(\text{simplify}(\text{linsolve}(WW, BB))) \\ ParaVar &:= \left[1 + \frac{5x e^x}{2} + 3 \sin(2x) e^x, \right. \\ &\quad \left. - \frac{(\cos(x) + \sin(x)) (12 \sin(x) \cos(x) + 2e^{-x} + 5x)}{2}, \right. \\ &\quad \left. \frac{(\cos(x) - \sin(x)) (12 \sin(x) \cos(x) + 2e^{-x} + 5x)}{2} \right] \end{aligned} \quad (26)$$

$$\begin{aligned} &> Aprima := ParaVar[1] \\ Aprima &:= 1 + \frac{5x e^x}{2} + 3 \sin(2x) e^x \end{aligned} \quad (27)$$

$$\begin{aligned} &> Bprima := ParaVar[2] \\ Bprima &:= - \frac{(\cos(x) + \sin(x)) (12 \sin(x) \cos(x) + 2e^{-x} + 5x)}{2} \end{aligned} \quad (28)$$

$$\begin{aligned} &> Dprima := ParaVar[3] \\ Dprima &:= \frac{(\cos(x) - \sin(x)) (12 \sin(x) \cos(x) + 2e^{-x} + 5x)}{2} \end{aligned} \quad (29)$$

$$\begin{aligned} &> SolFinal := y(x) = \text{simplify}(\text{expand}((\text{int}(Aprima, x) + _C1) \cdot yy[1] + (\text{int}(Bprima, x) + _C2) \\ &\quad \cdot yy[2] + (\text{int}(Dprima, x) + _C3) \cdot yy[3])) \\ SolFinal &:= y(x) = -\frac{29}{5} + (1 + x + _C1) e^{-x} + \frac{8 \cos(x)^2}{5} + \frac{(5 _C2 - 4 \sin(x)) \cos(x)}{5} \\ &\quad + \sin(x) _C3 + 5x \end{aligned} \quad (30)$$

$$\begin{aligned} &> SolFinalHom := y(x) = _C1 \cdot \exp(-x) + _C2 \cdot \cos(x) + _C3 \cdot \sin(x) \\ SolFinalHom &:= y(x) = _C1 e^{-x} + \cos(x) _C2 + \sin(x) _C3 \end{aligned} \quad (31)$$

$$\begin{aligned} &> SolFinalNoHom := y(x) = -\frac{29}{5} + x \cdot \exp(-x) + \frac{8}{5} \cdot \cos(x)^2 - \frac{4}{5} \cdot \sin(x) \cos(x) + 5 \cdot x \\ SolFinalNoHom &:= y(x) = -\frac{29}{5} + x e^{-x} + \frac{8 \cos(x)^2}{5} - \frac{4 \sin(x) \cos(x)}{5} + 5x \end{aligned} \quad (32)$$

$$\begin{aligned} &> Ecua := y''' + y'' + y' + y = 2e^{-x} + 5x + 6\sin(2x) \\ Ecua &:= \frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 2e^{-x} + 5x + 6\sin(2x) \end{aligned} \quad (33)$$

$$\begin{aligned} &> ComprobarUno := \text{simplify}(\text{eval}(\text{subs}(y(x) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0))) \\ ComprobarUno &:= 0 = 0 \end{aligned} \quad (34)$$

> *SolUltima* := *dsolve*(*Ecua*)

$$\text{SolUltima} := y(x) = x e^{-x} + 5x - 5 + e^{-x} + \frac{4 \cos(2x)}{5} - \frac{2 \sin(2x)}{5} + c_1 \cos(x) + c_2 \sin(x) \quad (35)$$
$$+ c_3 e^{-x}$$

> *SolFinal*

$$y(x) = -\frac{29}{5} + (1 + x + c_1) e^{-x} + \frac{8 \cos(x)^2}{5} + \frac{(5c_2 - 4 \sin(x)) \cos(x)}{5} + \sin(x) c_3 + 5x \quad (36)$$

>