$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\,dt$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i} \int_{a-i\infty}^{a+i\infty} e^{st} F(s)\,ds.$$

Teorema de existencia y unicidad $\mathcal{L}\{f(t)\}$.

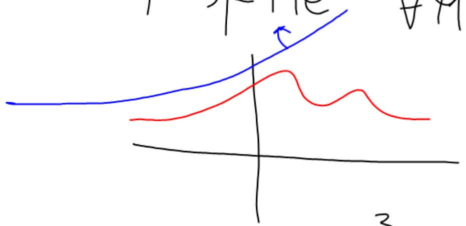
Si $f(t)$ es:

a) orden exponencial

b) seccionalmente continua

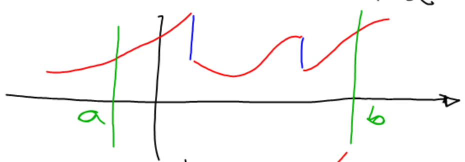
la transformada de Laplace existe y es única.

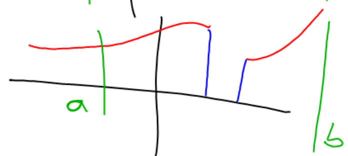
a) Si $|f(t)| \leq Me^{At}$ $\forall M, A \in \mathbb{R}$

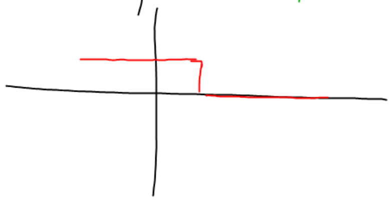


e^{x^2} ✗ $F(s)$

b) seccionalmente continua







① Lineal

$$\mathcal{L}\{af(t)+bg(t)\}=aF(s)+bG(s)$$

$a, b \in \mathbb{R}$

$$\mathcal{L}\{5e^{3t}+4t^2\}=5\mathcal{L}\{e^{3t}\}+4\mathcal{L}\{t^2\}.$$

$$=5\left(\frac{1}{s-3}\right)+4\left(\frac{2!}{s^3}\right)$$

$$=\frac{5}{s-3}+\frac{8}{s^3}$$

2 Semejanza

$$\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

$$\mathcal{L}\{e^{t}\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{4t}\} = \frac{1}{4}\left(\frac{1}{\frac{s}{4}-1}\right)$$

$$= \frac{1}{4}\left(\frac{1}{\frac{s-4}{4}}\right)$$

$$= \frac{1}{4}\left(\frac{4}{s-4}\right)$$

$$= \frac{1}{s-4}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

3) Derivada

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - s\cdot f(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2 f(0) - s\cdot f'(0) - f''(0)$$

$$\mathcal{L}\{f^{(IV)}(t)\} = s^4F(s) - s^3\cdot f(0) - s^2 f'(0) - sf''(0) - f'''(0).$$

4) Deriv. TL.

$$\mathcal{L}^{-1}\left\{F'(s)\right\} = -t\,f(t)$$

$$\mathcal{L}^{-1}\left\{F^{(n)}(s)\right\} = (-1)^n t^n f(t).$$

5) Integral.

$$\mathcal{L}\left\{ \int_0^t f(\tau)\, d\tau \right\} = \frac{F(s)}{s}$$

6) Integra TL.

$$\mathcal{L}^{-1}\left\{ \int_s^\infty F(\sigma)\, d\sigma \right\} = \frac{f(t)}{t}$$

7) Tardanza

$$\mathcal{L}\{f(t-a)\} = e^{-sa} F(s)$$

$$\mathcal{L}\{e^{5t}\} = \frac{1}{s-5}$$

$$\mathcal{L}\{e^{5(t-1)}\} = e^{-s}\left(\frac{1}{s-5}\right)$$

$$\mathcal{L}\{\cos(4t)\} = \frac{s}{s^2+16}$$

$$\mathcal{L}\{\cos(4(t-6))\} = e^{-6s}\left(\frac{s}{s^2+16}\right)$$

8) Desplazamiento

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$\mathcal{L}\{\operatorname{sen}(3t)\} = \frac{3}{s^2+9}$$

$$\mathcal{L}\{e^{5t}\operatorname{sen}(3t)\} = \frac{3}{(s-5)^2+9}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+s+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s^2+s+\frac{1}{4})+\frac{3}{4}}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right\}$$

$$= \frac{2}{\sqrt{3}}\mathcal{L}^{-1}\left\{\frac{\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right\}$$

$$= \frac{2}{\sqrt{3}}e^{-\frac{t}{2}}\operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right).$$

$$y'' + 4y = \cos(2x) \qquad \begin{aligned} y(0) &= 2 \\ y'(0) &= -2 \end{aligned}$$

$$\mathcal{L}\{y'' + 4y\} = \mathcal{L}\{\cos(2x)\}$$

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y\} = \frac{s}{s^2+4}$$

$$\left[s^2\mathcal{L}\{y\} - s(2) - (-2)\right] + 4\mathcal{L}\{y\} = \frac{s}{s^2+4}$$

$$(s^2+4)\mathcal{L}\{y\} - 2s + 2 = \frac{s}{s^2+4}$$

$$(s^2+4)\mathcal{L}\{y\} = \frac{s}{s^2+4} + 2s - 2$$

$$= \frac{s + (2s-2)(s^2+4)}{s^2+4}$$

$$= \frac{s + 2s^3 + 8s - 2s^2 - 8}{s^2+4}$$

$$(s^2+4)\mathcal{L}\{y\} = \frac{2s^3 - 2s^2 + 9s - 8}{s^2+4}$$

$$\mathcal{L}\{y\} = \frac{2s^3 - 2s^2 + 9s - 8}{(s^2+4)^2}$$

$$\mathcal{L}\{y\} = \frac{As+B}{(s^2+4)^2} + \frac{Ds+E}{(s^2+4)}$$