11) Producto TL.

convolución

$$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t)$$

$$f(t)*g(t) = \int_0^t f(z)\cdot g(t-z)\,dz$$

$$H(s) = \frac{s}{(s^2+4)^2} = \left(\frac{s}{s^2+4}\right)\cdot\left(\frac{1}{s^2+4}\right)$$

$$= \frac{1}{2}\left(\frac{s}{s^2+4}\right)\cdot\left(\frac{2}{s^2+4}\right)$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right)\cdot \mathcal{L}^{-1}\left(\frac{2}{s^2+4}\right)$$

$$\mathcal{L}^{-1}\{H(s)\} = \frac{1}{2}\cos(2t) * \operatorname{sen}(2t)$$

$$= \frac{1}{2}\int_0^t \cos(2z)\cdot \operatorname{sen}(2(t-z))\,dz$$

$$= \frac{1}{2}\int_0^t \cos(2z)\left[\operatorname{sen}(2t)\cos(2z) - \operatorname{sen}(2z)\cos(2t)\right]dz.$$

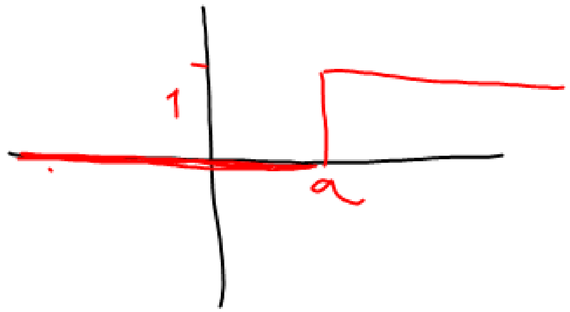
$$= \frac{1}{2}\left(\operatorname{sen}(2t)\int_0^t \cos^2(2z)\,dz - \cos(2t)\int_0^t \cos(2z)\operatorname{sen}(2z)\,dz\right)$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \frac{1}{4}t\operatorname{sen}(2t)$$

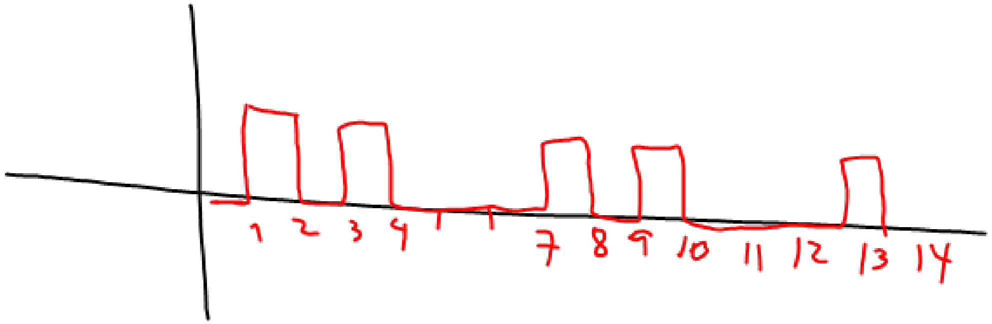
Seccionalmente continuas

función escalón unitario (Heaviside)

$$u(t-a) = \begin{cases} 0; & t < a \\ 1; & t \geq a \end{cases}$$



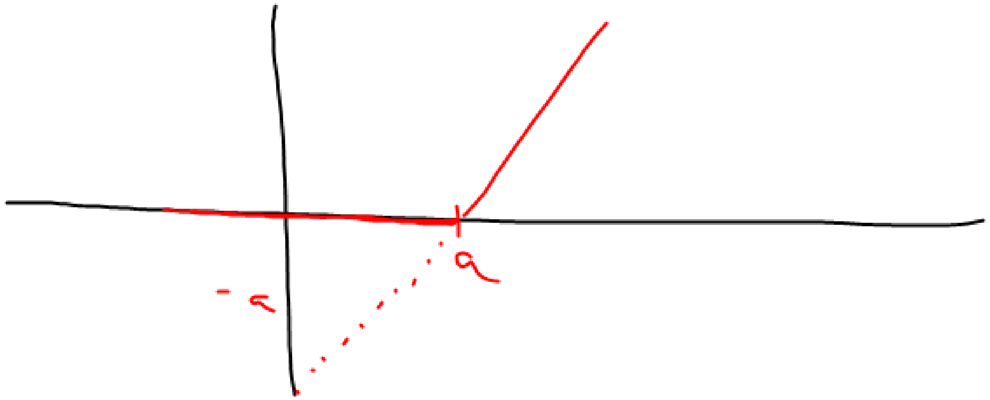
$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$



$$f = u(t-1) - u(t-2) + u(t-3) - u(t-4) + u(t-7) - u(t-8) +$$
$$+ u(t-9) - u(t-10) + u(t-13) - u(t+14)$$

función rampa unitaria

$$r(t-a) = \begin{cases} 0 & t < a \\ (t-a) & t \geq a \end{cases} \qquad \mathcal{L}\{t-a\} = \frac{1}{s^2}$$



$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s} \qquad u(0)=0$$

$$\mathcal{L}\{r(t-a)\} = \frac{e^{-as}}{s^2} \qquad r(0)=0$$

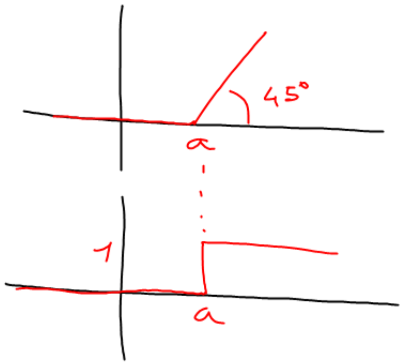
$$\mathcal{L}\left\{\frac{d}{dt}r(t-a)\right\} = s\,\mathcal{L}\{r\} - r(0)$$

$$\mathcal{L}\left\{\frac{d}{dt}r(t-a)\right\} = \cancel{s}\left(\frac{e^{-as}}{s^{\cancel{2}}}\right) - (0)$$

$$\mathcal{L}\left\{\frac{d}{dt}r(t-a)\right\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}\left\{\frac{d}{dt}r(t-a)\right\} = \mathcal{L}\{u(t-a)\}$$

$$\frac{d}{dt}r(t-a) = u(t-a)$$



delta de Dirac.

$$\delta(t-a) = \begin{cases} 0 \; ; & t \neq a \\ \displaystyle\int_{-\infty}^{\infty} \delta \, dt = 1 \end{cases}$$

$$\mathcal{L}\{\delta(t-a)\} = e^{-as}$$

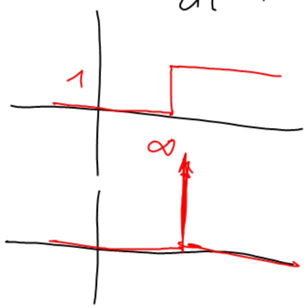
$$\mathcal{L}\left\{\frac{d}{dt} u(t-a)\right\} = s\mathcal{L}\{u\} - \cancel{u(0)}^{\,0}$$

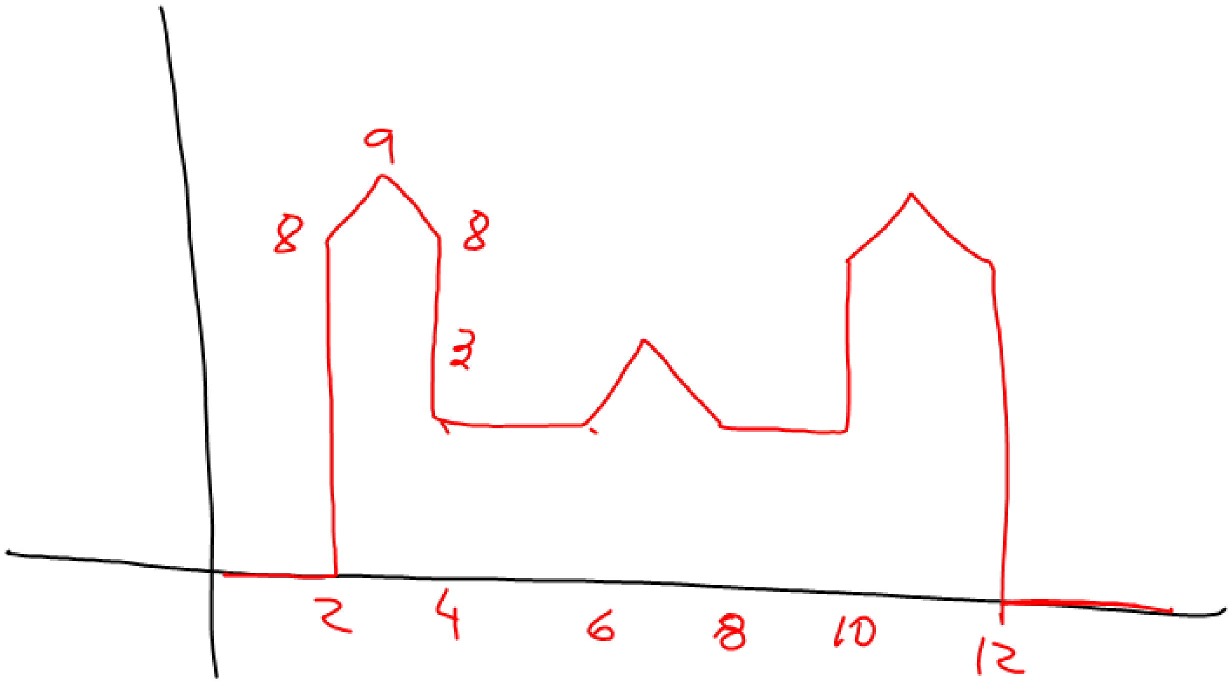
$$\mathcal{L}\left\{\frac{d}{dt} u(t-a)\right\} = \cancel{s}\left(\frac{e^{-as}}{\cancel{s}}\right)$$

$$\mathcal{L}\left\{\frac{d}{dt} u(t-a)\right\} = e^{-as}$$

$$\mathcal{L}\left\{\frac{d}{dt} u(t-a)\right\} = \mathcal{L}\{\delta(t-a)\}$$

$$\frac{d}{dt} u(t-a) = \delta(t-a)$$

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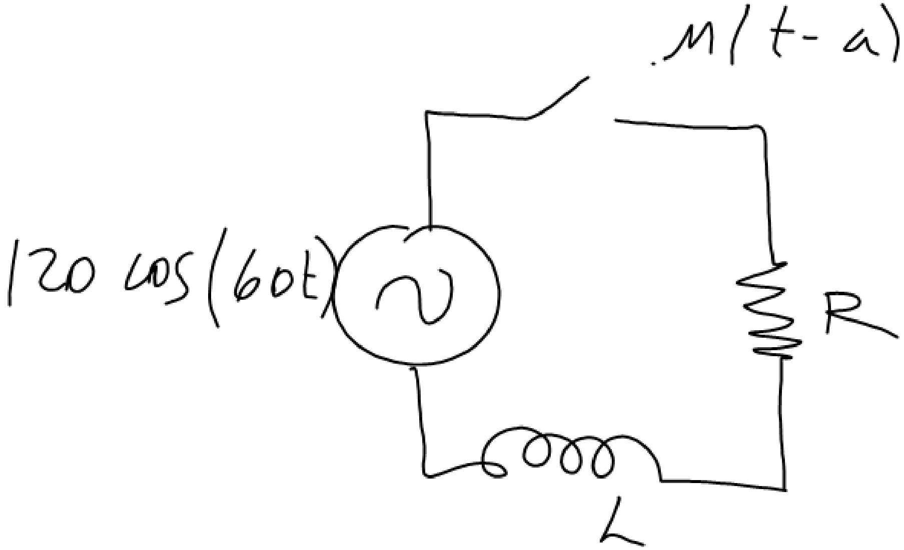

$$L\frac{di}{dt}+Ri = u(t-5)\,120\cos(60t).$$

$i(0)=0$

$$L\{s\mathcal{L}\{i\}-i(0)\}+R\mathcal{L}\{i\} = \frac{120\,e^{-5s}}{s^2+3600}$$

$$\left(s+\frac{R}{L}\right)\mathcal{L}\{i\} = \frac{120}{L}\left(\frac{e^{-5s}}{s^2+3600}\right)$$

$$\mathcal{L}\{i\} = \frac{120}{L}\left\{\frac{e^{-5s}}{\left(s+\frac{R}{L}\right)(s^2+3600)}\right\}.$$