> *restart*

> $Sistema := diff(x[1](t),t) = x[1](t) + x[2](t) + 5\cdot\exp(t), diff(x[2](t),t) = -x[1](t) + x[2](t) - 8\cdot t^2 : Sistema[1]; Sistema[2]$

$$\frac{d}{dt} x_1(t) = x_1(t) + x_2(t) + 5\,e^t$$

$$\frac{d}{dt} x_2(t) = -x_1(t) + x_2(t) - 8\,t^2 \qquad \textbf{(1)}$$

> $CondIni := x[1](0) = 2, x[2](0) = -3$

$$CondIni := x_1(0) = 2, x_2(0) = -3 \qquad \textbf{(2)}$$

> $AA := array([[1,1],[-1,1]])$

$$AA := \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \qquad \textbf{(3)}$$

> $Xcero := array([2,-3])$

$$Xcero := \begin{bmatrix} 2 & -3 \end{bmatrix} \qquad \textbf{(4)}$$

> $BB := array([5\cdot\exp(t), -8\cdot t^2])$

$$BB := \begin{bmatrix} 5\,e^t & -8\,t^2 \end{bmatrix} \qquad \textbf{(5)}$$

> $with(linalg):$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} e^t\cos(t) & e^t\sin(t) \\ -e^t\sin(t) & e^t\cos(t) \end{bmatrix} \qquad \textbf{(6)}$$

> $SolHom := evalm(MatExp \,\&*\, Xcero) : Xhom[1] = SolHom[1]; Xhom[2] = SolHom[2];$

$$Xhom_1 = 2\,e^t\cos(t) - 3\,e^t\sin(t)$$

$$Xhom_2 = -2\,e^t\sin(t) - 3\,e^t\cos(t) \qquad \textbf{(7)}$$

> $MatExpTau := map(rcurry(eval, t = 't - \mathrm{tau}'), MatExp)$

$$MatExpTau := \begin{bmatrix} e^{t-\tau}\cos(t-\tau) & e^{t-\tau}\sin(t-\tau) \\ -e^{t-\tau}\sin(t-\tau) & e^{t-\tau}\cos(t-\tau) \end{bmatrix} \qquad \textbf{(8)}$$

> $BBtau := map(rcurry(eval, t = '\mathrm{tau}'), BB)$

$$BBtau := \begin{bmatrix} 5\,e^{\tau} & -8\,\tau^2 \end{bmatrix} \qquad \textbf{(9)}$$

> $ProdTau := evalm(MatExpTau \,\&*\, BBtau) : ProdTau[1]; ProdTau[2]$

$$5\,e^{t-\tau}\cos(t-\tau)\,e^{\tau} - 8\,e^{t-\tau}\sin(t-\tau)\,\tau^2$$

$$-5\,e^{t-\tau}\sin(t-\tau)\,e^{\tau} - 8\,e^{t-\tau}\cos(t-\tau)\,\tau^2 \qquad \textbf{(10)}$$

> $SolNoHom := simplify(map(int, ProdTau, \mathrm{tau} = 0..t)) : XnoHom[1] = SolNoHom[1]; XnoHom[2] = SolNoHom[2]$

$$XnoHom_1 = (4\cos(t) + 9\sin(t))\,e^t - 4\,(t+1)^2$$

$$XnoHom_2 = -4 + 4\,t^2 + e^t\,(-5 + 9\cos(t) - 4\sin(t)) \qquad \textbf{(11)}$$

> $ComprobarUno[1] := simplify(subs(t=0, SolNoHom[1]))$

$$ComprobarUno_1 := 0 \tag{12}$$

> $ComprobarUno[2] := simplify(subs(t=0, SolNoHom[2]))$

$$ComprobarUno_2 := 0 \tag{13}$$

> $SolPartFinal := evalm(SolHom + SolNoHom) : x[1](t) = SolPartFinal[1]; x[2](t) = SolPartFinal[2]$

$$x_1(t) = 2\,e^t\cos(t) - 3\,e^t\sin(t) + (4\cos(t) + 9\sin(t))\,e^t - 4\,(t+1)^2$$

$$x_2(t) = -2\,e^t\sin(t) - 3\,e^t\cos(t) - 4 + 4\,t^2 + e^t\,(-5 + 9\cos(t) - 4\sin(t)) \tag{14}$$

> $ComprobarDos[1] := simplify(subs(t=0, SolPartFinal[1]))$

$$ComprobarDos_1 := 2 \tag{15}$$

> $ComprobarDos[2] := simplify(subs(t=0, SolPartFinal[2]))$

$$ComprobarDos_2 := -3 \tag{16}$$

> $Sistema[1]$

$$\frac{d}{dt}\,x_1(t) = x_1(t) + x_2(t) + 5\,e^t \tag{17}$$

> $ComprobarTres[1] := simplify(subs(x[1](t) = SolPartFinal[1], x[2](t) = SolPartFinal[2], Sistema[1]))$

$$ComprobarTres_1 := 12\,e^t\cos(t) - 8\,t - 8 = 12\,e^t\cos(t) - 8\,t - 8 \tag{18}$$

> $Sistema[2]$

$$\frac{d}{dt}\,x_2(t) = -x_1(t) + x_2(t) - 8\,t^2 \tag{19}$$

> $ComprobarTres[2] := simplify(subs(x[1](t) = SolPartFinal[1], x[2](t) = SolPartFinal[2], Sistema[2]))$

$$ComprobarTres_2 := (-12\sin(t) - 5)\,e^t + 8\,t = (-12\sin(t) - 5)\,e^t + 8\,t \tag{20}$$

> $restart$

> $Ecua := diff(z(x,y), x\$2) + 8\cdot diff(z(x,y), y\$2) = 0$

$$Ecua := \frac{\partial^2}{\partial x^2}\,z(x,y) + 8\,\frac{\partial^2}{\partial y^2}\,z(x,y) = 0 \tag{21}$$

> $SolGral := pdsolve(Ecua)$

$$SolGral := z(x,y) = f_1\left(y - 2\,I\sqrt{2}\,x\right) + f_2\left(y + 2\,I\sqrt{2}\,x\right) \tag{22}$$

> $restart$

> $Ecua := diff(z(x,y), x\$2) + 4\cdot diff(z(x,y), x, y) - 6\cdot diff(z(x,y), y\$2) = 0$

$$Ecua := \frac{\partial^2}{\partial x^2}\,z(x,y) + 4\,\frac{\partial^2}{\partial x \partial y}\,z(x,y) - 6\,\frac{\partial^2}{\partial y^2}\,z(x,y) = 0 \tag{23}$$

> $SolGral := pdsolve(Ecua)$

$$SolGral := z(x,y) = f_1\left(y - \left(2 + \sqrt{10}\right)x\right) + f_2\left(y - \left(2 - \sqrt{10}\right)x\right) \tag{24}$$

> $with(PDEtools)$

[*CanonicalCoordinates, ChangeSymmetry, CharacteristicQ, CharacteristicQInvariants,* (25) *ConservedCurrentTest, ConservedCurrents, ConsistencyTest, D_Dx, DeterminingPDE, Eta_k, Euler, FirstIntegralSolver, FromJet, FunctionFieldSolutions, InfinitesimalGenerator, Infinitesimals, IntegratingFactorTest, IntegratingFactors, InvariantEquation, InvariantSolutions, InvariantTransformation, Invariants, Laplace, Library, PDEplot, PolynomialSolutions, ReducedForm, SimilaritySolutions, SimilarityTransformation, Solve, SymmetryCommutator, SymmetryGauge, SymmetrySolutions, SymmetryTest, SymmetryTransformation, TWSolutions, ToJet, ToMissingDependentVariable, build, casesplit, charstrip, dchange, dcoeffs, declare, diff_table, difforder, dpolyform, dsubs, mapde, separability, splitstrip, splitsys, undeclare*]

> *restart*

> $Ecua := diff(z(x,y), y\$2) - 6 \cdot diff(z(x,y), x, y) + 8 \cdot diff(z(x,y), x) = z(x,y)$

$$Ecua := \frac{\partial^2}{\partial y^2} z(x,y) - 6 \frac{\partial^2}{\partial x \partial y} z(x,y) + 8 \frac{\partial}{\partial x} z(x,y) = z(x,y) \quad \textbf{(26)}$$

> $SolGral := pdsolve(Ecua)$

$$SolGral := z(x,y) = f_1(\_\xi 2)\, f_2(\_\xi 1) \ \textbf{where} \left[\left[\left\{ \frac{d}{d\_\xi 2} f_1(\_\xi 2) = \_c_1 f_1(\_\xi 2), \frac{d}{d\_\xi 1} f_2(\_\xi 1) = \frac{-8\,\_c_1 f_2(\_\xi 1) + f_2(\_\xi 1)}{6\,\_c_1 - \frac{4}{3}} \right\}, \textbf{and} \left\{ \_\xi 1 = -\frac{x}{6}, \_\xi 2 = 6\,y + x \right\} \right] \right] \quad \textbf{(27)}$$

> $with(PDEtools):$

> $SolGralDos := build(SolGral)$

$$SolGralDos := z(x,y) = \frac{c_1 c_2 \left(e^{\frac{\_c_1^2 (6y+x)}{9\_c_1 - 2}} \right)^9 e^{-\frac{x}{4(9\_c_1 - 2)}}}{\left(e^{-\frac{x\_c_1}{6(9\_c_1 - 2)}} \right)^{12} \left(e^{\frac{\_c_1 (6y+x)}{9\_c_1 - 2}} \right)^2} \quad \textbf{(28)}$$

>