

TEMA 3.- Matriz exponencial.

SERIE 3. Entregar 15 MAYO

SERIE 4: SUBIR 14  $\rightarrow$  ENTREGAR 21

EXAMEN PARCIAL 2  $\Rightarrow$  TEMAS 3 & 4  $\rightarrow$  23 MAYO.

$$\frac{d\bar{x}}{dt} = A \bar{x}$$

$$\bar{x} = e^{At} \bar{x}(0)$$

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$$\frac{d}{dt} \bar{x} = A \bar{x}(t) + b(t)$$

$$\bar{x} = e^{At} \bar{x}(0) + \int_0^t \underline{e^{(t-z)}} b(z) dz$$

# TEMA 4 : ECUACIÓN DIFERENCIAL EN DERIVADAS PARCIALES.

$$F(\underbrace{x, y, z(x, y)}_{\text{dos o más var. independ.}}, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}, \dots) = 0$$

incognita

$$\frac{\partial^2 z}{\partial x^2} + 8 \frac{\partial^2 z}{\partial y^2} = 0$$

$$\text{orden} = 2$$

$$z(x, y) = \underset{g}{F_1(x, y)} + F_2(x, y)$$

$$z = f(y + mx)$$

$$\frac{\partial z}{\partial x} = f' \cdot m \quad \frac{\partial z}{\partial y} = f' \cdot (1)$$

$$\frac{\partial^2 z}{\partial x^2} = f'' m^2 \quad \frac{\partial^2 z}{\partial y^2} = f'' (1)$$

$$f'' m^2 + 8 f'' = 0$$

$$(m^2 + 8) f'' = 0 \quad f''(y + mx) = 0$$

$$m^2 + 8 = 0$$

$$m^2 = -8$$

$$m_{1,2} = \pm \sqrt{8}i$$

$$f'(y + mx) = C_1$$

$$\int f(y + mx) = C_1(y + mx) + C_2$$

UTIL.

$$z(x, y) = F_1(y + \sqrt{8}i x) + F_2(y - \sqrt{8}i x).$$

$$\frac{\partial^2 z}{\partial x^2} + 4 \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = 0$$

	TEMARIO	REAL
EDO	80%	20%
ED en DP	20%	80%

## Método de Variables Separables

Prueba y error

$$H_0: z(x, y) = F(x) \cdot G(y)$$

$$\frac{\partial^2 z}{\partial y^2} - 6 \frac{\partial^2 z}{\partial x \partial y} + 8 \frac{\partial^2 z}{\partial x^2} = z$$

$$z = F \cdot G$$

$$\frac{\partial z}{\partial y} = F G' \quad \frac{\partial z}{\partial x} = F' G$$

$$\frac{\partial^2 z}{\partial y^2} = F G'' \quad \frac{\partial^2 z}{\partial x \partial y} = F' G'$$

$$F G'' - 6 F' G' + 8 F' G = F G$$

$$F G'' - F G = 6 F' G' - 8 F' G$$

$$F(G'' - G) = F'(6G' - 8G)$$

$$\frac{G'' - G}{6G' - 8G} = \frac{F'}{F}$$

$$\frac{F'}{F} = \alpha \quad \frac{G'' - G}{6G' - 8G} = \alpha$$

$$\alpha = 0$$

$$\frac{F'}{F} = 0 \quad F' = 0 \quad \boxed{F(x) = C_1}$$

$$\frac{G'' - G}{6G' - 8G} = 0 \quad G'' - G = 0$$

$$m^2 - 1 = 0$$

$$m = \pm 1$$

$$G(y) = C_1 e^y + C_2 e^{-y}$$

$$z(x, y) = C_{10} (C_1 e^y + C_2 e^{-y})$$

$$\boxed{z(x, y) = C_{100} e^y + C_{200} e^{-y}} \\ \alpha = 0$$

para  $\alpha > 0$   $\alpha = +\beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F'}{F} = \beta^2 \quad F' = \beta^2 F \quad F' - \beta^2 F = 0$$

$$m - \beta^2 = 0$$

$$m = \beta^2$$

$$F(x) = c_1 e^{\beta^2 x}$$

$$\frac{G'' - G}{6G' - 8G} = \beta^2 \quad G'' - G = \beta^2 (6G' - 8G)$$

$$G'' - G - 6\beta^2 G' + 8\beta^2 G = 0$$

$$m_1 = 6\beta^2 + \frac{\sqrt{36\beta^4 - 32\beta^2 + 4}}{2}$$

$$G'' - 6\beta^2 G' + (8\beta^2 - 1)G = 0$$

$$m_2 = 6\beta^2 - \frac{\sqrt{36\beta^4 - 32\beta^2 + 4}}{2}$$

$$m^2 - 6\beta^2 m + (8\beta^2 - 1) = 0$$

$$G(y) = c_1 e^{m_1 y} + c_2 e^{m_2 y}$$

$$Z(x, y) = e^{\beta^2 x} (c_{10} e^{m_1 y} + c_{20} e^{m_2 y})$$

$\alpha > 0$