

# ECUACIONES DIFERENCIALES

Es una expresión matemática que basa la forma de "ecuación"

$$F\left(x, y, \frac{dy}{dx}\right) = 0$$

que contiene al menos una de las derivadas de una función desconocida.

" $y/x$ " se conoce como "incógnita" al menos una variable independiente

" $x$ " y cuya finalidad al

resolverla es encontrar la

forma de la incógnita.

Método

$$\frac{dy}{dx} = y$$

$$y = C_1 e^x$$

$$[C_1 e^x] = [C_1 e^x]$$

$$\frac{dy}{dx} = g \frac{d}{dx}(e^x)$$

$$C_1 e^x - C_1 e^x = 0$$

$\cancel{0 = 0}$

$$\frac{dy}{dx} = C_1 e^x$$

Soluciones:

a) general (I)

b) particular ( $x_0$ )

c) singular (#)

$\exists, \emptyset.$  {

ED. Ondinaria

$$F(x, y(x), \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots) = 0 \quad \text{Cap. I, II, III}$$

$y(x)$  una y sólo una variable depend.)

ED. en Derivadas Parciales

$$F(x, y, z(x, y), \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial x^2}, \dots) = 0$$

$z(x, y)$  (dos o más var. ind.) Cap IV

	V.D. REAL	ESTA MATERIA
EDO	20%	80%
ED en DP	80%	20%

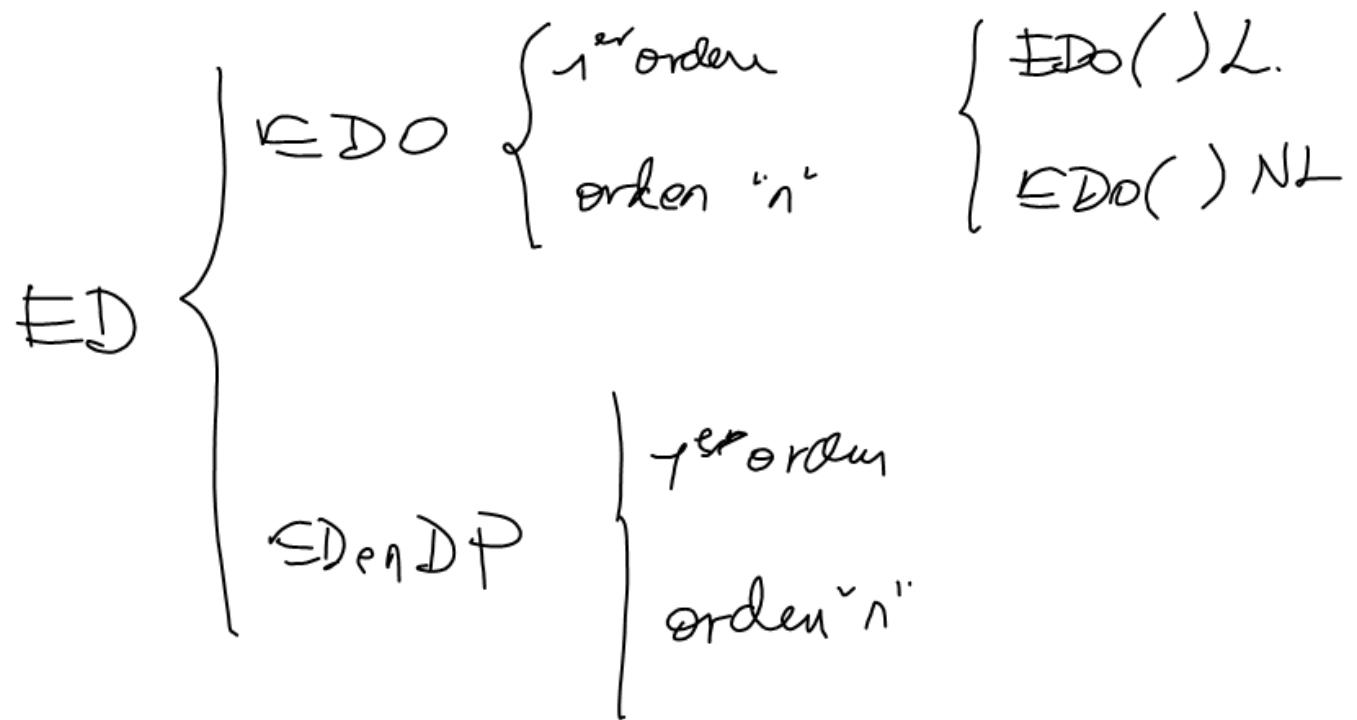
ED { primer orden CAP I EDO(1)  
 orden superior a 1. CAP<sub>s</sub> II & III EDO(n)  
 "El orden ED corresponde al orden  
 de la derivada de mayor orden"

$$\frac{dy}{dx} + 5y = 3 \cos(4x) \quad \text{orden} = 1.$$

$$\frac{d^3y}{dx^3} + 6x \frac{d^2y}{dx^2} + 4x^2 \frac{dy}{dx} - 8y = 0 \quad \text{orden} = 3$$

$$\frac{\partial z}{\partial x} - 6 \frac{\partial z}{\partial y} + 8z = 0 \quad \text{orden} = 1$$

$$\frac{\partial^2 z}{\partial x^2} - 8 \frac{\partial^2 z}{\partial x \partial y} + 6 \frac{\partial^2 z}{\partial y^2} = z \quad \text{orden} = 2$$



$$\text{EDO} \Rightarrow a_0(x) \frac{d^n y}{dx^n} + a_1(x) \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1}(x) \frac{dy}{dx} + a_n(x)y = Q(x)$$

LINÉAL EDO(n) L CC.

$$\text{Si } a_i(x) = a_i$$

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_n y = Q(x)$$

$$\frac{dy}{dx} = \frac{1}{y} \quad \frac{dy}{dx} - \frac{1}{y} = 0 \quad NL$$

$$y \frac{dy}{dx} = 1$$

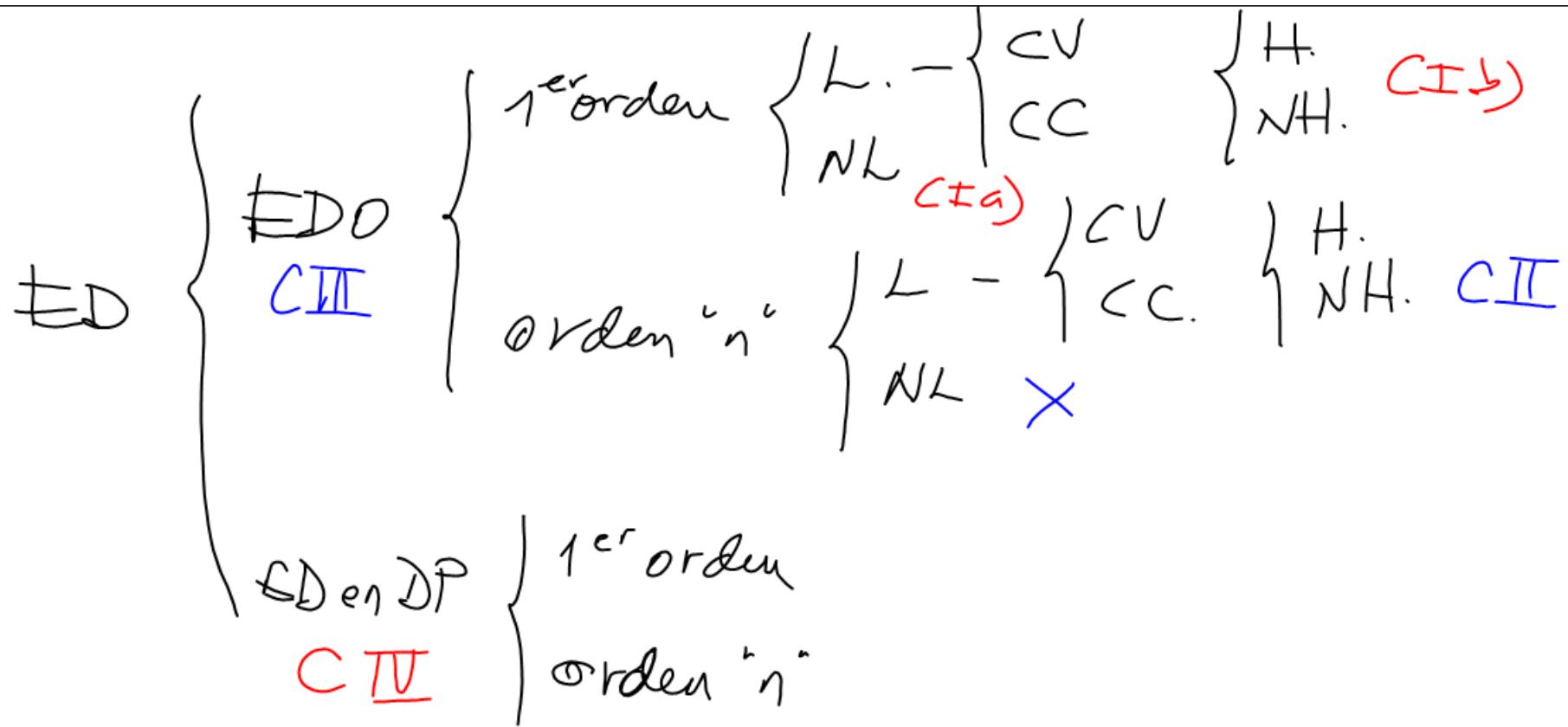
$$\left( \frac{dy}{dx} \right)^2 + 6y = 0 \quad NL$$

$$\frac{dy}{dx} + y^3 = 0 \quad NL$$

$$\frac{d\theta}{dt^2} + \operatorname{sen}(\theta) = 0 \quad NL$$

$$\frac{d\theta}{dt^2} + \theta = 0 \quad L$$

$\operatorname{sen}(\theta) \doteq \theta$  [rad]  $0 \leq \theta \leq 90^\circ$



- Sábado
- 1<sup>er</sup> Examen Parcial = 29 Mar 9:00
- 2<sup>o</sup> Examen Parcial = 26 Abril 10:15.
- 3<sup>er</sup> Examen Parcial = 22 Mayo 11:00

$$y(0)=4$$

$$\frac{dy}{dx} - a_1 y = 0 \quad y = C_1 e^{a_1 x}$$

$$y \Rightarrow C_1 e^{a_1(0)} = 4$$

$$y_p = e^{mx} \quad \frac{dy}{dx} = m e^{mx}$$

$$C_1(1) = 4 \\ C_1 = 4$$

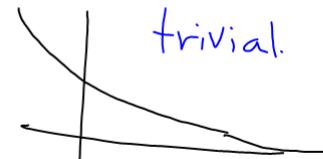
$$[m e^{mx}] - a_1 [e^{mx}] = 0$$

$$y_p = 4 e^{a_1 x} \quad (m - a_1) e^{mx} = 0 \quad e^{mx} = 0 \quad m x \rightarrow -\infty$$

Sol. ecuación particular característica  $m - a_1 = 0$

$$m = a_1$$

$$y = e^{a_1 x}$$



trivial.

$$\frac{dy}{dx} - a_1 y = 0 \quad y = C_1 e^{a_1 x}$$

$$[C_1 a_1 e^{a_1 x}] - a_1 [C_1 e^{a_1 x}] = 0 \quad \frac{dy}{dx} = C_1 a_1 e^{a_1 x}$$

$$(C_1 a_1 - C_1 a_1) e^{a_1 x} = 0$$

$$(0) e^{a_1 x} = 0$$

$$\underbrace{\textcircled{1}}_{\sim} \equiv 0$$

}

orden EDO  $\Rightarrow$  parámetros SG.

$$\frac{dy}{dx} - a_1 y = 0 \quad \text{orden} = 1$$

$$y_g = C_1 e^{a_1 x}$$

$$\text{orden} = 2 \quad \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad M_1 = \frac{a_1}{a_2}$$

$$y_g = C_1 e^{a_1 x} + C_2 e^{a_2 x}$$

$$\begin{vmatrix} e^{a_1 x} & e^{a_2 x} \\ a_1 e^{a_1 x} & a_2 e^{a_2 x} \end{vmatrix} \neq 0$$

$$(a_1 - a_2) e^{a_1 x} e^{a_2 x} \neq 0$$

$$y_g = C_1 y_1 + C_2 y_2 + C_3 y_3$$

$$e^{a_1 x} \neq 0$$

$$e^{a_2 x} \neq 0$$

$$\text{orden} = 3 \quad \frac{d^3y}{dx^3} + a_1 \frac{d^2y}{dx^2} + a_2 \frac{dy}{dx} + a_3 y = 0$$

$$(a_1 - a_2) \neq 0$$

$$a_1 \neq a_2$$

$$y = C_1 e^{2x} + C_2 e^{-x} + C_3 e^{4x}$$

$$\frac{dy}{dx} = 2C_1 e^{2x} - C_2 e^{-x} + 4C_3 e^{4x}$$

$$\frac{d^2y}{dx^2} = 4C_1 e^{2x} + C_2 e^{-x} + 16C_3 e^{4x}$$

$$\frac{d^3y}{dx^3} = 8C_1 e^{2x} - C_2 e^{-x} + 64C_3 e^{4x}$$

$$\frac{d^3y}{dx^3} - 5 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 8y = 0$$