> restart

> $Ecua := \left(\dfrac{x}{\text{sqrt}(x^2+y^2)} + \dfrac{1}{x} + \dfrac{1}{y} \right) + \left(\dfrac{y}{\text{sqrt}(x^2+y^2)} + \dfrac{1}{y} - \dfrac{x}{y^2} \right) \cdot y' = 0$

$$Ecua := \frac{x}{\sqrt{x^2+y(x)^2}} + \frac{1}{x} + \frac{1}{y(x)} + \left(\frac{y(x)}{\sqrt{x^2+y(x)^2}} + \frac{1}{y(x)} - \frac{x}{y(x)^2} \right) \left(\frac{d}{dx}\, y(x) \right) = 0 \quad \textbf{(1)}$$

> $M := \dfrac{x}{\sqrt{x^2+y^2}} + \dfrac{1}{x} + \dfrac{1}{y}$

$$M := \frac{x}{\sqrt{x^2+y^2}} + \frac{1}{x} + \frac{1}{y} \quad \textbf{(2)}$$

> $N := \dfrac{y}{\sqrt{x^2+y^2}} + \dfrac{1}{y} - \dfrac{x}{y^2}$

$$N := \frac{y}{\sqrt{x^2+y^2}} + \frac{1}{y} - \frac{x}{y^2} \quad \textbf{(3)}$$

> $DerMy := diff(M, y)$

$$DerMy := - \frac{x\,y}{(x^2+y^2)^{3/2}} - \frac{1}{y^2} \quad \textbf{(4)}$$

> $DerNx := diff(N, x)$

$$DerNx := - \frac{x\,y}{(x^2+y^2)^{3/2}} - \frac{1}{y^2} \quad \textbf{(5)}$$

> $Comprobar := \left(- \dfrac{x \cdot y}{(x^2+y^2)^{3/2}} - \dfrac{1}{y^2} \right) - \left(- \dfrac{x \cdot y}{(x^2+y^2)^{3/2}} - \dfrac{1}{y^2} \right) = 0$

$$Comprobar := 0 = 0 \quad \textbf{(6)}$$

Por lo tanbto es Exacta

> $IntMx := int(M, x)$

$$IntMx := \ln(x) + \frac{x}{y} + \sqrt{x^2+y^2} \quad \textbf{(7)}$$

> $SolGralUno := IntMx + int((N - diff(IntMx, y)), y) = \_C1$

$$SolGralUno := \ln(x) + \frac{x}{y} + \sqrt{x^2+y^2} + \ln(y) = \_C1 \quad \textbf{(8)}$$

> $IntNy := int(N, y)$

$$IntNy := \ln(y) + \sqrt{x^2+y^2} + \frac{x}{y} \quad \textbf{(9)}$$

> $SolGralDos := IntNy + int((M - diff(IntNy, x)), x) = \_C1$

$$SolGralDos := \ln(x) + \frac{x}{y} + \sqrt{x^2+y^2} + \ln(y) = \_C1 \quad \textbf{(10)}$$

> $SolFinal := \ln(x) + \dfrac{x}{y(x)} + \sqrt{x^2+y(x)^2} + \ln(y(x)) = \_C1$

$$\quad \textbf{(11)}$$

$$SolFinal := \ln(x) + \frac{x}{y(x)} + \sqrt{x^2 + y(x)^2} + \ln(y(x)) = \_C1 \tag{11}$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx} y(x) = \frac{y(x)\left((y(x) + x)\sqrt{x^2 + y(x)^2} + y(x)\,x^2\right)}{x\left((x - y(x))\sqrt{x^2 + y(x)^2} - y(x)^3\right)} \tag{12}$$

> $Ecua$

$$\frac{x}{\sqrt{x^2 + y(x)^2}} + \frac{1}{x} + \frac{1}{y(x)} + \left(\frac{y(x)}{\sqrt{x^2 + y(x)^2}} + \frac{1}{y(x)} - \frac{x}{y(x)^2}\right)\left(\frac{d}{dx} y(x)\right) = 0 \tag{13}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx} y(x) = \frac{y(x)\left((y(x) + x)\sqrt{x^2 + y(x)^2} + y(x)\,x^2\right)}{x\left((x - y(x))\sqrt{x^2 + y(x)^2} - y(x)^3\right)} \tag{14}$$

> $ComprobarDos := simplify(rhs(DerSolFinal) - rhs(DerEcua)) = 0$
$$ComprobarDos := 0 = 0 \tag{15}$$

> $restart$

> $Ecua := \left(1 - x^2 \cdot y\right) + x^2 \cdot (y - x) \cdot y' = 0$

$$Ecua := 1 - x^2 y(x) + x^2 (y(x) - x)\left(\frac{d}{dx} y(x)\right) = 0 \tag{16}$$

> $with(DEtools):$

> $odeadvisor(Ecua)$
$$[\_rational, [\_1st\_order, \_with\_symmetry\_[F(x),G(x)]], [\_Abel, 2nd\ type,\ class\ B]] \tag{17}$$

> $FacInt := intfactor(Ecua)$

$$FacInt := \frac{1}{x^2} \tag{18}$$

> $M := 1 - x^2 y$

$$M := -x^2 y + 1 \tag{19}$$

> $N := x^2 (y - x)$

$$N := x^2 (y - x) \tag{20}$$

> $MM := expand(FacInt \cdot M)$

$$MM := -y + \frac{1}{x^2} \tag{21}$$

> $NN := expand(FacInt \cdot N)$

$$NN := y - x \tag{22}$$

> $EcuaExacta := \left(-y(x) + \frac{1}{x^2}\right) + (y(x) - x) \cdot diff(y(x), x) = 0$

$$EcuaExacta := -y(x) + \frac{1}{x^2} + (y(x) - x)\left(\frac{d}{dx} y(x)\right) = 0 \tag{23}$$

> $odeadvisor(EcuaExacta)$
$$[\_exact, \_rational, [\_1st\_order, \_with\_symmetry\_[F(x),G(x)]], [\_Abel, 2nd\ type,\ class\ B]] \tag{24}$$

> $IntMMx := int(MM, x)$

$$IntMMx := -\frac{1}{x} - y\,x \tag{25}$$

> $diff\,(IntMMx, y)$

$$-x \tag{26}$$

> $SolGral := IntMMx + int((NN - diff\,(IntMMx, y)), y) = \_C1$

$$SolGral := -\frac{1}{x} - y\,x + \frac{y^2}{2} = \_C1 \tag{27}$$

> $SolFinal := -\frac{1}{x} - y(x)\cdot x + \frac{y(x)^2}{2} = \_C1$

$$SolFinal := -\frac{1}{x} - y(x)\,x + \frac{y(x)^2}{2} = \_C1 \tag{28}$$

> $DerSolFinal := expand(isolate(diff\,(SolFinal, x), diff\,(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\,y(x) = \frac{y(x)}{y(x) - x} - \frac{1}{x^2\,(y(x) - x)} \tag{29}$$

> $Ecua$

$$1 - x^2\,y(x) + x^2\,(y(x) - x)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{30}$$

> $DerEcua := expand(isolate(Ecua, diff\,(y(x), x)))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{y(x)}{y(x) - x} - \frac{1}{x^2\,(y(x) - x)} \tag{31}$$

> $Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua)) = 0$

$$Comprobar := 0 = 0 \tag{32}$$

> $restart$

> $Ecua := (2\cdot x\cdot y^2 - 3\cdot y^3) + (7 - 3\cdot x\cdot y^2)\cdot y' = 0$

$$Ecua := 2\,x\,y(x)^2 - 3\,y(x)^3 + (7 - 3\,x\,y(x)^2)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{33}$$

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_rational] \tag{34}$$

> $intfactor(Ecua)$

$$\frac{1}{y(x)^2} \tag{35}$$

> $FacInt := \frac{1}{y^2}$

$$FacInt := \frac{1}{y^2} \tag{36}$$

> $M := 2\,x\,y^2 - 3\,y^3$

$$M := 2\,x\,y^2 - 3\,y^3 \tag{37}$$

> $N := \left(7 - 3\,x\,y^2\right)$

$$N := -3\,x\,y^2 + 7 \tag{38}$$

> $MM := expand(FacInt \cdot M)$

$$MM := 2\,x - 3\,y \tag{39}$$

> $NN := expand(FacInt \cdot N)$

$$NN := -3\,x + \frac{7}{y^2} \tag{40}$$

> $Comprobar := diff(MM, y) = diff(NN, x)$

$$Comprobar := -3 = -3 \tag{41}$$

> $IntNNy := int(NN, y)$

$$IntNNy := -3\,x\,y - \frac{7}{y} \tag{42}$$

> $SolGral := IntNNy + int((MM - diff(IntNNy, x)), x) = \_C1$

$$SolGral := -3\,x\,y - \frac{7}{y} + x^2 = \_C1 \tag{43}$$

> $SolFinal := -3\,x \cdot y(x) - \dfrac{7}{y(x)} + x^2 = \_C1$

$$SolFinal := -3\,x\,y(x) - \frac{7}{y(x)} + x^2 = \_C1 \tag{44}$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\,y(x) = \frac{(-3\,y(x) + 2\,x)\,y(x)^2}{3\,x\,y(x)^2 - 7} \tag{45}$$

> $Ecua$

$$2\,x\,y(x)^2 - 3\,y(x)^3 + \left(7 - 3\,x\,y(x)^2\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{46}$$

> $DerEcua := simplify(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{(-3\,y(x) + 2\,x)\,y(x)^2}{3\,x\,y(x)^2 - 7} \tag{47}$$

> $Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua)) = 0$

$$Comprobar := 0 = 0 \tag{48}$$

>