

EQUACIÓN LINEAL CON COEFICIENTES CONSTANTES

$$\frac{dy}{dx} + a_1 y + a_2 = 0$$

$$y = e^{mx}$$

$$m^2 + a_1 m + a_2 = 0$$

$$1.- \quad m_1 \neq m_2 \in \mathbb{R}$$

$$2.- \quad m_1 = m_2 \in \mathbb{R}$$

$$3.- \quad m_1 = a \pm bi \in \mathbb{C} \quad m_1 \neq m_2$$

Caso I. -  $m_1 \neq m_2 \in \mathbb{R}$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$W = \begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0$$

Caso III. -  $m_1 = m_2 \in \mathbb{C}$

$$y = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x} \quad x \in \mathbb{R} \quad y \in \mathbb{R}$$

$$\text{Euler} \quad e^{\pi i} = -1$$

$$y = e^{ax} \left( C_1 e^{bx+i} + C_2 e^{-bx-i} \right)$$

$$e^{wi} = \cos(w) + \operatorname{sen}(w)i$$

$$e^{-wi} = \cos(w) - \operatorname{sen}(w)i$$

$$y = e^{ax} \left( C_1 (\cos(bx) + \operatorname{sen}(bx)i) + C_2 (\cos(bx) - \operatorname{sen}(bx)i) \right)$$

$$y = e^{ax} \left( [C_1 + C_2] \cos(bx) + [C_1 - C_2] \operatorname{sen}(bx) \right)$$

Caso III)

$$y = C_{10} e^{ax} \cos(bx) + C_{20} e^{ax} \operatorname{sen}(bx)$$

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0 \quad EDO(2) LCC H.$$

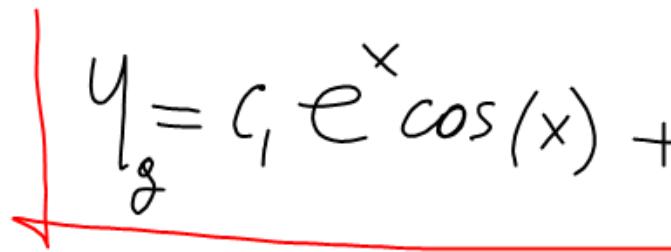
$$m^2 - 2m + 2 = 0$$

$$m_{1,2} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2}$$

$$m_{1,2} = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$m_{1,2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$m_{1,2} = \frac{2 \pm 2i}{2} \Rightarrow 1 \pm i$$

  $y_g = C_1 e^{x \cos(x)} + C_2 e^{x \operatorname{sen}(x)}$

CASO II.-  $m_1 = m_2$

$$y_1 = e^{m_1 x}$$

$$\frac{d}{dm} \left( \begin{array}{l} m^2 + a_1 m + a_2 = 0 \\ m_1 = m_2 \\ 2m + a_1 = 0 \end{array} \right)$$

$$\frac{d}{dm} \left( \begin{array}{l} m^2 + a_1 m + a_2 = 0 \\ m_1 \neq m_2 \\ (m - m_1) + (m - m_2) = 0 \end{array} \right)$$

$$W = \begin{bmatrix} e^{m_1 x} & e^{m_1 x} \\ m_1 e^{m_1 x} & m_1 e^{m_1 x} \end{bmatrix} = 0$$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \quad m_1 = m_2$$

Diagram illustrating the solution of a second-order linear homogeneous differential equation. The equation is given as:

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

The characteristic equation is:

$$m^2 + a_1 m + a_2 = 0$$

Solving for  $m$ , we get  $m_1 = m_2$ .

Two linearly independent solutions are derived from this repeated root:

- $y_1 = e^{m_1 x}$
- $y_2 = x e^{m_1 x}$

A blue circle highlights the term  $\frac{d}{dm}$  in the denominator of the first term of the original equation.

$$\frac{dy}{dx} = \frac{\operatorname{sen}(y)}{x \cos(y) - \operatorname{sen}^2(y)}$$

$$y(0) = \frac{\pi}{2}$$