$y''' - y'' + y' - y = x^2 + x$

EDO(3) Lcc NH.

Método Operador Diferencial.

Hom $\rightarrow y''' - y'' + y' - y = 0$

MOD

$(D^3 - D^2 + D - 1)y = 0$

$(D-1)(D^2+1)y = 0$

$$\boxed{y_{g/H} = C_1 e^x + C_2 \cos(x) + C_3 \operatorname{sen}(x)}$$

$(D-1)(D^2+1)y = x^2 + x$ EDO(3)Lcc NH.

$(D-1)(D^2+1)\underset{(A)}{D^3} y = 0$ EDO(6)Lcc H.

MOD

$y_g = C_1 e^x + C_2 \cos(x) + C_3 \operatorname{sen}(x) + C_4 + C_5 x + C_6 x^2$

$\Rightarrow$

$y_g = C_1 e^x + C_2 \cos(x) + C_3 \operatorname{sen}(x) + A + Bx + Dx^2$

$y_p = A + Bx + Dx^2$

$(D^3 - D^2 + D - 1)y = x^2 + x$

$Dy = B + 2Dx$

$D^2 y = 2D$

$D^3 y = 0$

$(0) - (2D) + (B + 2Dx) - (A + Bx + Dx^2) = x^2 + x$

$(-2D + B - A) + (2D - B)x + (-D)x^2 = x^2 + x$

$$\boxed{\begin{aligned} -A + B - 2D &= 0 \\ -B + 2D &= 1 \\ -D &= 1 \end{aligned}}$$

$\boxed{D = -1}$

$-B = 1 - 2(-1)$

$-B = 3$

$\boxed{B = -3}$

$-A = -(-3) + 2(-1)$

$-A = 1$ $\boxed{A = -1}$

$y_p = -1 - 3x - x^2$

$y_g = C_1 e^x + C_2 \cos(x) + C_3 \operatorname{sen}(x) - 1 - 3x - x^2$

$(D^3 - D^2 + D - 1)y = x^2 + x$

$$(D-2)(D-3)^2 y = 5e^{3x} + 7x^2$$

$$(D-2)(D-3)^2 y = 0$$

$$\boxed{y = C_1 e^{2x} + C_2 x e^{3x} + C_3 e^{3x}}$$

$$(D-2)(D-3)^2 (D-3)_A D^3_A y = 0$$

$$(D-2)(D-3)^3 D^3 y = 0 \quad \text{EDO}(7)\text{ LCCH.}$$

$$y_{g/H_A} = C_1 e^{2x} + C_2 e^{3x} + C_3 x e^{3x} + C_4 x^2 e^{3x} + C_5 + C_6 x + C_7 x^2$$

$$y_{g/NH.} = C_1 e^{2x} + C_2 e^{3x} + C_3 x e^{3x} + A x^2 e^{3x} + B + Dx + Ex^2$$

$$y_{p/A} = A x^2 e^{3x} + B + Dx + Ex^2$$

$$Q(x) = 5e^{3x} + 7x^2$$