> *restart*

> $Ecua := y'' + 5 \cdot y' + 6 \cdot y = \sin(x)^2$

$$Ecua := \frac{d^2}{dx^2}\, y(x) + 5\, \frac{d}{dx}\, y(x) + 6\, y(x) = \sin(x)^2 \tag{1}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) + 5\, \frac{d}{dx}\, y(x) + 6\, y(x) = 0 \tag{2}$$

> $Q := rhs(Ecua)$

$$Q := \sin(x)^2 \tag{3}$$

> $EcuaCarac := m^2 + 5 \cdot m + 6 = 0$

$$EcuaCarac := m^2 + 5\, m + 6 = 0 \tag{4}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -2, -3 \tag{5}$$

> $yy[1] := \exp(Raiz[1] \cdot x);\ yy[2] := \exp(Raiz[2] \cdot x)$

$$yy_1 := e^{-2x}$$

$$yy_2 := e^{-3x} \tag{6}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{-2x} & e^{-3x} \\ -2\,e^{-2x} & -3\,e^{-3x} \end{bmatrix} \tag{7}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \sin(x)^2 \end{bmatrix} \tag{8}$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} \dfrac{\sin(x)^2}{e^{-2x}} & -\dfrac{\sin(x)^2}{e^{-3x}} \end{bmatrix} \tag{9}$$

> $Aprima := ParaVar[1];\ Bprima := ParaVar[2]$

$$Aprima := \frac{\sin(x)^2}{e^{-2x}}$$

$$Bprima := -\frac{\sin(x)^2}{e^{-3x}} \tag{10}$$

> $SolGralHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGralHom := y(x) = \_C1\, e^{-2x} + \_C2\, e^{-3x} \tag{11}$$

> $SolGralNoHom := y(x) = A \cdot yy[1] + B \cdot yy[2]$

$$SolGralNoHom := y(x) = A\, e^{-2x} + B\, e^{-3x} \tag{12}$$

> $A := simplify(int(Aprima, x)) + \_C1$

$$A := -\frac{e^{2x}\left(\cos(x)^2 + \sin(x)\cos(x) - \dfrac{3}{2}\right)}{4} + \_C1 \tag{13}$$

> $B := simplify(int(Bprima, x)) + \_C2$

$$B := -\frac{e^{3x}\left(9\sin(x)^2 - 6\sin(x)\cos(x) + 2\right)}{39} + \_C2 \tag{14}$$

> $simplify(SolGralNoHom)$

$$y(x) = -\frac{3\sin(x)^2}{13} - \frac{5\sin(x)\cos(x)}{52} - \frac{\cos(x)^2}{4} + \frac{101}{312} + \_C2\,e^{-3x} + \_C1\,e^{-2x} \tag{15}$$

> $Ecua$

$$\frac{d^2}{dx^2}\,y(x) + 5\,\frac{d}{dx}\,y(x) + 6\,y(x) = \sin(x)^2 \tag{16}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralNoHom), Ecua)))$

$$Comprobar := \sin(x)^2 = \sin(x)^2 \tag{17}$$

> $restart$

EJEMPLO PRIMER EXAMEN PARCIAL

> $restart$

1)

> $Ecua := x \cdot y' = 2 \cdot y + x^3 \cdot \cos(x)$

$$Ecua := x\left(\frac{d}{dx}\,y(x)\right) = 2\,y(x) + x^3\cos(x) \tag{18}$$

> $CondIni := y(\text{Pi}) = 2$

$$CondIni := y(\pi) = 2 \tag{19}$$

> $EcuaDos := \dfrac{lhs(Ecua)}{x} = expand\left(\dfrac{rhs(Ecua)}{x}\right)$

$$EcuaDos := \frac{d}{dx}\,y(x) = \frac{2\,y(x)}{x} + x^2\cos(x) \tag{20}$$

> $EcuaTres := lhs(EcuaDos) - \dfrac{2\,y(x)}{x} = rhs(EcuaDos) - \dfrac{2\,y(x)}{x}$

$$EcuaTres := \frac{d}{dx}\,y(x) - \frac{2\,y(x)}{x} = x^2\cos(x) \tag{21}$$

> $p := -\dfrac{2}{x}; q := rhs(EcuaTres)$

$$p := -\frac{2}{x}$$

$$q := x^2\cos(x) \tag{22}$$

> $SolGralNoHom := y(x) = \_C1 \cdot \exp(int(-p, x)) + \exp(int(-p, x)) \cdot int(\exp(int(p, x)) \cdot q, x)$

$$SolGralNoHom := y(x) = \_C1\,x^2 + x^2\sin(x) \tag{23}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralNoHom), lhs(EcuaTres)$
> $- rhs(EcuaTres) = 0)))$

$$Comprobar := 0 = 0 \tag{24}$$

> $Para := simplify(isolate(subs(x = \mathrm{Pi}, rhs(SolGralNoHom) = 2), \_C1))$

$$Para := \_C1 = \frac{2}{\pi^2} \tag{25}$$

> $SolPart := subs(\_C1 = rhs(Para), SolGralNoHom)$

$$SolPart := y(x) = \frac{2\,x^2}{\pi^2} + x^2 \sin(x) \tag{26}$$

> $ComprobarDos := simplify(subs(x = \mathrm{Pi}, SolPart))$

$$ComprobarDos := y(\pi) = 2 \tag{27}$$

> $CondIni$

$$y(\pi) = 2 \tag{28}$$

> $restart$

2)

> $Ecua := y' - 3 \cdot y = \dfrac{(1 - \exp(4\,x))}{\exp(x)}$

$$Ecua := \frac{d}{dx}\, y(x) - 3\,y(x) = \frac{1 - e^{4\,x}}{e^x} \tag{29}$$

> $CondIni := y(0) = 0$

$$CondIni := y(0) = 0 \tag{30}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d}{dx}\, y(x) - 3\,y(x) = 0 \tag{31}$$

> $Q := rhs(Ecua)$

$$Q := \frac{1 - e^{4\,x}}{e^x} \tag{32}$$

> $yy[1] := \exp(3 \cdot x)$

$$yy_1 := e^{3\,x} \tag{33}$$

> $Comprobar := eval(subs(y(x) = yy[1], EcuaHom))$

$$Comprobar := 0 = 0 \tag{34}$$

> $SolGralHom := y(x) = \_C1 \cdot yy[1]$

$$SolGralHom := y(x) = \_C1\, e^{3\,x} \tag{35}$$

> $SolGralNoHom := y(x) = simplify(\_C1 \cdot yy[1] + yy[1] \cdot int(\exp(-3\,x) \cdot Q, x))$

$$SolGralNoHom := y(x) = (\_C1 - x)\, e^{3\,x} - \frac{e^{-x}}{4} \tag{36}$$

> $Para := isolate(subs(x = 0, rhs(SolGralNoHom) = 0), \_C1)$

$$Para := \_C1 = \frac{1}{4} \tag{37}$$

> $SolPart := subs(\_C1 = rhs(Para), SolGralNoHom)$

(38)

$$SolPart := y(x) = \left(\frac{1}{4} - x\right) e^{3x} - \frac{e^{-x}}{4} \tag{38}$$

> *Ecua*

$$\frac{d}{dx} y(x) - 3 y(x) = \frac{1 - e^{4x}}{e^x} \tag{39}$$

> *Comprobar := simplify(eval(subs(y(x) = rhs(SolPart), lhs(Ecua) − rhs(Ecua) = 0)))*

$$Comprobar := 0 = 0 \tag{40}$$

> *with(DEtools) :*
> *odeadvisor(Ecua)*

$$[[\_linear, class\ A]] \tag{41}$$

> *FactInt := simplify(intfactor(Ecua))*

$$FactInt := e^{-3x} \tag{42}$$

> *EcuaDos := lhs(Ecua) − rhs(Ecua) = 0*

$$EcuaDos := \frac{d}{dx} y(x) - 3 y(x) - \frac{1 - e^{4x}}{e^x} = 0 \tag{43}$$

> $M := -3y - \dfrac{1 - e^{4x}}{e^x}$

$$M := -3y - \frac{1 - e^{4x}}{e^x} \tag{44}$$

> $N := 1$

$$N := 1 \tag{45}$$

> *diff (M, y) ≠ diff (N, x)*

$$-3 \neq 0 \tag{46}$$

> *MM := M·FactInt*

$$MM := \left(-3y - \frac{1 - e^{4x}}{e^x}\right) e^{-3x} \tag{47}$$

> *NN := N·FactInt*

$$NN := e^{-3x} \tag{48}$$

> *diff (MM, y) = diff (NN, x)*

$$-3 e^{-3x} = -3 e^{-3x} \tag{49}$$

> *IntMMx := int(MM, x)*

$$IntMMx := x + \frac{1}{4 \left(e^x\right)^4} + \frac{y}{\left(e^x\right)^3} \tag{50}$$

> *SolGral := expand(isolate(IntMMx + int((NN − diff (IntMMx, y)), y) = _C1, y))*

$$SolGral := y = \_C1 \left(e^x\right)^3 - \left(e^x\right)^3 x - \frac{1}{4 e^x} \tag{51}$$

> *expand(SolGralNoHom)*

$$\tag{52}$$

$$y(x) = \_C1 \left(e^x\right)^3 - \left(e^x\right)^3 x - \frac{1}{4\,e^x} \tag{52}$$

> *restart*

3)

> $Ecua := y'' + 4 \cdot y = \csc(2\,x)$

$$Ecua := \frac{d^2}{dx^2}\,y(x) + 4\,y(x) = \csc(2\,x) \tag{53}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\,y(x) + 4\,y(x) = 0 \tag{54}$$

> $Q := rhs(Ecua)$

$$Q := \csc(2\,x) \tag{55}$$

> $EcuaCarac := m^2 + 4 = 0$

$$EcuaCarac := m^2 + 4 = 0 \tag{56}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 2\,I,\ -2\,I \tag{57}$$

> $yy[1] := \cos(\text{Im}(Raiz[1]) \cdot x);\ yy[2] := \sin(\text{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \cos(2\,x)$$

$$yy_2 := \sin(2\,x) \tag{58}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \cos(2\,x) & \sin(2\,x) \\ -2\,\sin(2\,x) & 2\,\cos(2\,x) \end{bmatrix} \tag{59}$$

> $BB := array([0\,,\,Q])$

$$BB := \begin{bmatrix} 0 & \csc(2\,x) \end{bmatrix} \tag{60}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\dfrac{1}{2} & \dfrac{\cot(2\,x)}{2} \end{bmatrix} \tag{61}$$

> $Aprima := ParaVar[1];\ Bprima := ParaVar[2]$

$$Aprima := -\frac{1}{2}$$

$$Bprima := \frac{\cot(2\,x)}{2} \tag{62}$$

> $SolGralNoHom := y(x) = (simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2)$
$\cdot yy[2]))$

$$SolGralNoHom := y(x) = \left(-\frac{x}{2} + \_C1\right)\cos(2\,x) + \left(-\frac{\ln(\csc(2\,x)^2)}{8} + \_C2\right)\sin(2\,x) \tag{63}$$

> $SolFinal := y(x) = \_C1 \cdot \cos(2 \cdot x) + \_C2 \cdot \sin(2\,x) - \dfrac{x \cdot \cos(2 \cdot x)}{2} - \dfrac{\log\left(\csc(2 \cdot x)^2\right)}{8} \cdot \sin(2\,x)$

$$SolFinal := y(x) = -\frac{\sin(2\,x)\ln\left(\csc(2\,x)^2\right)}{8} + \sin(2\,x)\,\_C2 + \cos(2\,x)\,\_C1 - \frac{\cos(2\,x)\,x}{2} \qquad \textbf{(64)}$$

> $Ecua$

$$\frac{d^2}{dx^2}\,y(x) + 4\,y(x) = \csc(2\,x) \qquad \textbf{(65)}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralNoHom), lhs(Ecua) - rhs(Ecua) = 0)))$
$$Comprobar := 0 = 0 \qquad \textbf{(66)}$$

> $restart$

4)

> $Ecua := simplify(x \cdot diff(y(x), x\$2) + (x - 1) \cdot diff(y(x), x) + diff(x \cdot diff(y(x), x), x)) = 6 \cdot (x^2 + x \cdot \exp(-x))$

$$Ecua := x\left(2\,\frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x)\right) = 6\,x^2 + 6\,x\,e^{-x} \qquad \textbf{(67)}$$

> $EcuaDos := \dfrac{lhs(Ecua)}{2 \cdot x} = simplify\left(\dfrac{rhs(Ecua)}{2 \cdot x}\right)$

$$EcuaDos := \frac{d^2}{dx^2}\,y(x) + \frac{\frac{d}{dx}\,y(x)}{2} = 3\,e^{-x} + 3\,x \qquad \textbf{(68)}$$

> $EcuaDosHom := lhs(EcuaDos) = 0$

$$EcuaDosHom := \frac{d^2}{dx^2}\,y(x) + \frac{\frac{d}{dx}\,y(x)}{2} = 0 \qquad \textbf{(69)}$$

> $Q := rhs(EcuaDos)$

$$Q := 3\,e^{-x} + 3\,x \qquad \textbf{(70)}$$

> $EcuaCarac := m^2 + \dfrac{m}{2} = 0$

$$EcuaCarac := m^2 + \frac{1}{2}\,m = 0 \qquad \textbf{(71)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 0, -\frac{1}{2} \qquad \textbf{(72)}$$

> $yy[1] := \exp(Raiz[1] \cdot x); yy[2] := \exp(Raiz[2] \cdot x)$
$$yy_1 := 1$$

$$yy_2 := e^{-\frac{x}{2}} \qquad \textbf{(73)}$$

> $with(linalg):$
> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} 1 & e^{-\frac{x}{2}} \\ 0 & -\dfrac{e^{-\frac{x}{2}}}{2} \end{bmatrix} \qquad (74)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & 3\,e^{-x} + 3\,x \end{bmatrix} \qquad (75)$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} 6\,e^{-x} + 6\,x & -\dfrac{6\,(e^{-x}+x)}{e^{-\frac{x}{2}}} \end{bmatrix} \qquad (76)$$

> $Aprima := ParaVar[1]; Bprima := simplify(ParaVar[2])$

$$Aprima := 6\,e^{-x} + 6\,x$$

$$Bprima := -6\,x\,e^{\frac{x}{2}} - 6\,e^{-\frac{x}{2}} \qquad (77)$$

> $SolGralNoHom := y(x) = simplify((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)$
> $\cdot yy[2])$

$$SolGralNoHom := y(x) = -12\,x + 24 + 6\,e^{-x} + e^{-\frac{x}{2}}\_C2 + 3\,x^2 + \_C1 \qquad (78)$$

> $Ecua$

$$x\left(2\,\frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x)\right) = 6\,x^2 + 6\,x\,e^{-x} \qquad (79)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralNoHom), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \qquad (80)$$

> $restart$

8) Serie 2

> $Ecua := y'' + 2\cdot y' + 2\cdot y = \exp(-x)\cdot \sec(x)$

$$Ecua := \frac{d^2}{dx^2}\,y(x) + 2\,\frac{d}{dx}\,y(x) + 2\,y(x) = e^{-x}\sec(x) \qquad (81)$$

RESPUESTA

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2}\,y(x) + 2\,\frac{d}{dx}\,y(x) + 2\,y(x) = 0 \qquad (82)$$

> $Q := rhs(Ecua)$

$$Q := e^{-x}\sec(x) \qquad (83)$$

> $EcuaCarac := m^2 + 2\cdot m + 2 = 0$

$$EcuaCarac := m^2 + 2\,m + 2 = 0 \qquad (84)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -1 + I, -1 - I \qquad (85)$$

Caso III Raíces complejas

> $yy[1] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \cos(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := \mathrm{e}^{-x} \cos(x) \tag{86}$$

> $yy[2] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_2 := \mathrm{e}^{-x} \sin(x) \tag{87}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} \mathrm{e}^{-x} \cos(x) & \mathrm{e}^{-x} \sin(x) \\ -\mathrm{e}^{-x} \cos(x) - \mathrm{e}^{-x} \sin(x) & -\mathrm{e}^{-x} \sin(x) + \mathrm{e}^{-x} \cos(x) \end{bmatrix} \tag{88}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & \mathrm{e}^{-x} \sec(x) \end{bmatrix} \tag{89}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\tan(x) & 1 \end{bmatrix} \tag{90}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\tan(x)$$
$$Bprima := 1 \tag{91}$$

> $SolPart := y(x) = int(Aprima, x) \cdot yy[1] + int(Bprima, x) \cdot yy[2]$

$$SolPart := y(x) = \ln(\cos(x))\, \mathrm{e}^{-x} \cos(x) + x\, \mathrm{e}^{-x} \sin(x) \tag{92}$$

> $SolGralHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGralHom := y(x) = \_C1\, \mathrm{e}^{-x} \cos(x) + \_C2\, \mathrm{e}^{-x} \sin(x) \tag{93}$$

> $SolGralNoHom := y(x) = simplify(rhs(SolGralHom) + rhs(SolPart))$

$$SolGralNoHom := y(x) = (\cos(x) \ln(\cos(x)) + \cos(x)\, \_C1 + \sin(x)\, (\_C2 + x))\, \mathrm{e}^{-x} \tag{94}$$

> $Ecua$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) + 2\, \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) + 2\, y(x) = \mathrm{e}^{-x} \sec(x) \tag{95}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralNoHom), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \tag{96}$$

> $restart$

>

>