TEMA 3. TRANSFORMADA DE LAPLACE.

+ PROBLEMAS DE ECUACIONES DIFERENCIALES CON CONDICIONES INICIALES

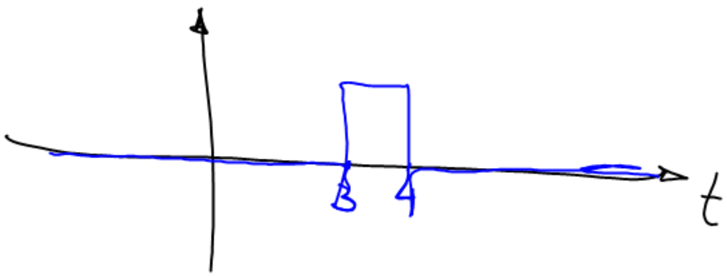
+ SE PUEDEN TENER FUNCIONES SECCIONALMENTE CONTINUAS

ESCALÓN UNITARIO.

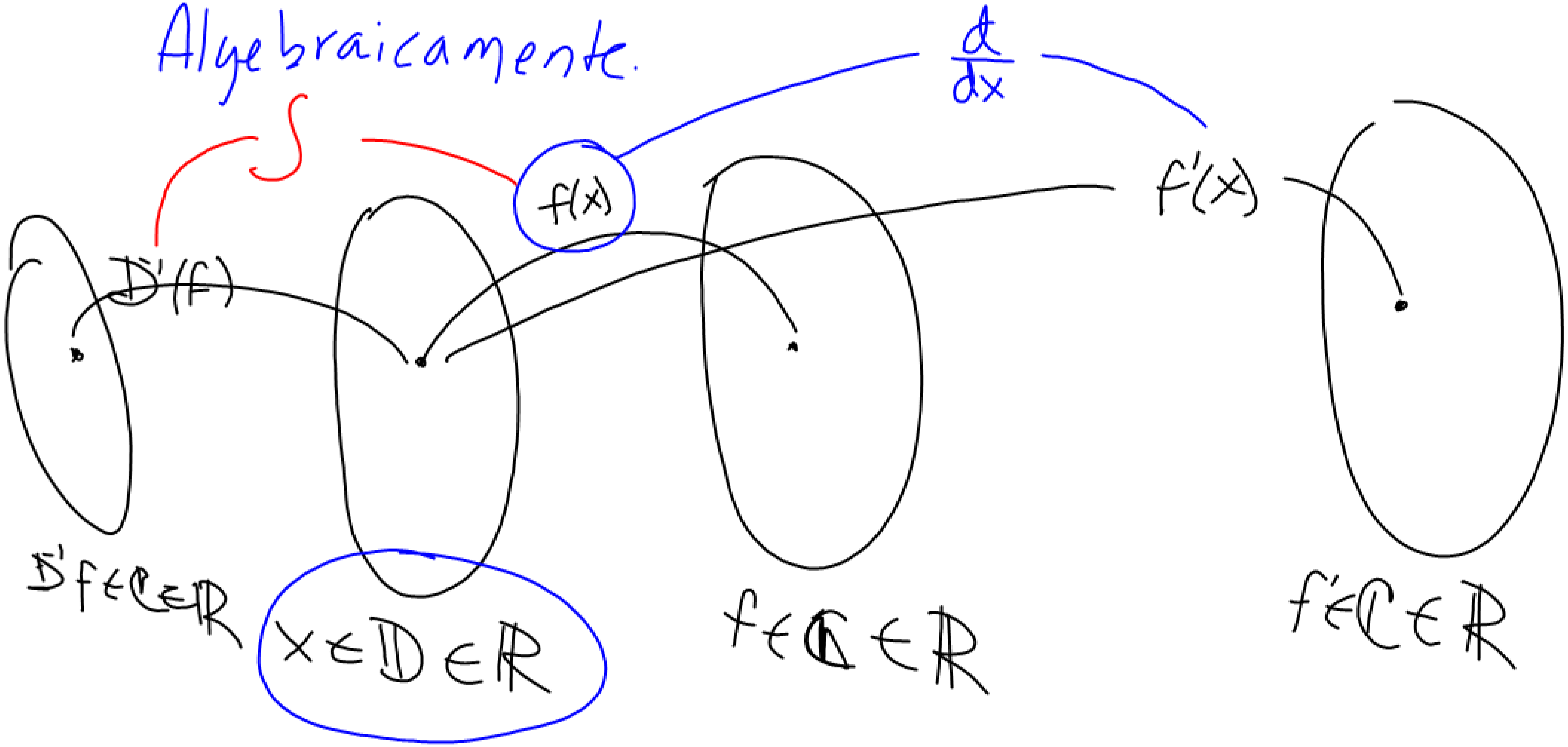
$$u\begin{cases} 0; & t<a \\ 1; & t\geq a \end{cases}$$

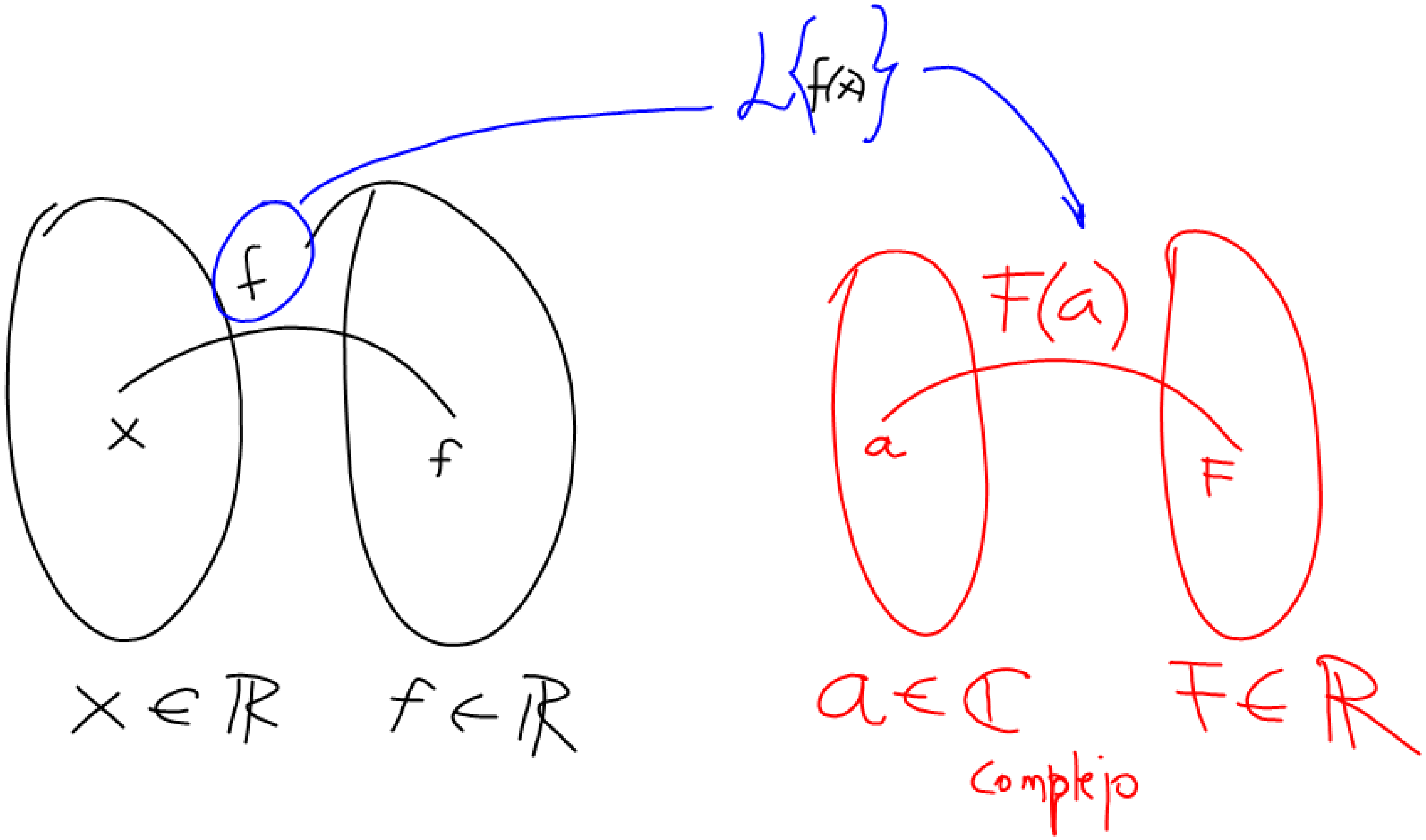
$u(t-a)$

$u(t-3)-u(t-4)=$ pulso



Algebraicamente.





$\mathcal{L}\{f(x)\}$
f
x
f
$F(a)$
a
F
$x \in \mathbb{R}$
$f \in \mathbb{R}$
$a \in \mathbb{C}$
complejo
$F \in \mathbb{R}$

$$T\{f(t)\} = \int_{-\infty}^{\infty} N(t,s)\, f(t)\, dt \Rightarrow F(s)$$

$f \in \mathbb{R}$

$t \in \mathbb{R}$

$F \in \mathbb{R}$

$s \in \mathbb{C}$

$$N(t,s) = \begin{cases} 0 \;; & t \leq 0 \\ e^{-st} \;; & t > 0 \end{cases}$$

Laplace

$$\mathcal{L}\{f(t)\} = F(s)$$

$$= \int_{0}^{\infty} e^{-st} f(t)\, dt$$

$$\mathcal{L}\{f(t)\} = F(s)$$

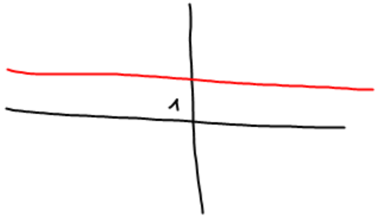
$$af(t) + bg(t) \iff aF(s) + bF(s)$$

$a, b \in \mathbb{R}$

$$\frac{d}{dt}f(t) \iff sF(s) - f(0)$$

$$\int f(t)\,dt \iff \frac{F(s)}{s}$$

$y(t) = 1 \qquad \mathcal{L}\{1\} = \int_0^\infty e^{-st}(1)\,dt$



$$= \left[-\frac{1}{s}\int -s e^{-st}\,dt\right]_0^\infty$$

$$= -\frac{1}{s}\left[e^{-st}\right]_0^\infty$$

$$= -\frac{1}{s}\left(\lim_{a\to\infty} e^{-sa} - 1\right)$$

$$\lim_{a\to\infty} e^{-sa} = \lim_{a\to\infty} \frac{1}{e^{sa}}$$

$$\lim_{a\to\infty} e^{as} \to \infty$$

$$= \lim_{b\to\infty} \frac{1}{b} = 0$$

$$= -\frac{1}{s}\left((0) - 1\right)$$

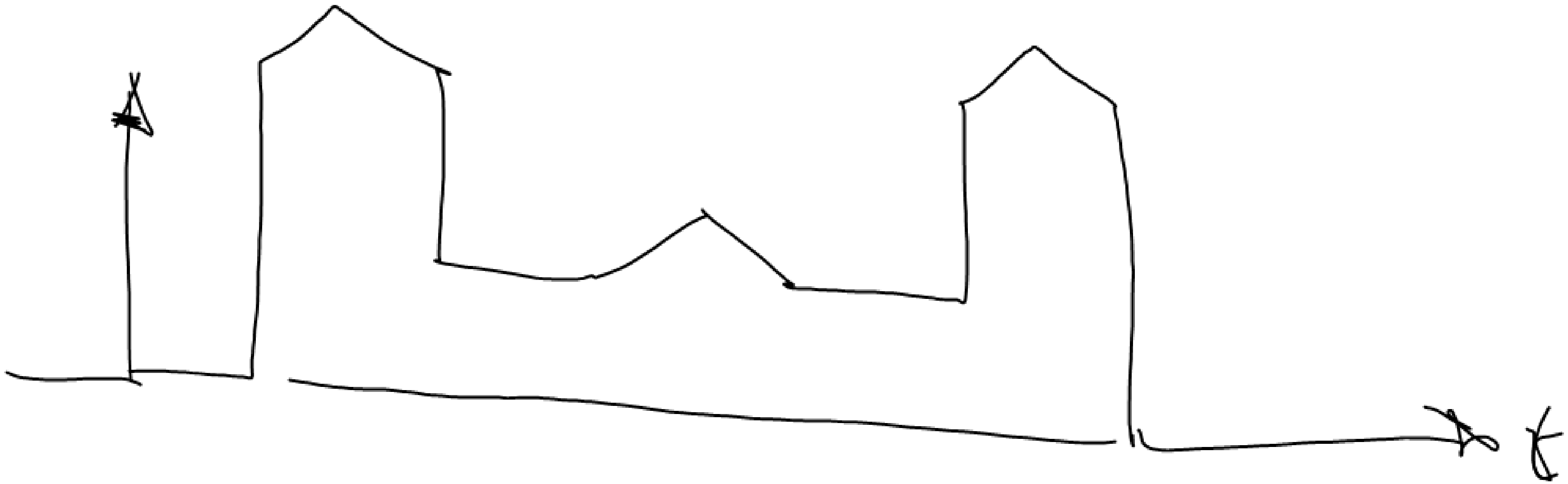
$$= \frac{1}{s}$$

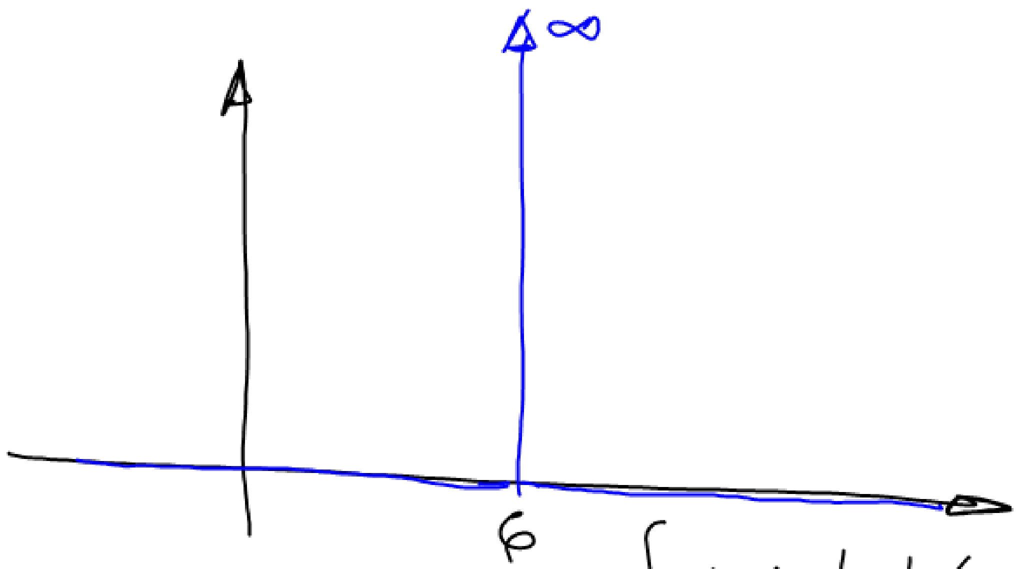
$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2} \qquad \mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{t^2\} = \frac{2!}{s^3} \qquad \mathcal{L}\{\operatorname{sen}(bt)\} = \frac{b}{s^2+b^2}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$





$$\mathrm{Dirac}(6)\begin{cases} 0\,; & t \neq 6 \\ \displaystyle\int_{-\infty}^{\infty} \mathrm{Dirac}(6) = 1 \end{cases}$$

Propiedades de Transf. de Laplace.

① $\mathcal{L}\{af(t)+bg(t)\} = aF(s)+bG(s)$

② $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

$$\mathcal{L}\{e^{t}\} = \frac{1}{s-1}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{a}\left(\frac{1}{\left(\frac{s}{a}\right)-1}\right)$$

$$= \frac{1}{a}\left(\frac{a}{s-a}\right)$$

$$= \frac{1}{s-a}$$

3

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - s\cdot f(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2\cdot f(0) - s\cdot f'(0) - f''(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - \left(\sum s^{i-1} f^{(i)}(0)\right)$$

$$(D-1)(D-2)y(t)=e^{4t} \qquad y(0)=2 \quad y(0)=3$$

$$(D^2-3D+2)y(t)=e^{4t}$$

$$\mathcal{L}\{(D^2-3D+2)y(t)\}=\mathcal{L}\{e^{4t}\}.$$

$$\mathcal{L}\{D^2y(t)\}-3\mathcal{L}\{Dy(t)\}+2\{y(t)\}=\frac{1}{s-4}$$

$$\left[s^2\mathcal{L}\{y(t)\}-s(2)-(3)\right]-3\left[s\mathcal{L}\{y\}-2\right]+2\{y\}=\frac{1}{s-4}$$

$$(s^2-3s+2)\mathcal{L}\{y\}=\frac{1}{s-4}+2s-3$$

$$(s^2-3s+2)\mathcal{L}\{y\}=\frac{1+(2s-3)(s-4)}{(s-4)}$$

$$=\frac{1+2s^2-8s-3s+12}{(s-4)}.$$

$$\mathcal{L}\{y\}=\frac{2s^2-11s+13}{(s-1)(s-2)(s-4)}$$

$$\mathcal{L}\{y\}=\frac{A}{s-1}+\frac{B}{s-2}+\frac{D}{s-4}$$

$$y(t)=Ae^{t}+Be^{2t}+De^{4t}$$