Transformada de Laplace.

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{(-st)} f(t)\,dt \Rightarrow F(s)$$

$f, t \in \mathbb{R}$

$F \in \mathbb{R}$

$s \in \mathbb{C}$ Compleja.

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} F(s)\,ds \Rightarrow f(t)$$

NO ES ÚNICA.

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\operatorname{sen}(bt)$	$\frac{b}{s^2+b^2}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$

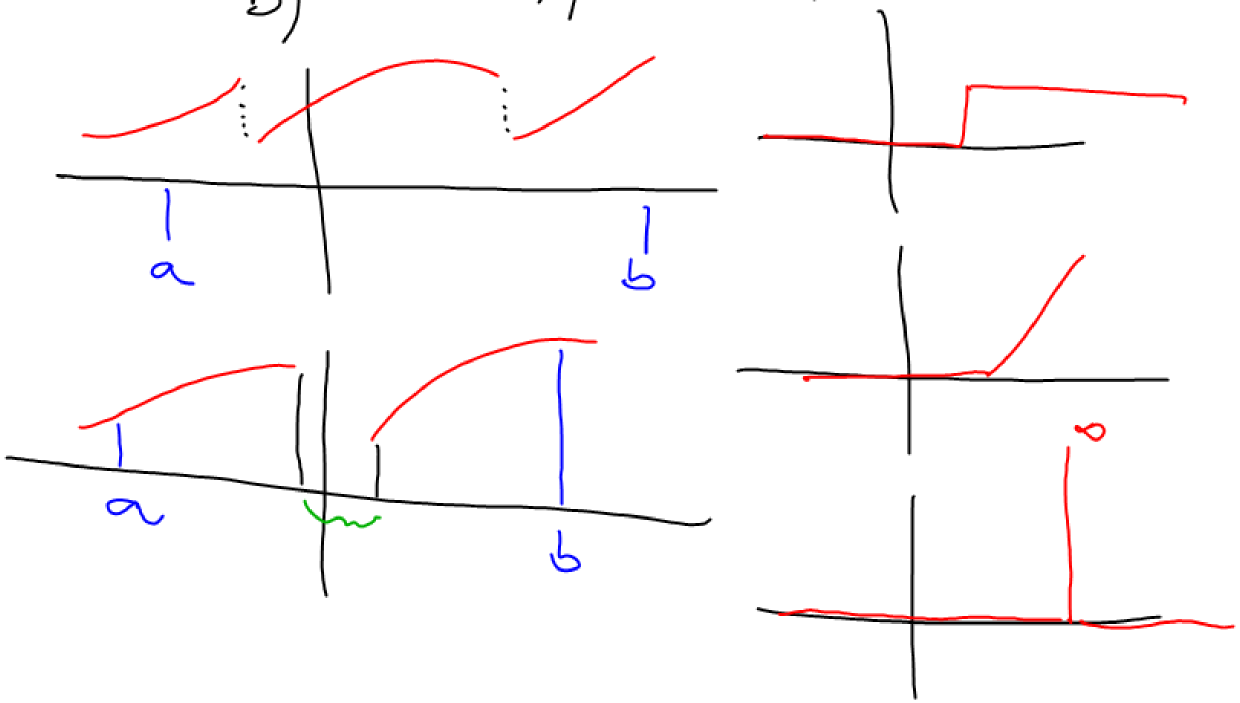
Teorema de Existencia y Unicidad TL.

La f(t) debe ser de clase "A" para que la TL exista y sea única

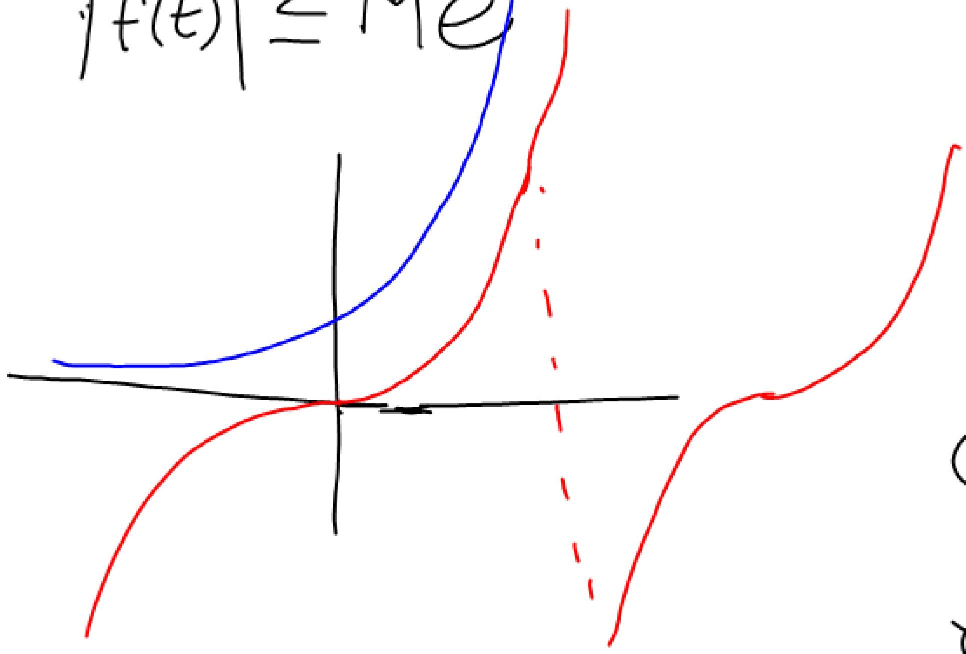
una f(t) es de clase "A"

a) seccionalmente continua

b) orden exponencial.



$$|f(t)| \leq Me^{at}$$



$$e^{t^n} \qquad n>1$$

$$e^{t^2} \qquad e^{t^3}$$

1. $$\mathcal{L}\{af(t)+bg(t)\} = aF(s)+bG(s)$$

2. $$\mathcal{L}\{at\} = \frac{1}{a}F\left(\frac{s}{a}\right)$$

3. $$\mathcal{L}\{f'(t)\} = sF(s)-f(0)$$

4. $$\mathcal{L}^{-1}\{F'(s)\} = -tf(t)$$

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

(5) $$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

(6) $$\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \frac{f(t)}{t}$$

(7) $$\mathcal{L}\{f(t-\tau)\} = e^{-s\tau}F(s)$$

(8) $$\mathcal{L}^{-1}\{F(s-\sigma)\} = u(t-\sigma)\,e^{\sigma t} f(t)$$

(9) $$\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t)$$ operación convolución

$$f(t) * g(t) = \int_0^t f(\tau)\cdot g(t-\tau)\,d\tau.$$

$$\mathcal{L}\{f(t-2)\} = e^{-2s}F(s)$$

$$\mathcal{L}\{e^{(t-a)}\} = e^{-as}\frac{1}{s}$$

$$\mathcal{L}\{e^{3(t-2)}\} = \frac{e^{-2s}}{s-3}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{\cos(2t)\} = \frac{s}{s^2+4}$$

$$\mathcal{L}\{e^{5t}\cos(2t)\} = \frac{(s-5)}{(s-5)^2+4}$$

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$\mathcal{L}\{f(t)\} = F(s)$$

$$\mathcal{L}\{t^2e^{2t}\} = \frac{2}{(s-2)^3}$$

$$\mathcal{L}\{t^2\} = \frac{2!}{s^3}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s+(1))+2-(1)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+1-1}{(s+1)^2+1}\right\}=\mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+1^2}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)+1^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\}=e^{-t}\cos(t)-e^{-t}\operatorname{sen}(t)$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\cdot\frac{1}{s^2+4}\right\}$$

$$\mathcal{L}^{-1}\left\{F(s)\cdot G(s)\right\} = f(t) * g(t).$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\cdot\frac{2}{s^2+4}\right\}$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} * \mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}.$$

$$\frac{1}{2}\cos(2x) * \operatorname{sen}(2x)$$

$$\frac{1}{2}\cos(2x) * \operatorname{sen}(2x) = \frac{1}{2}\int_0^t \cos(2\tau)\operatorname{sen}(2(t-\tau))\,d\tau$$

$$= \frac{1}{2}\int_0^t \cos(2\tau)\cdot\left[\operatorname{sen}(2t)\cdot\cos(2\tau) - \operatorname{sen}(2\tau)\cos(2t)\right]d\tau$$

$$\mathcal{L}\left\{\frac{s}{(s^2+4)^2}\right\} = \frac{1}{2}\operatorname{sen}(2t)\int_0^t \cos(2\tau)\cdot\cos(2\tau)\,d\tau - \frac{1}{2}\cos(2t)\int_0^t \cos(2\tau)\operatorname{sen}(2\tau)\,d\tau.$$

$$\frac{1}{2}\cos(2t) * \operatorname{sen}(2t) = \frac{t\operatorname{sen}(2t)}{4}$$